



Faculty of Engineering

**Type-1 and Interval Type-2 Monotone Takagi-Sugeno-Kang Fuzzy
Inference Systems and Their Practical Application**

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Type-1 and Interval Type-2 Monotone Takagi-Sugeno-Kang Fuzzy Inference Systems and Their Practical Application

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DECLARATION

I declare that the work in this thesis was carried out in accordance with the regulations of Universiti Malaysia Sarawak. Except where due acknowledgements have been made, the work is that of the author alone. The thesis has not been accepted for any degree and is not concurrently submitted in candidature of any other degree.



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P/S: The experiment in Chapter 4 was conducted at Neuon Ai Sdn Bhd with approval. Consent from Neuon Ai Sdn Bhd to report the experimental data in Chapter 4 is attached in Appendix A.

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ABSTRACT

This research focuses on the analysis and application of Type-1 (T1) and Interval Type-2 (IT2) Fuzzy Inference Systems (FIS) to ensure monotonicity, a fundamental concept delineating functions with consistent behavior. Even though mathematical conditions to ensure the monotonicity property of T1 and IT2 FIS are available, it is not clear how to design the fuzzy membership functions (FMFs) at the antecedent part of a fuzzy rule base such that the resulting FIS model is always monotone. This research addresses the uncertainties and inconsistencies in fuzzy rules affecting the monotonicity of FIS outputs. The study reviews sufficient conditions for T1 and IT2 FIS to satisfy monotonicity, analyzing T1 FIS through simulation experiments and exploring its applications in image processing (edge detection) and COVID-19 mitigation strategies. IT2 FIS is similarly reviewed and analyzed. The research investigates a Transmission Cause and Effect Analysis (TCEA) procedure using monotone Interval Type-2 Takagi-Sugeno-Kang (TSK) FIS and tests its applicability in image processing. MATLAB is used for simulation experiments, measuring effectiveness through performance metrics specific to image processing and COVID-19 mitigation. This multidisciplinary approach combines theoretical analysis with practical applications, contributing to the understanding of how fuzzy logic can be leveraged in various domains. The findings are expected to benefit researchers and practitioners addressing complex problems and underline the potential of FIS in nonlinear systems modelling.

Keywords: Fuzzy Inference Systems (FIS), Monotonicity, Type-1 (T1) FIS and Interval Type-2 (IT2) FIS, Edge detection, COVID-19

Sistem Sumberan Kabur Monoton Jenis-1 dan Interval Jenis-2 dengan Aplikasi

ABSTRAK

Kajian ini memfokuskan pada analisis dan aplikasi Sistem Inferens Kabur Jenis-1 (T1) dan Jenis-2 Selang (IT2) untuk memastikan monoton, konsep asas yang menentukan fungsi dengan tingkah laku yang konsisten. Walaupun syarat matematik untuk memastikan sifat monoton T1 dan IT2 FIS tersedia, tidak jelas bagaimana untuk mereka bentuk fungsi keahlian kabur (FMFs) supaya model FIS yang terhasil sentiasa monoton. Kajian ini menangani ketidakpastian dan ketidakkonsistenan dalam peraturan kabur yang mempengaruhi monoton output FIS. Kajian ini mengkaji syarat-syarat untuk T1 dan IT2 FIS untuk mematuhi monoton, menganalisis T1 FIS melalui eksperimen simulasi dan meneroka aplikasinya dalam pemprosesan imej (pengesanan tepi) dan strategi mitigasi COVID-19. IT2 FIS dikaji dan dianalisis dengan cara yang sama. Penyelidikan ini juga menyiasat prosedur Analisis Penyebab dan Kesan Transmisi (TCEA) menggunakan Sistem Inferens Kabur Takagi-Sugeno-Kang (TSK) Jenis-2 Selang dan menguji kebolehgunaannya dalam pemprosesan imej. MATLAB digunakan untuk eksperimen simulasi, mengukur keberkesanan melalui metrik prestasi khusus untuk pemprosesan imej dan mitigasi COVID-19. Pendekatan ini menggabungkan analisis teori dengan aplikasi praktikal, menyumbang kepada pemahaman bagaimana logik kabur dapat dimanfaatkan dalam pelbagai domain. Penemuan kajian ini diharapkan dapat memberi manfaat kepada penyelidik dan pengamal dalam menangani masalah kompleks dan menekankan potensi FIS dalam pemodelan sistem bukan linear.

Kata kunci: *Sistem Sumberan Kabur (FIS), Monotonisiti, Jenis-1 (T1) FIS dan Interval Jenis-2 (IT2) FIS, Pengesanan Tepi, COVID-19*

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LIST OF ABBREVIATIONS

CMFCA	Conventional Mamdani Fuzzy Control Algorithm
Det	Detect
FIS	Fuzzy Inference System
FMEA	Failure Mode and Effects Analysis
FMF	Fuzzy Membership Functions
FOU	Footprint of Uncertainty
FRPN	Fuzzy Risk Priority Number
FS	Fuzzy Set
IT2	Interval Type-2
IT2FS	Interval Type-2 Fuzzy Set
KM	Karnik-Mendel
MFm	Fuzzy Membership Function
MISO	Multi-Input-Single Output
MOI	Mean-of-Inversion
Occ	Occurrence
RPN	Risk Priority Number
Sev	Severity
SISO	Single-Input-Single-Output
T1	Type-1
T1FS	Type-1 Fuzzy Set
T2	Type-2
T2FS	Type-2 Fuzzy Set
TC	Transmission Course

TCEA	Transmission Cause and Effect Analysis
TEs	Transmission Effects
TPs	Transmission Potentials
TSK	Takagi-Sugeno-Kang

CHAPTER 1

INTRODUCTION

1.1 Background

Inference plays an important role in our everyday activities (Evans et al., 1993). Inference in reasoning is defined as the act of forming a conclusion, judgment, or prediction from presented evidence or premises (Fisher, 2001). It is a critical element of deductive reasoning and inductive reasoning, facilitating the formation of judgments or decisions grounded in the provided information (Baron, 2008). Deductive reasoning permits the derivation of conclusions that maintain logical coherence with the given premises and propositions (Johnson-Laird, 2006; Rips, 1990). In contrast to deductive inference, inductive inference entails drawing a generalized conclusion from a set of specific instances or observations (Rips, 1990; Russell and Norvig, 1995). Numerous inferential methodologies, such as Bayesian inference (Box and Tiao, 1973), probabilistic inference (Pearl, 1988), automated logical inference (Harrison, 2009), and fuzzy inference (Jang and Sun, 1996), have been proposed in the scientific literature to model the process of inference.

This investigation in this research prioritizes fuzzy inference for its proficient accommodation of ambiguity and uncertainty inherent in human linguistic labels, employing a Fuzzy Inference System (FIS) to draw conclusions utilizing fuzzy set theory (Zadeh, 1965) and fuzzy logic (Zadeh, 1988). The applications of fuzzy set theory, initiated by Zadeh in 1965, have seen substantial evolution and dispersion across a multitude of domains, including but not limited to pattern recognition (Ajiboye and Weir, 2005), risk assessment (Tay and Lim, 2006), image processing (Lee et al., 2005), and robotics (Chatterjee et al., 2008). The FIS, one of the pioneering and most successful implementations of fuzzy sets

theory (Feng, 2006), equipped with fuzzy if-then rules, exemplifies the unique capability to replicate qualitative aspects of human cognition and reasoning without the necessity for comprehensive quantitative analyses (Jang, 1993). Furthermore, the FIS has asserted its effectiveness as a controlling strategy for various complex nonlinear or nonanalytic systems (Feng, 2006). A typical FIS model incorporates three central components i.e., a fuzzy rule base featuring a collection of fuzzy IF-THEN rules, a database designating the fuzzy membership functions (FMFs) used in the fuzzy IF-THEN rules, and an inference mechanism (Jang and Sun, 1996)

Remarkably, in the realm of fuzzy sets and systems, especially within FIS models, the principle of monotonicity holds considerable significance (Lindskog and Ljung, 2000; Tay and Lim, 2008a, 2008b; Kouikoglou and Phillis, 2009; Van Broekhoven and De Baets, 2009; Seki, Ishii and Mizumoto, 2010; Won and Karray, 2014). Monotonicity, a fundamental concept in mathematics, delineates a function that either consistently ascends or descends as its independent variable intensifies (Apostol, 1974; Ross, 2010). This property, embodied by a monotone function, ensures an uninterrupted direction of progression, which could be either ascending or descending. Notably, for any two components x_1 and x_2 in set X , a monotonically increasing function satisfies $f(x_2) \geq f(x_1)$ whenever $x_1 < x_2$. Conversely, a monotonically decreasing function validates $f(x_2) \leq f(x_1)$ under the same conditions (Stewart, 2015; O'Neill, 2006).

Monotone functions are integral in several mathematical domains, including real analysis (Rudin, 1976), calculus (Adams and Essex, 2013), and optimization theory (Luenberger and Ye, 2008). They play a crucial role in preserving or inverting order, providing key insights for mathematical proofs and algorithmic development in both

mathematics and computer science (Boyd and Vandenberghe, 2004). In real analysis, the Monotone Convergence Theorem, positing the convergence of every bounded, monotone sequence of real numbers, is central (Rudin, 1976). Monotone functions also contribute to calculus by aiding in determining the integrability of functions, understanding limits and continuity, and solving differential equations (Adams and Essex, 2013). Furthermore, monotonicity is a fundamental property in optimization theory, guiding iterative optimization algorithms such as gradient descent towards consistent improvements (Luenberger and Ye, 2008).

The importance of an FIS model that preserves the monotonicity property between its input and output relationship (hereafter denoted as a monotone FIS model) has been widely studied in the literature, with the key reasons that demand the monotonicity property are summaries as follows:

- (i) Many real-world systems and control problems obey the monotonicity property between the input and output variables (Seki et al., 2010; Won and Karray, 2014);
- (ii) The validity of the FIS output needs to be ensured for undertaking comparison, selection, and decision-making problems (Seki et al., 2010; Kouikoglou and Phillis, 2009; Tay and Lim, 2008a, 2008b; Tay and Lim, 2011a; Pang, Tay and Lim, 2016)
- (iii) in the case where the number of data samples is small or the fuzzy rule set is incomplete, it is important to fully exploit the available qualitative information or knowledge related to the system (Van Broekhoven and De Baets, 2009).
- (iv) additional qualitative information or knowledge of the system is useful for overcoming issues related to noise and inconsistency in the data samples, as well as the over-fitting problem (Van Broekhoven and De Baets, 2009).

Various useful mathematical conditions conducive to satisfying the monotonicity property have been established for different FIS models, including T1 Mamdani FIS (Wu and Sung, 1994, 1996; Wu, 1997; Zhao and Zhu, 2000; Van Broekhoven and De Baets, 2008, 2009), T1 Takagi-Sugeno-Kang (TSK) FIS (Won et al., 2002; Won and Karray, 2014; Tay and Lim, 2008a, 2011a), and Interval Type-2 (IT2) FIS (Li et al., 2014). In order to construct a monotone T1 and IT2 FIS model, it necessitates a fuzzy membership function (FMF) that adheres to particular mathematical conditions at the antecedent part, in conjunction with a consistent, complete, and monotone fuzzy rule base at the consequent part (Li et al., 2014).

1.2 Problem Statements

Even though mathematical conditions to ensure the monotonicity property of T1 and IT2 FIS (Won et al., 2002; Won and Karray, 2014; Tay and Lim, 2008a, 2011a; Li et al., 2014) are available, it is not clear how to design the FMFs at the antecedent part of a fuzzy rule base such that the resulting FIS model is always monotone. In other words, when such a design of FMFs at the antecedent part is coupled with the consequent part of a fuzzy rule base, a consistent, complete, and monotone fuzzy rule base (which is denoted as a fuzzy rule base with certain fuzzy rules in this research) can be formed to produce a monotone FIS model.

Nevertheless, achieving such ideal circumstances in practice can be challenging. Fuzzy rules employed for constructing monotone FIS models are prone to uncertainties including inconsistency (Van Broekhoven and De Baets, 2009) and non-monotonicity (Pang et al., 2016). These flawed fuzzy rules, termed as uncertain fuzzy rules in this research, can lead to issues. In particular, when the fuzzy rule base used for constructing the FIS model

fails to produce monotone output relative to the inputs, the validity of the inference outcomes from the resultant FIS model may not always be guaranteed.

1.3 Key Research Questions

The research focuses on establishing reliable FIS that maintain monotonicity, which ensures consistent outputs with varying inputs. The study explores both T1 and IT2 FIS models, aiming to identify the necessary conditions for monotonicity and their practical applications. The key research questions guiding this study are:

- (i) What are the specific parametric conditions that should be met for a T1 TSK FIS to exhibit monotonicity, with a particular focus on functional consequents?
- (ii) How can the parametric conditions ensuring monotonicity in a T1 TSK FIS-based Risk Priority Number (RPN) risk assessment tool be employed in devising a novel systematic approach to combat COVID-19?
- (iii) How can the parametric condition for monotonicity in a T1 TSK FIS be effectively applied to address a problem in image processing?
- (iv) What are the parametric conditions that ensure monotonicity in an IT2 FIS, and how can parametric conditions be utilized to develop systematic approaches for COVID-19 prevention?
- (v) How can monotonic IT2 FIS models be applied to address image processing problem?

1.4 Research Objectives

The main aim of this research is to design and develop monotone FIS models and study their applicability. The specific research objectives (ROs) are:

- (i) RO1: To analyses the parametric condition required for TSK FIS to exhibit monotonicity, with focusing on functional consequents.
- (ii) RO2: To apply the parametric conditions of monotone T1 TSK FIS in a systematic approach for COVID-19 prevention.
- (iii) RO3: To apply the parametric conditions of monotone T1 TSK FIS and solve problems in image processing, focusing on edge detection.
- (iv) RO4: To analysis parametric conditions of monotone IT2 FIS and apply the parametric conditions in a systematic approach for COVID-19 prevention.
- (v) RO5: To apply the parametric conditions of monotone IT2 FIS and solve problem in image processing, focusing on edge detection.

Monotonicity ensures consistent outputs with varying inputs, which is crucial for reliability. This research investigates the parametric conditions for achieving monotonicity and applies these conditions in COVID-19 prevention and image processing. COVID-19 prevention faces challenges in effective risk assessment due to data uncertainty, which monotone FIS models can address by providing reliable outputs for prioritizing interventions. In medical image processing, accurate edge detection is critical but challenging due to noise and image quality variations. Monotone FIS models improve reliability, crucial for effective diagnosis and treatment.

1.5 Research Flow

The ROs are accomplished through step-by-step approach as outlined in Figure 1.1. The process initiates with a comprehensive review of relevant literatures, specifically the sufficient conditions for T1 and IT2 FIS to satisfy the monotonicity property.

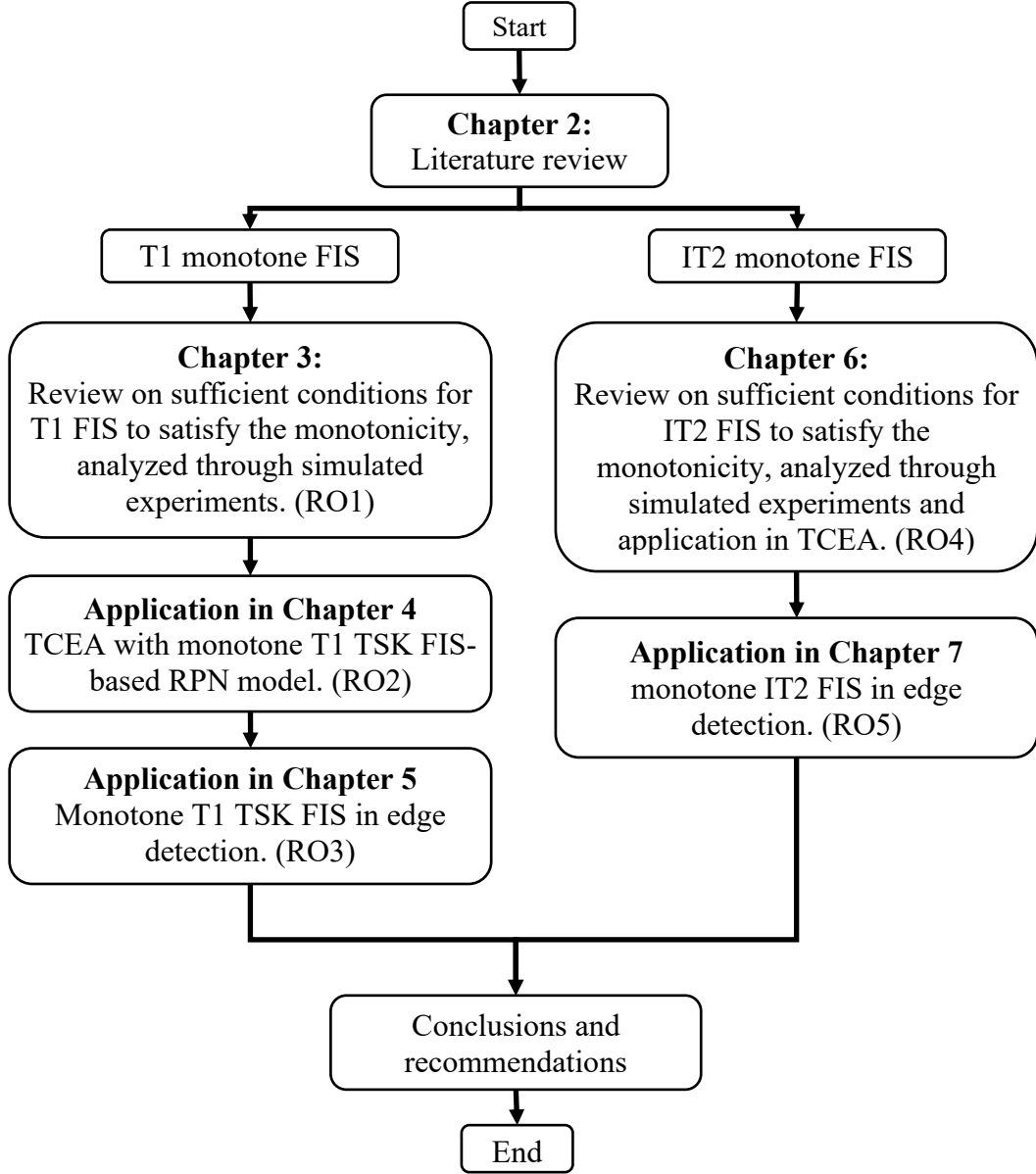


Figure 1.1: Research flow

Regarding the T1 FIS model, the efficacy of the mathematical conditions and analysis laid out in works by Won et al. (2002), Tay and Lim (2011a), and Won and Karray (2014) for the construction of a monotone T1 TSK FIS are reviewed, as detailed in Chapter 3. The utility of these mathematical conditions is analyzed through simulated experiments. Further, applicability of the T1 monotone FIS is probed through two applications. Chapter 4 sees the implementation of a novel strategy i.e., the Transmission Cause and Effect

Analysis (TCEA), which incorporates the monotone T1 TSK FIS for proposing strategies to combat COVID-19. In chapter 5, monotone T1 TSK FIS is utilized in image processing, particularly in edge detection, to convert digitized mammograms into edge images.

In respect to the IT2 FIS model, the mathematical conditions and analysis contributed by Li et al. (2014) towards the establishment of a monotone IT2 FIS are reviewed, as discussed in Chapter 6. The utility of these mathematical conditions is analyzed via simulated experiments. Additionally, an application with TCEA procedure that incorporates the monotone IT2 TSK FIS is demonstrated in Chapter 6. Chapter 7 explores the application of monotone IT2 FIS in the field of image processing, specifically for edge detection, enabling the transformation of digitized mammograms into edge images.

1.6 Research Scopes

The scope of this research involves a comprehensive evaluation of the mathematical conditions necessary to ensure monotonicity in T1 and IT2 FIS models. It reviews the contributions of Won et al. (2002), Tay and Lim (2011a), and Won and Karray (2014) in the context of T1 FIS models, as well as the work of Li et al. (2014) in relation to IT2 FIS models. Following this, the research broadens its scope to explore the practical implementation of these conditions in two distinct applications. The first case study utilizes an innovative approach, TCEA, employing both monotone T1 TSK FIS based RPN model and IT2 monotone FIS based RPN model to propose strategies for managing the COVID-19 pandemic. The second application delves into the application of monotone T1 TSK and IT2 FIS in the field of image processing, particularly for edge detection, enabling the transformation of digitized mammograms into edge images.

1.7 Thesis Outline

The organization of this thesis is summarized, as follows.

Chapter 1 presents the essential information of monotonicity and monotone function, with the focus on monotone T1 and IT2 FIS. The motivations, ROs, scope and methodologies are described.

Chapter 2 introduces the fundamental concepts of fuzzy set theory, including the variety of fuzzy sets and their functional operations. This chapter further discusses the intricacies of FIS models, specifically the paradigms of T1 FIS and IT2 FIS. It also probes into the monotonicity property of FIS models, providing an in-depth exploration of the testing procedures for monotonicity.

Chapter 3 delves into the foundational concepts of monotonicity, monotonic functions, and FIS, with a focus on monotone FIS models. It outlines the sufficient conditions for monotone T1 Single Input Single Output (SISO) and Multiple Input Single Output (MISO) TSK FIS models, based on the simplified theorem from notable works of Won et al. (2002), Tay and Lim (2011a), and Won and Karray (2014), and encapsulated as Theorem 3.1 and Theorem 3.2. The chapter also explores the design of T1 Trapezoidal and Gaussian FMF guided by these theorems. Furthermore, it highlights how these theorems are applied in formulating consequent functions of fuzzy rules for monotone TSK FIS, specifically introducing first and second-order polynomial functions.

Chapter 4 unfolds with the introduction of a novel TCEA procedure, formulated to combat COVID-19 pandemic. The chapter presents the aims, challenges and aspirations linked with this innovative approach. The transmission risk concept is brought to the fore,

incorporating a causal map and a detailed table scaling severity, occurrence, and detection. The intricacies of the TCEA procedure are outlined, and the design elements of the monotone FIS-based RPN model are detailed. The chapter then navigates through a real-world case study, using data gleaned from an IT company. The chapter concludes with an in-depth discussion on the presented topics.

Chapter 5 illuminates the basics and background of object edge detection, including a detailed description of the edge detection framework proposed by Bezdek et al. (1998). The chapter delves into the use of waterfall blending functions as a monotonic function, illustrating how the monotonic T1 TSK FIS is incorporated within the realm of fuzzy blending functions. The chapter culminates with a practical application of the edge detection framework, demonstrating its potential in analyzing digitized mammograms.

Chapter 6 serves as an introduction to the IT2 FIS model, outlining the monotone IT2 FIS and its pertinent fuzzy parameter conditions. The chapter explores parametric conditions for designing trapezoidal and Gaussian IT2 FMFs in the context of the monotone IT2 TSK FIS. Subsequently, Chapter 6 conducts simulated experiments on SISO Monotone IT2 FIS, using both trapezoidal and Gaussian FMFs. Building upon the groundwork of Chapter 4, Chapter 6 applies the Gaussian and trapezoidal IT2 FMF designs to MISO Monotone IT2 FIS models, through practical application.

Chapter 7 provides a comprehensive explanation of the fundamental theory and application context of the monotone IT2 FIS within the realm of edge detection. It thoroughly describes the integration of a proposed fuzzy blending function with a monotone IT2 FIS Model. Two distinct designs of IT2 FMF in relation to the IT2 Fuzzy Blending function, namely the IT2 Trapezoidal FMF and the IT2 Gaussian FMF, are showcased. The

chapter further delves into the practical application of the IT2 Fuzzy Blending function, using a digitized mammogram from a standard patient sourced from the Mammographic Image Analysis Society's (MIAS) database. The mammogram is subsequently transformed into edge images.

Chapter 8 summarizes the main contributions of the research and presents suggestions for future work. It provides a comprehensive review of the key findings and their implications within the field. Additionally, the chapter outlines limitations of the research and highlights the potential areas for further exploration, offering directions for future research initiatives.

CHAPTER 2

LITERATURE REVIEW

2.1 Overview

Chapter 1 provides a comprehensive overview of the research project, including problem statements, objectives, project scope, research methodology, and the organization of the thesis. In this chapter, a review on FIS, which include the background and the related studies, is presented. The chapter is structured as follows.

Section 2.2 provides an overview of the basic concepts in fuzzy set theory, including various fuzzy sets and their operations. Section 2.3 reviews the background of FIS models. Sections 2.4 and 2.5 present the paradigms of Type-1 (T1) FIS and IT2 FIS, respectively. Section 2.6 discusses the monotonicity property of FIS models. The procedure for the monotonicity test is elaborated in Section 2.7. Section 2.8 reviews Monotone FIS and related work in engineering. The chapter concludes with final remarks at Section 2.9.

2.2 Fuzzy Set Theory

Fuzzy set theory, introduced by Zadeh (1965) as an extension of classical set theory, provides a powerful framework for handling uncertainties and vagueness in natural language (Zadeh, 1965; Zadeh, 1975; Klir and Yuan, 2009). Fuzzy set theory employs a membership function that assigns a degree of membership between 0 and 1 to elements within a fuzzy set. Unlike classical sets with distinct boundaries separating members from non-members, fuzzy sets enable a gradual transition from "belonging" to "not belonging" (Jang and Sun, 1996). A fundamental characteristic of a fuzzy set lies in its membership function, which

allows for various degrees of membership for the elements of a given set (Bien and Yu, 1995).

It is worth noting that the concept of fuzzy set, as demonstrated in Zadeh's work (1965), is commonly referred to as T1 fuzzy sets (FS). Building upon this foundation, Type-2 (T2) and IT2 FSs are developed as extensions of T1 FSs, enabling enhanced treatment of uncertainty and flexibility in managing vagueness. In this study, the focus lies on both T1 FS and IT2 FS. To distinguish between T1 FS and T2 FS symbolically, the tilde symbol ($\sim$) is placed above the fuzzy set symbol. For instance, A represents a T1 FS, while $\tilde{A}$ represents a T2 FS

2.2.1 Type-1 Fuzzy Set

This section presents the basic definitions, terminology, and operations of T1 FS. T1 FS are defined by a crisp membership function assigning a single membership grade to each element, with operations such as complement, union, and intersection. These foundational concepts enable handling uncertainties and vagueness in various applications.

(A) Basic Definition and Terminology of T1 FS

Introduced by Zadeh (1965), T1 FS employs a crisp membership function, assigning a single membership grade to each element. A T1 FS A within a space of objects X is considered. A can be expressed as ordered pairs $(x, \mu_A(x))$, where $x \in X$ and $\mu_A(x)$ is the degree of membership, or membership grade, of x in A . The closer the value of $\mu_A(x)$ is to 1, the stronger the membership of x in T1 FS A . If $A(x) = 1$ signifies full membership of x in A , or $A(x) = 0$ signifies non-membership, then the T1 FS A is considered a crisp set. The definition of a T1 FS is presented in Definition 2.1.

Definition 2.1: (Jang and Sun, 1996) A T1 FS $A \subset X$ is defined as a mapping $\mu_A(x): X \rightarrow [0,1]$.

Next, concepts of support and height within the context of a fuzzy set are presented in Definition 2.2 and Definition 2.3, respectively. Definition 2.4 and Definition 2.5 provide the definitions of normal and convex fuzzy sets, respectively.

Definition 2.2: (Klir and Yuan, 1995; Jang and Sun, 1996; Mendel and Wu, 2010) The support of a T1 FS is the set of elements in A with non-zero membership grade, represented as $supp(A)$ in Equation 2.1.

$$supp(A) = \{x \in X | \mu_A(x) > 0\} \quad \text{Equation 2.1}$$

A T1 FS is classified as a fuzzy singleton when its support consists of a single point in X , with $A(x) = 1$.

Definition 2.3: (Jang and Sun, 1996; Mendel and Wu, 2010) The height of a T1 FS A is the largest membership grade of any element in A , represented as $hieght(A)$ as in Equation 2.2

$$hieght(A) = \max(\mu_A(x)) \text{ for } \forall x \in X. \quad \text{Equation 2.2}$$

Definition 2.4: (Jang and Sun, 1996; Mendel and Wu, 2010;) A normal T1 FS is one with $supp_{x \in X} A(x) = 1$

Definition 2.5: (Jang and Sun, 1996; Mendel and Wu, 2010; Klir and Yuan, 1995) A T1 FS is convex if $A(x_1\lambda + (1 - \lambda)x_2) = \min[A(x_1), A(x_2)]$, where $(x_1, x_2) \in X$ and $\lambda \in [0,1]$.

(B) Fuzzy Operation on T1 FS

Zadeh (1965) extended the fundamental operations of complement, intersection, and union from classical set theory to fuzzy set theory as follows. Consider T1 FSs A and B depicted

in Figure 2.1, which are subsets of a universal set X , where $x \in X$. Definition of complement, intersection, and union in fuzzy set theory are presented in Definition 2.6, Definition 2.7 and Definition 2.8, respectively.

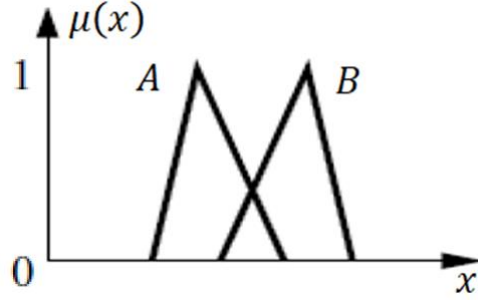


Figure 2.1: Illustration of T1 FS A and B

Definition 2.6: (Jang et al. 1997) The **complement** (also known as negation) of T1 FS A , denoted $\bar{A}$ is defined in Equation 2.3.

$$\mu_{\bar{A}} = 1 - \mu_A$$

Equation 2.3

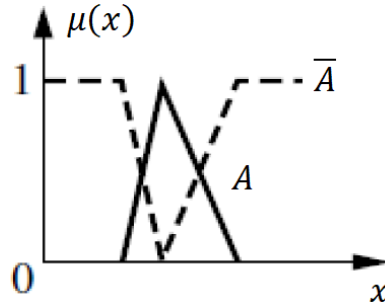


Figure 2.2: Complement of T1 FS A

Definition 2.7: (Jang and Sun, 1996) The **union** (also known as disjunction) of two T1 FSs A and B is another T1 FSs $A \cup B$ represented by Equation 2.4. $\mu_C(x)$ is the membership function of $A \cup B$ relates the membership functions of A and B .

$$\mu_C(x) = \mu_A(x) \vee \mu_B(x) = \max(\mu_A(x), \mu_B(x))$$

Equation 2.4

Figure 2.3 illustrates the union T1 FSs A and B , and the grey region represents the membership function of C .

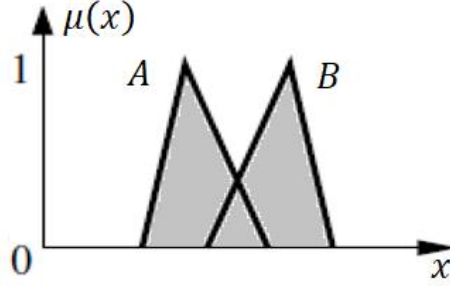


Figure 2.3: Union of T1 FS A and B

Definition 2.8: (Jang and Sun, 1996) The **intersection** (also known as conjunction) of T1 FSs A and B is another T1 FSs $A \cap B$ represented by Equation 2.5. $\mu_C(x)$ is the membership function of $A \cap B$ relates the membership functions of A and B .

$$\mu_C(x) = \mu_A(x) \wedge \mu_B(x) = \min(\mu_A(x), \mu_B(x)) \quad \text{Equation 2.5}$$

Figure 2.4 illustrates the intersection T1 FSs A and B , and the grey region represents the membership function of C .

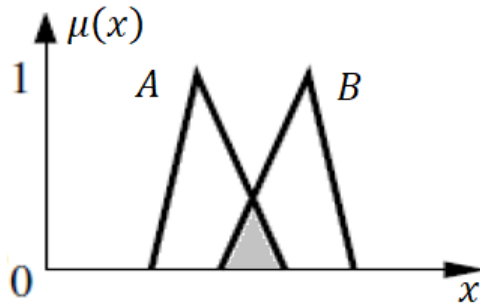


Figure 2.4: Intersection of T1 FS A and B

2.2.2 Type-2 Fuzzy Set

This section introduces the fundamental definitions and terminology related to T2 FS and IT2 FS. Fuzzy Operation on IT2 FS is presented in this section.

(A) Basic Definition and Terminology of T2 FS and IT2 FS

The concepts of T2 FS and IT2 FS were first proposed by Lotfi Zadeh (1975), further developed and popularized by Jerry Mendel and others (Karnik and Mendel, 2001; Mendel 2002; Mendel and John, 2002). T2 FS is introduced to enhance the representation of uncertainty by incorporating a second dimension of uncertainty into the membership function (Mendel, 2002). Instead of a single membership grade associated with each element as in T1 FS, T2 FS introduces a range of possible membership values, resulting in a footprint of uncertainty (FOU). The definition of a T2 FS is presented in Definition 2.9.

Definition 2.9: (Mendel, 2002) $\tilde{A}$ represents the T2 FS, x represents the element in the universe of discourse X , and $\mu_{\tilde{A}}(x, u)$ denotes the membership grade associated with x and the secondary variable u . Mathematically, a Type-2 fuzzy set is defined as follows:

$$\tilde{A} = \{(x, \mu_{\tilde{A}}(x, u)) \mid x \in X, u \in J_x \subseteq [0, 1]\} \quad \text{Equation 2.6}$$

in which $0 < \mu_{\tilde{A}}(x, u) \leq 1$. $\tilde{A}$ is a three-dimensional representation as presented Figure 2.5 T2 FS extends the traditional two-dimensional fuzzy set by including a third dimension i.e., u to represents the membership function values at each point on its two-dimensional domain. The additional dimension is known as the FOU and provides a measure of the uncertainty associated with the membership function of the T2 FS $\tilde{A}$.

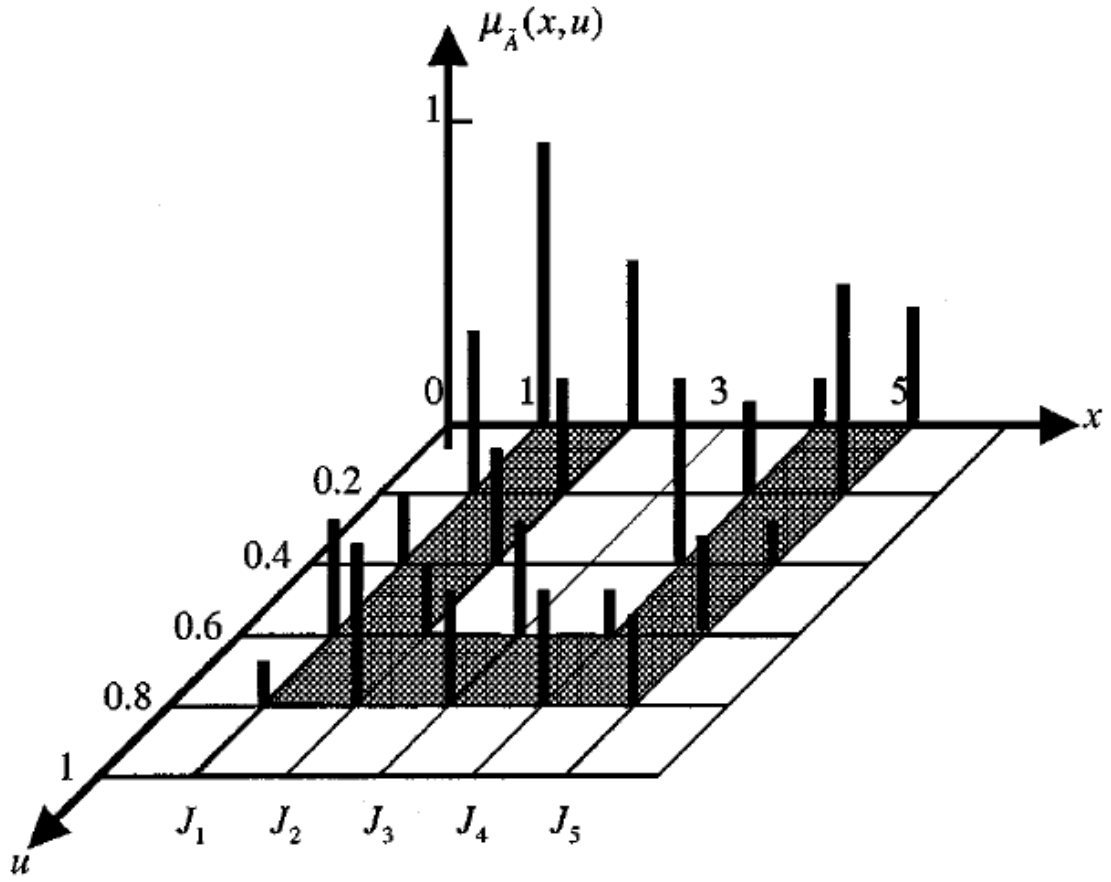


Figure 2.5: Illustration of T2 FS $\tilde{A}$ (Mendel, 2002)

IT2 FS further simplified the representation of T2FS as in Definition 2.10

Definition 2.10: (Mendel, 2002) $\tilde{A}$ represents the IT2 FS. The FOU is approximated by an upper and lower membership functions, defining an interval of possible membership values. Mathematically, an IT2 FS $\tilde{A}$ is defined as in Equation 2.7.

$$\tilde{A} = \{x, [\underline{\mu}_{\tilde{A}}(x), \bar{\mu}_{\tilde{A}}(x)] \mid x \in X\} \quad \text{Equation 2.7}$$

where $[\underline{\mu}_{\tilde{A}}(x), \bar{\mu}_{\tilde{A}}(x)]$ denotes the interval membership values associated with x . An example of IT2 FS is illustrated in Figure 2.6.

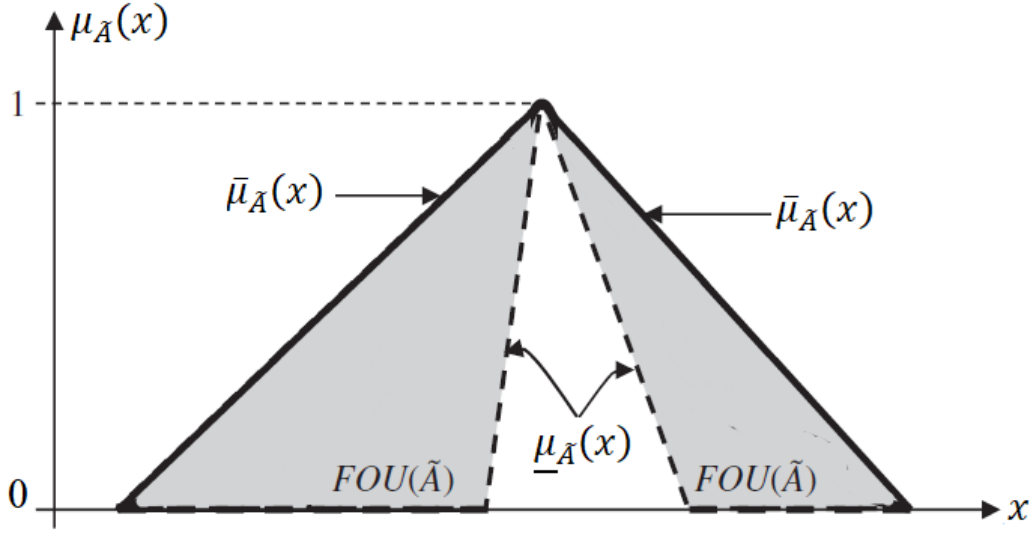


Figure 2.6: Illustration of IT2 FS $\tilde{A}$

The definition of FOU of an IT2 FS (e.g., $\tilde{A}$) is detailed in Definition 2.11

Definition 2.11: (Mendel et al., 2006; Mendel and Wu, 2010) An IT2 FS, $\tilde{A}$, as depicted in Figure 2.6, is described by its FOU, i.e., $FOU(\tilde{A})$. $FOU(\tilde{A})$ is bounded by two T1 membership function $\underline{\mu}_{\tilde{A}}(x)$ and $\bar{\mu}_{\tilde{A}}(x)$. Mathematically, an IT2 FS $\tilde{A}$ is defined as in Equation 2.8

$$FOU(\tilde{A}) = \bigcup_{\forall x \in X} [\underline{\mu}_{\tilde{A}}(x), \bar{\mu}_{\tilde{A}}(x)] \quad \text{Equation 2.8}$$

where $\cup$ denotes set-theoretic union. Alternatively, IT2 FS is described in Equation 2.9

$$\tilde{A} = \frac{1}{FOU(\tilde{A})} \quad \text{Equation 2.9}$$

Next, concepts of support and convex within the context of an IT2 FS are presented in Definition 2.12 and Definition 2.13, respectively.

Definition 2.12: (Mendel and Wu, 2010) The support of $\underline{\mu}_{\tilde{A}}(x)$ and $\bar{\mu}_{\tilde{A}}(x)$ is the crisp set of all points $x \in X$ such that $\underline{\mu}_{\tilde{A}}(x) > 0$ and $\bar{\mu}_{\tilde{A}}(x) > 0$. The support of IT2 FS $\tilde{A}$ is same as the support of $\bar{\mu}_{\tilde{A}}(x)$

Definition 2.13: (Mendel and Wu, 2010) IT2 FS $\tilde{A}$ is convex if both $\underline{\mu}_{\tilde{A}}(x)$ and $\bar{\mu}_{\tilde{A}}(x)$ which are T1 FSs, are convex over their respective supports (see Definition 2.5).

In general, the supports of $\underline{\mu}_{\tilde{A}}(x)$ and $\bar{\mu}_{\tilde{A}}(x)$ are different, and the support of $\underline{\mu}_{\tilde{A}}(x)$ is contained within the support of the $\bar{\mu}_{\tilde{A}}(x)$.

(B) Fuzzy Operation on IT2 FS

This section is dedicated to exploring fuzzy operations specifically on IT2 FS. Consider IT2 FSs $\tilde{A}$ and $\tilde{B}$ depicted in Figure 2.7, which are subsets of a universal set X , where $x \in X$. Definition of complement, intersection, and union in Fuzzy Operation on IT2 FS are presented in Definition 2.14. Definition 2.15 and Definition 2.16, respectively.

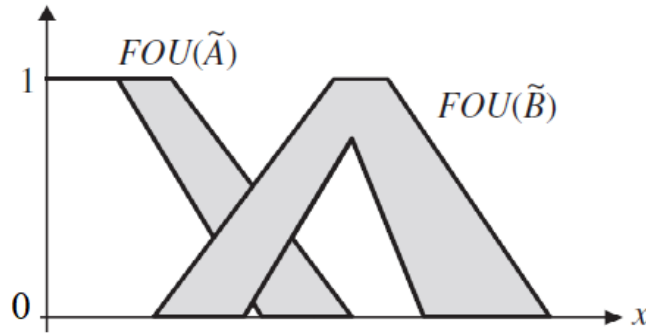


Figure 2.7: Illustration of IT2 FS $\tilde{A}$ and $\tilde{B}$ (Mendel and Wu, 2010)

Definition 2.14: (Mendel and Wu, 2010) The **complement** of IT2 FS $\tilde{A}$ is another IT2 FS denoted as $\tilde{A}^c$ with $FOU(\tilde{A}^c)$ as defined in Equation 2.10.

$$\tilde{A}^c = \frac{1}{FOU(\tilde{A}^c)} = \frac{1}{[1 - \underline{\mu}_{\tilde{A}}(x), 1 - \bar{\mu}_{\tilde{A}}(x)]} \quad \text{Equation 2.10}$$

Definition 2.15: (Mendel and Wu, 2010) The **union** (also known as disjunction) of IT2 FSs $\tilde{A}$ and $\tilde{B}$ is another IT2 FS $\tilde{A} \cup \tilde{B}$ with $FOU(\tilde{A} \cup \tilde{B})$ as represented by Equation 2.11.

$$\tilde{A} \cup \tilde{B} = \frac{1}{FOU(\tilde{A} \cup \tilde{B})} = \frac{1}{[\underline{\mu}_{\tilde{A}}(x) \vee \underline{\mu}_{\tilde{B}}(x), \bar{\mu}_{\tilde{A}}(x) \vee \bar{\mu}_{\tilde{B}}(x)]} \quad \text{Equation 2.11}$$

where $\vee$ denotes the disjunction operator (e.g., maximum). Figure 2.8 illustrates the union IT2 FSs $\tilde{A}$ and $\tilde{B}$.

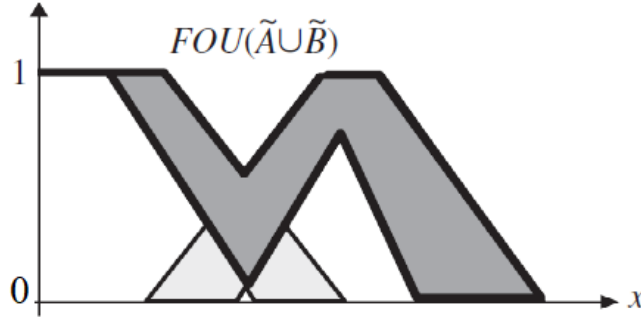


Figure 2.8: Union of IT2 FSs $\tilde{A}$ and $\tilde{B}$

Definition 2.16: (Mendel and Wu, 2010) The **intersection** (also known as conjunction) of IT2 FSs $\tilde{A}$ and $\tilde{B}$ is another IT2 FS $\tilde{A} \cap \tilde{B}$ with $FOU(\tilde{A} \cap \tilde{B})$ as represented by Equation 2.12.

$$\tilde{A} \cap \tilde{B} = \frac{1}{FOU(\tilde{A} \cap \tilde{B})} = \frac{1}{[\underline{\mu}_{\tilde{A}}(x) \wedge \underline{\mu}_{\tilde{B}}(x), \bar{\mu}_{\tilde{A}}(x) \wedge \bar{\mu}_{\tilde{B}}(x)]} \quad \text{Equation 2.12}$$

where $\wedge$ denotes the conjunction operator (e.g., minimum). Figure 2.9 illustrates the union IT2 FSs $\tilde{A}$ and $\tilde{B}$

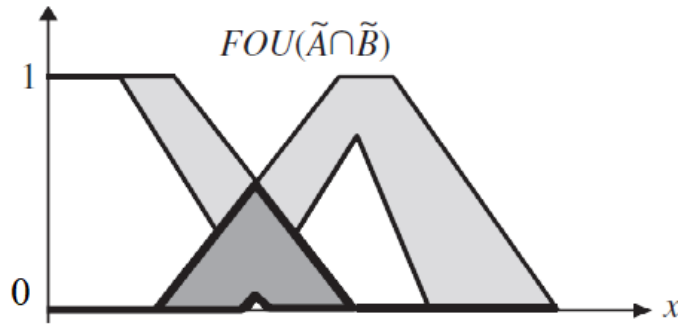


Figure 2.9: Intersection of IT2 FSs $\tilde{A}$ and $\tilde{B}$

2.3 Background on Fuzzy Inference Systems

A FIS, alternatively called a fuzzy rule-based system, fuzzy expert system, fuzzy model, fuzzy associative memory, fuzzy logic controller, or simply as a fuzzy system (Melin and Castillo, 2001), is one of the pivotal applications of fuzzy set theory. A FIS employs fuzzy set theory, fuzzy IF-THEN rules, and fuzzy reasoning for nonlinear mapping, enabling discernible decision-making and pattern recognition (Zadeh, 1965; Mamdani and Assilian, 1975; Takagi and Sugeno, 1985). Expert knowledge is assimilated within an FIS via a comprehensive knowledge base incorporating a database and an array of fuzzy IF-THEN rules, providing nuanced interpretations where traditional binary logic may fall short (Zadeh, 1965). This section focuses on review of integral roles of fuzzy IF-THEN rules and fuzzy reasoning within FIS models and elucidates different types of FIS models.

2.3.1 Fuzzy If-Then Rules

An FIS encodes expert knowledge using fuzzy IF-THEN rules (or known as “fuzzy production rules”, “fuzzy implication”, “fuzzy conditional statement”) (Melin and Castillo, 2001). Each rule is a two-part proposition, an antecedent (input) and a consequent (output), representing imprecise, vague, or uncertain knowledge in natural language expressions (Ross, 2009).

A typical FIS model, i.e., $y = f(\mathbf{x})$ is consider where $\mathbf{x} = (x_1, \dots x_n)$. A fuzzy If-Then rule accommodate set-theoretic operations like AND, OR, and NOT, and can depict an n -input FIS model $y = f(\mathbf{x})$ as follows:

$$IF (x_1 \text{ is } A_1) \text{ AND } (x_2 \text{ is } A_2) \dots \text{AND } (x_n \text{ is } A_n) \text{ THEN } (y \text{ is } B)$$

Here, A_n and B are the linguistic variables/fuzzy sets of input $\mathbf{x}$ and output y domains, respectively, and the antecedent and consequent are “IF $(x_1 \text{ is } A_1) \text{ AND } (x_2 \text{ is } A_2) \dots \text{AND } (x_n \text{ is } A_n)$ ” and “ $y \text{ is } B$ ” correspondingly. The AND operator in the rule antecedent is the product function.

2.3.2 Fuzzy Reasoning

Modus Ponens is a fundamental rule of inference in classical logic (Łukasiewicz, 1957). The rule states that if a certain proposition is true and if that proposition implies a second proposition, then the second proposition must also be true. For instance, given the truth of proposition A and the implication $A \rightarrow B$, the truth of proposition B can be inferred as follows:

Premise 1 (Rule):	If A , then B ($A \rightarrow B$)
Premise 2 (Fact):	A is true
Consequent	Therefore, B must also be true

Fuzzy reasoning, also known as approximate reasoning, is an inference procedure that derives a conclusion from a set of fuzzy rules (Zadeh, 1965). Modus ponens is employed in an approximate manner; therefore, it is also known as generalized modus ponens (GMP) since modus ponens becomes a special case. It can be written as

Premise 1 (rule): *If x is A Then y is B*

Premise 2 (Fact): *x is A'*

Consequent *y is B'*

where A' is close to A , and B' is close to B .

2.4 General Paradigm of T1 FIS

General Paradigm of T1 FIS is introduced in this section. The architecture of a T1 FIS model comprises four main components: a rule base, fuzzifier, inference engine, and defuzzifier (Klir and Yuan, 2009; Mendel, 2001; Ross, 2009; Feng, 2006). This structure is represented in the block diagram as shown in Figure 2.10.

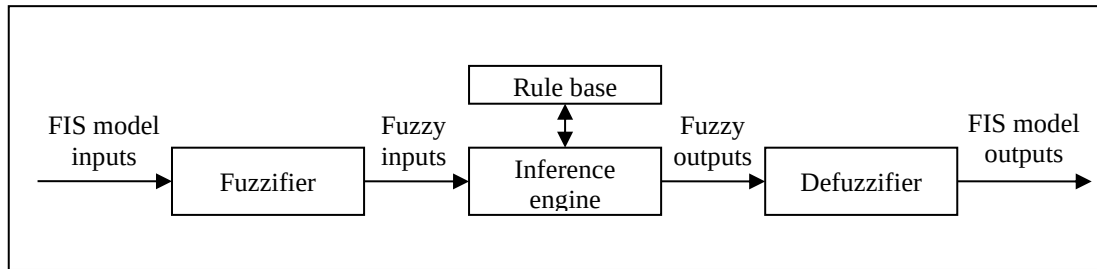


Figure 2.10: A general T1 FIS (Melin and Castillo, 2001)

As in Figure 2.10, the T1 FIS model begins with the process at fuzzifier, where crisp inputs is mapped to the corresponding membership functions of T1 FSs defined in the rule base. The rule base incorporates fuzzy IF-THEN rules and database. The database outlines the membership functions of the T1 FSs utilized within the fuzzy IF-THEN rules. For The inference engine, receiving these fuzzy inputs, generates fuzzy outputs, each corresponding to the result of a specific fuzzy IF-THEN rule. Finally, the defuzzification interface converts the fuzzy outputs into a single crisp output.

The T1 Mamdani FIS (Mamdani and Assilian, 1975; Mamdani, 1977) and T1 Takagi-Sugeno-Kang (TSK) FIS models (Takagi and Sugeno, 1985; Sugeno and Kang, 1988) are two popular types of T1 FIS model. Both FIS models are characterized by fuzzy IF-THEN rules with identical antecedent structures. The primary distinction between T1 Mamdani and T1 TSK FIS models resides in the structure of their rule consequents, with Mamdani leveraging a fuzzy set and T1 TSK FIS using a real-valued function. This variation leads to distinct aggregation and defuzzification methods. T1 Mamdani FIS uses defuzzification to obtain a clear output, as explained in Section 2.4.1. On the other hand, T1 TSK employs a weighted average for faster processing, as outlined in Section 2.4.2.

2.4.1 Mamdani FIS

Introduced in 1975, the T1 Mamdani FIS model was the inaugural FIS designed by Mamdani and Assilian for controlling a steam engine and boiler system. In their experiment, the FIS model utilized a set of linguistic control rules sourced from experienced human operators. Two fuzzy rules as summarized in Figure 2.11 are used for demonstrated the aggregation and defuzzification methods of a T1 Mamdani FIS model, denoted by $y = f(\mathbf{x})$, where $\mathbf{x} = (x_1, x_2) \in X_1 \times X_2$. A_1^n and A_2^n are the T1 FSs of the n -th fuzzy rule, where $n = 1, 2$. C^n is fuzzy consequent of the n -th fuzzy rule. The fuzzy IF-THEN rules of the T1 Mamdani FIS model is described as follows:

$$\text{IF } (x_1 \text{ is } A_1^n) \text{ AND } (x_2 \text{ is } A_2^n) \text{ THEN } (y \text{ is } C^n)$$

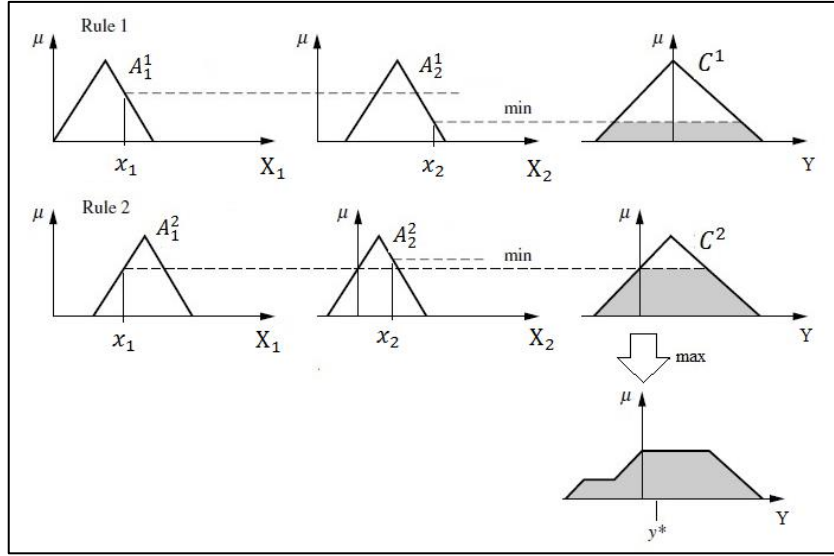


Figure 2.11: Fuzzy reasoning procedure of T1 Mamdani FIS model operation (Ross, 2009)

As in Figure 2.6, the fuzzification process converts crisp values, specifically x_1 and x_2 , into fuzzy inputs. These fuzzy inputs are then employed in the inference process, utilizing min and max operators. In the case that a crisp value is needed as the output, the defuzzification process is performed to convert the aggregated output fuzzy set into a crisp value. Various approach of defuzzifiers has been suggested in the literature of Mendel (1995), including the Maximum, Mean of Maxima, Centroid, Height, and Modified Height Defuzzifiers, have been suggested.

2.4.2 Takagi and Sugeno

The T1 TSK FIS model is a highly regarded alternative to the Mamdani-type FIS model, characterized by its employment of linear functions in the consequence part of the fuzzy rules (Takagi and Sugeno, 1985; Sugeno and Kang, 1988). The T1 TSK FIS offers significant computational efficiency compared to the T1 Mamdani FIS, largely because it eliminates the need for a defuzzification step by employing weighted averages to compute the final output (Sugeno and Kang, 1988). The T1 TSK FIS is well-regarded for its ability to model complex systems with interpretable relationships between variables, offering

transparency and understandability in many practical applications. However, the model's accuracy hinges on the precise tuning of parameters, a potentially complex task with high-dimensional data (Jang, 1993).

Two fuzzy rules as summarized in Figure 2.12 are used for demonstrating the aggregation and defuzzification methods of a T1 TSK FIS model, denoted by $y = f(\mathbf{x})$, where $\mathbf{x} = (x_1, x_2) \in X_1 \times X_2$. A_1^n and A_2^n are the T1 FSs of the n -th fuzzy rule, where $n = 1, 2$. y^n is the consequent of the n -th fuzzy rule. The fuzzy IF-THEN rules of the T1 TSK FIS model is described as follows:

$$\text{IF } (x_1 \text{ is } A_1^n) \text{ AND } (x_2 \text{ is } A_2^n) \text{ THEN } (y^n \text{ is } f^n(x_1, x_2)) \quad n = 1, 2$$

In the consequent of the fuzzy rule, denoted by $y^n = f_n(x_1, x_2)$, is a polynomial function of the input variables x_1 and x_2 . In a first-order Sugeno FIS model, $f_n(x_1, x_2)$ represents a first-order polynomial function (i.e., $f^n(x_1, x_2) = c_0^n + c_1^n x_1 + c_2^n x_2$), where c_0^n, c_1^n and c_2^n are constants (Jang and Sun, 1996). In a zero-order Sugeno model, $f^n(x_1, x_2)$ is a zero-order polynomial, or a constant ($f^n(x_1, x_2) = c^n$). Note that c^n can also be interpreted as a fuzzy singleton (Sugeno, 1999; Ross, 2009).

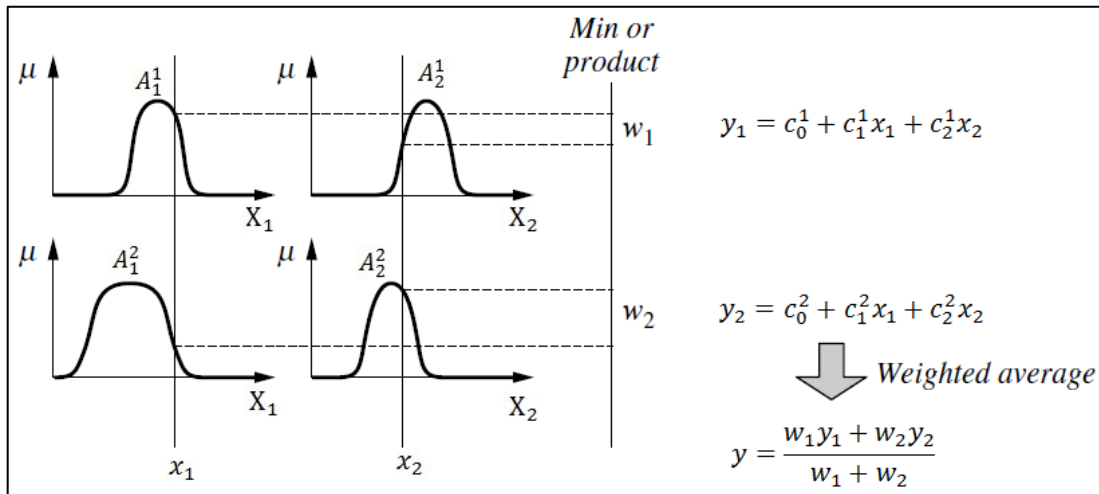


Figure 2.12: An illustration of Sugeno FIS model operation (Jang and Sun, 1996)

In Figure 2.6, illustrated first-order T1 TSK FIS model operation. Fuzzification process converts crisp values, specifically x_1 and x_2 , into fuzzy inputs. These fuzzy inputs are then processed through the inference mechanism, calculating w_1 and w_2 for each rule using the minimum operator. The result of $y^n = f_n(x_1, x_2)$ is computed for each rule. Given that each rule generates a crisp output via a function, the final output is determined by a weighted average.

2.5 General Paradigm of IT2 FIS Model

The operation of an IT2 FIS is depicted in Figure 2.13. During the process, each input is fuzzified into an IT2 FS, which then activates a certain set of rules. The output for each rule is determined using an extended sup-star composition as proposed by Mendel (2001). This process begins with the calculation of a firing interval and then identification of the fired-rule output FOU. Subsequently, these output FOUs from all fired rules are amalgamated. This amalgamation is followed by a reduction from IT2 FSs to a singular number through two steps, i.e., type reduction and defuzzification. Type reduction project step projects an IT2 FS into an interval-valued set. Defuzzification calculates the average of the endpoints of the interval, hence producing a crisp value as output.

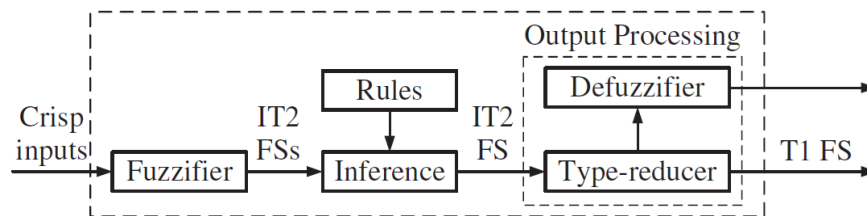


Figure 2.13: An IT2 FIS model (Mendel, 2001)

A detail inference process of IT2 FIS is presented at Figure 2.14. Left-hand part of Figure 2.14 details the computation of the firing interval in an Interval Type-2 Fuzzy

Inference System (IT2 FIS) with singleton inputs. In this example, two antecedents are involved, each represented by an IT2 FS, specifically $\tilde{F}_1$ and $\tilde{F}_2$. For input $x_1 = x'_1$, the $FOU(\tilde{F}_1)$ intersects at every point within the interval $[\underline{\mu}_{\tilde{F}_1}(x'_1), \bar{\mu}_{\tilde{F}_1}(x'_1)]$. Similarly, for input $x_2 = x'_2$, the $FOU(\tilde{F}_2)$ intersects at every point within the interval $[\underline{\mu}_{\tilde{F}_2}(x'_2), \bar{\mu}_{\tilde{F}_2}(x'_2)]$. The firing interval, denoted as $F(\mathbf{x}') = [\underline{f}(\mathbf{x}'), \bar{f}(\mathbf{x}')]$, is then calculated, where $\underline{f}(\mathbf{x}') = \min[\underline{\mu}_{\tilde{F}_1}(x'_1), \underline{\mu}_{\tilde{F}_2}(x'_2)]$ and $\bar{f}(\mathbf{x}') = \min[\bar{\mu}_{\tilde{F}_1}(x'_1), \bar{\mu}_{\tilde{F}_2}(x'_2)]$.

At left-hand part of Figure 2.14 illustrates the process of generating the fired-rule output FOU through the extended sup-star composition as proposed by Mendel (2001). The inference process uses the minimum t-norm operator, which truncates the consequent represented by $FOU(\tilde{G})$ to produce the fired-rule output $FOU(\tilde{B})$. Here, $FOU(\tilde{B})$ results from the t-norm operation between $\underline{f}(\mathbf{x}')$ and $\underline{G}$, and between $\bar{f}(\mathbf{x}')$ and $\bar{G}$. Observe that $\underline{B}$ and $\bar{B}$ are clipped versions of $\underline{G}$ and $\bar{G}$

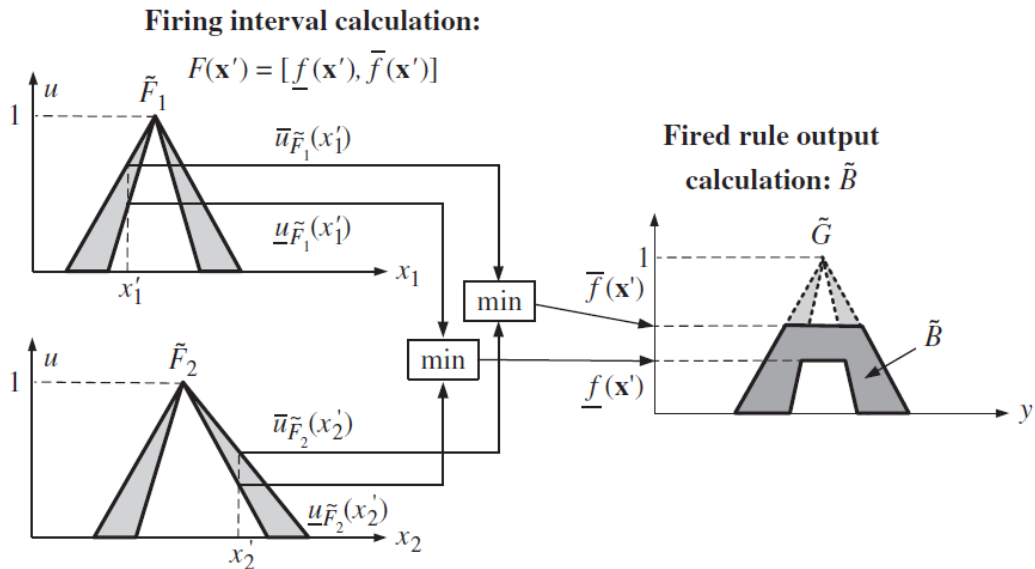


Figure 2.14: IT2 FIS inference process (Mendel, 2007)

2.6 Summary of Review on FIS Model

As discussed in Sections 2.4 and 2.5, the evolution of FIS began with the T1 Mamdani FIS in 1975, which used linguistic control rules and required defuzzification. This was followed by the T1 TSK FIS in 1985, which improved computational efficiency by eliminating the defuzzification step. Further advancements included the first-order TSK FIS in 1996 and various defuzzification methods proposed in 2001. The Interval Type-2 FIS (IT2 FIS) introduced in 2007 marked a significant leap by effectively handling uncertainty through IT2 fuzzy sets, enhancing the applicability of FIS models in complex systems.

Table 2.1 highlights the key milestones and main findings in the development of different FIS models, reflecting the continuous effort to improve computational efficiency, model accuracy, and practical applicability in complex systems.

Table 2.1: Chronological Development of FIS Models

Author (Year)	FIS Type	Main Finding
Mamdani and Assilian (1975)	T1 Mamdani FIS	Introduced the first FIS for controlling a steam engine and boiler system using linguistic control rules from human operators. It requires a defuzzification step to convert fuzzy outputs into crisp values.
Mamdani (1977)	T1 Mamdani FIS	Enhanced the initial model, focusing on defuzzification methods for obtaining a clear output from fuzzy sets.
Takagi and Sugeno (1985)	T1 TSK FIS	Developed an alternative FIS model that uses linear functions in the consequence part of fuzzy rules, offering significant computational efficiency by eliminating the need for defuzzification.
Sugeno and Kang (1988)	T1 TSK FIS	Further improved the T1 TSK model, highlighting its capability to model complex systems with interpretable relationships and computational efficiency.
Jang (1993)	T1 TSK FIS	Demonstrated the accuracy of the T1 TSK FIS model, emphasizing the need for precise tuning of parameters, especially with high-dimensional data.
Jang and Sun (1996)	First-order TSK FIS	Illustrated the first-order TSK FIS model operation, showing the use of first-order polynomial functions in the fuzzy rules.
Sugeno (1999)	Zero-order TSK FIS	Proposed the zero-order Sugeno model, using constants or fuzzy singletons in the consequent part of the fuzzy rules.

Mendel (2001)	T1 Mamdani FIS	Suggested various approaches to defuzzifiers, including Maximum, Mean of Maxima, Centroid, Height, and Modified Height methods.
Mendel (2007)	IT2 FIS	Introduced the IT2 FIS model, which extends the capabilities of traditional FIS by handling uncertainty more effectively through IT2 fuzzy sets.

2.7 Monotonicity Property of FIS Model

This section provides a comprehensive definition of monotone Fuzzy Inference Systems, followed by an exploration of the latest studies concerning the monotonicity property of FIS.

2.7.1 Monotone Fuzzy Inference Systems

Monotone FIS satisfies the monotonicity property between the input and output spaces. Monotone FIS is defined as follows:

Definition 2.17: (Tay and Lim, 2008a) (Monotone FIS) Consider an FIS, $y = f(x_1, \dots, x_i, \dots, x_n)$, that fulfils the condition of monotonicity between its output, y , with respect to its i^{th} input, x_i within the universe of discourse. Consider $x_i^1 < x_i^2 \in x_i$, the model output is either monotonically increases or decreases as x_i increases, as shown in Equation 2.13 and Equation 2.14, respectively. For this research, the focus was predominantly on the case where the function's output is monotonically increasing with its input variables.

$$f(x_1, \dots, x_i^1, \dots, x_n) \leq f(x_1, \dots, x_i^2, \dots, x_n) \quad \text{Equation 2.13}$$

$$f(x_1, \dots, x_i^1, \dots, x_n) \geq f(x_1, \dots, x_i^2, \dots, x_n) \quad \text{Equation 2.14}$$

Investigating the mathematical conditions for an FIS model to ensure monotonicity is a crucial area in the study of monotonicity properties in FIS. This research has examined several FIS models, including T1 Mamdani FIS, T1 TSK FIS, T2 FIS, and IT2 FIS,

identifying the necessary conditions to preserve monotonicity. Relevant and significant findings are reviewed and discussed in the following sections.

2.7.2 Monotonicity Property of T1 Mamdani FIS and Related Works

(A) Wu and Sung (1994, 1996) and Wu (1997)

Early research on monotonicity in T1 Mamdani FIS, conducted by Wu and Sung (1994, 1996) and Wu (1997), highlighted its importance for stability analysis in FIS-based control issues, as well as its relevance for JPEG models in image compression. Research in Wu and Sung (1994, 1996) and Wu (1997) proposed a new defuzzification operator, the mean-of-inversion (MOI), to maintain the monotonicity of T1 Mamdani FIS models. Notably, the MOI operator independently defuzzifies each activated rule, instead of aggregating them prior to defuzzification.

(B) Zhao and Zhu (2000)

Zhao and Zhu (2000) examined process control problems including single-input single-output and multi-input multi-output configurations of T1 Mamdani FIS. They asserted that the output can be expressed as a monotone function of the input variables, fulfilling the monotonicity property which is crucial for stability and steady state error in FIS-based control. Their research derived sufficient conditions for single-input single-output and two-input single-output conventional Mamdani fuzzy control algorithm (CMFCA) to meet the monotonicity property, concluding that if the rule base is monotonically ordered, the CMFCA systems maintain (rough) monotonicity.

(C) Van Broekhoven and De Baets (2008, 2009)

Van Broekhoven and De Baets (2008, 2009) emphasized the value of the monotonicity property as qualitative information, especially with a small sample size. They examined the input-output monotonicity of T1 Mamdani FIS with trapezoidal membership functions and mean of maximum defuzzification. They asserted that a monotone fuzzy rule base alone doesn't guarantee the monotonicity of a T1 Mamdani FIS, but a smooth, monotone fuzzy rule base is necessary. Guidelines were provided for designing monotonicity-preserving FIS models with different mathematical operators, stressing the significance of operator selection and output membership function properties.

2.7.3 Monotonicity property of T1 TSK FIS and related works

(A) Won et al. (2002) and Won and Karray (2014)

Research by Won et al. (2002) and Won and Karray (2014) sheds light on the monotonicity property of T1 TSK FIS models. These studies established necessary conditions for a TSK FIS model by examining the output of an FIS model concerning its input(s). This suggests that to preserve the monotonicity of a T1 TSK FIS model, both antecedent and consequent parts of the fuzzy rules must adhere to specific constraints. These necessary conditions were subsequently integrated with evolutionary computation-based learning (Won et al., 2001) and least-square learning (Koo et al., 2004). Further, in Won and Karray (2014), it was found that the proposed parametric conditions are both necessary and sufficient for creating a monotone single-input zero-order T1 TSK FIS with two rules and suffice for constructing monotone multi-input first- or higher-order TSK FIS models.

(B) Tay and Lim (2008a, 2011a)

Following the approach used by Won et al. (2002), Tay and Lim (2008a, 2011a) employed a similar method to address the monotonicity property in T1 TSK FIS with Gaussian membership functions. In their research, Tay and Lim (2011a) scrutinized the sufficient conditions of T1 TSK FIS and found that adhering to two mathematical conditions at the antecedent and consequent parts of the fuzzy rules was crucial for constructing a FIS model that preserves monotonicity. Additionally, they explained the sufficient conditions for T1 TSK FIS with Gaussian membership functions, using the projection of these functions onto a linear function to demonstrate the input variable range within which the fuzzy model remains monotone.

2.7.4 Monotonicity Property of IT2 FIS and Related Works

In their work, Li et al. (2009, 2014) emphasized the significance of the monotonicity property in system modelling and control problems, underscoring its utility as a form of qualitative or prior knowledge for FIS modelling. They investigated the sufficient conditions required for a multi-input IT2 FIS model under the monotonicity constraint. Li et al. (2014) evaluated five type-reduction and defuzzification methods, including the Karnik-Mendel method, Du-Ying method, Begian-Melek-Mendel method, Wu-Tan method, and Nie-Tan method. The conditions they established hold true for triangular, trapezoidal, and Gaussian IT2 FS fuzzy sets and can also be applied to T1 TSK FIS models.

2.8 Review of Monotone FIS and Related Work in Engineering

This section explores the application of monotone FIS across various engineering disciplines. It highlights key contributions and implementations in reliability engineering

and risk assessment, electrical and electronic engineering, biomedical engineering, mechanical and control engineering, and civil engineering.

2.8.1 Electrical and Electronic Engineering

Milanés et al. (2010) developed a fuzzy controller for urban intersections that utilizes vehicle-to-vehicle (V2V) communications. This system ensures monotonic speed adjustments, helping to manage vehicle flow efficiently and reduce traffic jams. Their real-world tests demonstrated the system's capability to improve traffic management at intersections, ensuring safety and efficiency.

Llorca et al. (2011) designed an autonomous pedestrian collision avoidance system that employs a fuzzy steering controller. This system uses inputs such as lateral displacement and vehicle speed, processed through fuzzy rules, to make monotonic steering adjustments. Their research proved that such a fuzzy logic-based system could effectively prevent collisions, mimicking human-like maneuvers and providing safer navigation for intelligent vehicles.

Wang et al. (2016) implemented a fuzzy logic control method combined with genetic algorithms to optimize traffic light timings based on real-time traffic data. This integration ensured that the traffic control system could adapt monotonically to changing conditions, significantly reducing vehicle waiting times and improving the overall efficiency of urban traffic management.

Alcalá-Fdez et al. (2017) introduced evolutionary fuzzy rule-based classification systems (FRBCSs) designed to handle monotonic classification problems. By incorporating a non-monotonicity index (NMI) and using genetic algorithms for tuning, they ensured that

the monotonic relationships between input features and output variables were preserved. Their methods showed competitive performance in ordinal classification problems and on various benchmark datasets.

Wu et al. (2022) developed a monotonic intuitionistic fuzzy TOPSIS (Technique for Order Preference by Similarity to Ideal Solution) method. Their method uses admissible distance measures to maintain the natural partial order of intuitionistic fuzzy values, ensuring more accurate and rational decision-making. By ensuring monotonicity with general linear orders, they addressed limitations in existing TOPSIS methods, demonstrating the efficiency and reliability of their approach in various decision-making scenarios.

2.8.2 Mechanical and Control Engineering

Kouikoglou and Phillis (2009) provided conditions for ensuring monotonicity in hierarchical sum-product fuzzy systems. Their research was crucial for applications in queueing systems, sustainable development assessments, and material recyclability measurements, demonstrating the broad utility of monotonic fuzzy systems in various control processes.

Azadeh et al. (2010) developed a fuzzy inference system for diagnosing pump failures in the petrochemical industry. Their system preserved monotonicity through expert-derived linguistic rules and historical data, enhancing the maintenance process by reducing errors and improving reliability.

He et al. (2011) proposed a fuzzy logic algorithm for the multi-state control of wet friction clutches in automotive drivelines. Their strategy ensured monotonic pressure

adjustments, leading to smoother engagement and improved vehicle performance, validated through experimental results.

Birle et al. (2013) reviewed the application of fuzzy logic control in food and beverage processes, emphasizing the importance of monotonicity in handling uncertainty and variability. Their work demonstrated the benefits of fuzzy systems in optimizing fermentation processes and improving quality control in the food industry.

Zhang et al. (2013) utilized prior information to ensure monotonicity in SIRMs-connected fuzzy logic controllers for overhead crane control. By encoding continuity, symmetry, and monotonicity into the design process, they developed controllers that effectively managed the stabilization of overhead cranes, highlighting the practical benefits of their approach in mechanical control systems.

Zhu et al. (2018) proposed a fuzzy neural-network inverse system for decoupling control in fermentation processes, maintaining monotonic control actions. Their method achieved high accuracy and robust performance in managing complex biological systems.

2.8.3 Biomedical Engineering

Wang and Winters (2005) developed a dynamic recurrent neuro-fuzzy system with monotonic state transitions for accurate context-aware rehabilitation in stroke patients and muscle strength training. This system combined expert-based and evidence-based reasoning to manage wearable applications in rehabilitation by understanding the current context and predicting outcomes.

In the field of cardiovascular diagnostics, Tsipouras et al. (2007) transformed crisp models into fuzzy models, ensuring monotonicity through parameter optimization. Their

methodology improved the accuracy and interpretability of diagnostic systems, specifically in the classification of arrhythmic and ischemic beats.

Expanding the scope of medical diagnosis, Seki and Mizumoto (2012) proved the monotonicity properties of the SIRMs method with consequent fuzzy sets. Their approach ensured that the inference results were monotonic if the antecedent and consequent parts were monotonically increasing, improving accuracy in medical diagnostics. Further enhancing the SIRMs method, Seki and Nakashima (2014) incorporated functional weights into SIRMs-connected fuzzy inference models. This ensured monotonicity in high-dimensional medical diagnosis systems, particularly in the classification of diabetes data.

Ding et al. (2022) applied a higher-order FIS with a correntropy induced loss (C-loss) function for robust, monotonic identification of DNA N4-methylcytosine sites. By leveraging the kernel method and C-loss function, their model demonstrated superior performance in genetic data analysis.

2.8.4 Reliability Engineering and risk assessment

In the research by Tay and Lim (2008a; 2008b), the fuzzy Failure Mode and Effect Analysis (FMEA) methodology was introduced, where monotone FIS were proposed for calculating the fuzzy Risk Priority Number (FRPN), i.e., the monotone FIS-based RPN model. The monotone FIS-based RPN model they developed was designed to ensure both monotone output and output resolution properties, providing logical, consistent, and meaningful risk prioritization. They demonstrated the application of this methodology across various sectors, including Semiconductor Manufacturing (specifically in the Wafer Mounting and Underfill Dispensing processes), Public Hospital operations, and Sewage

Treatment Plants, illustrating the utility of the monotone FIS-based RPN model in these areas.

Jong et al. (2013) further applied the fuzzy FMEA methodology to Edible Bird Nest (EBN) production. By utilizing fuzzy rules and membership functions for risk prioritization, they ensured reliable risk assessment and quality control throughout the EBN production stages, including Swiftlets Farming, Harvesting, Cleaning, Drying and Reshaping, and Storing and Packaging.

Kerk et al. (2015) proposed an analytical interval FIS that transformed incomplete and nonmonotone rules into interval-valued rules. This innovative approach was applied to the monotone FIS-based RPN model in fuzzy FMEA, specifically for risk prioritization in the Test Handler Process in Semiconductor Manufacturing. This allowed for the appropriate ranking of failure modes even with incomplete and nonmonotone fuzzy rules.

Kerk et al. (2019) introduced Monotone Interval FIS (MIFIS) with interval-valued fuzzy rules to better handle uncertainties. This model was successfully applied to the monotone FIS-based RPN model in fuzzy FMEA, with demonstrations of its application in risk assessment for a Public Hospital and a Sewage Treatment Plant. The demonstration showed the ability of the FIS-based RPN model to provide reliable and rational prioritization in the presence of uncertain fuzzy rules.

Pang et al. (2020) developed a Monotone Fuzzy Rule Relabeling (MFRR) framework for TSK FIS in FMEA. This framework involves relabeling fuzzy rules to restore monotonicity, and it was applied to FMEA in a Sewage Treatment Plant, a Public Hospital, and a Semiconductor Plant. The MFRR framework achieved minimal computational cost

while ensuring the monotonicity of the fuzzy rule bases, thereby enhancing the reliability of the FMEA process.

Most recently, Pang et al. (2020) developed a Monotone Fuzzy Rule Relabeling (MFRR) framework for monotone TSK FIS, which involves relabeling fuzzy rules to restore the monotonicity of the TSK FIS. This model was successfully applied to the monotone FIS-based RPN model in fuzzy FMEA, with demonstrations of its application in risk assessment for a Sewage Treatment Plant, a Public Hospital, and a Semiconductor Plant. The MFRR framework achieved minimal computational cost while ensuring the monotonicity of the fuzzy rule bases, thereby enhancing the reliability of the FMEA process.

2.8.5 Civil Engineering

Chiew, Lau, and Ng (2018) developed a SIRMs-connected FIS with a monotonicity index for predicting the compressive strength of high-performance fly-ash concrete. Their model ensured accurate and monotonic relationships between inputs and outputs, making it valuable for designing materials with specific performance characteristics in the construction industry.

2.8.6 Summary of Monotone FIS Applications and Related Work in Engineering

Previous Sections 2.8.1 to 2.8.2 summarize the key applications of monotone FIS across various engineering disciplines, including reliability engineering and risk assessment, electrical and electronic engineering, biomedical engineering, mechanical and control engineering, and civil engineering. Table 2.2 highlights significant contributions and implementations of monotone FIS in these fields.

Table 2.2: Summary of Monotone FIS Applications and Related Work in Engineering

Author (year)	Finding
Electrical and Electronic Engineering	
Milanés et al. (2010)	Developed a fuzzy controller for urban intersections, ensuring monotonic speed adjustments using vehicle-to-vehicle communications.
Llorca et al. (2011)	Designed a fuzzy steering controller that respects monotonic properties by using fuzzy rules based on lateral displacement and vehicle speed inputs.
Alcalá-Fdez et al. (2017)	Developed evolutionary FRBCSs for monotonic classification, ensuring monotonicity through genetic algorithms and a non-monotonicity index (NMI).
Wu et al. (2022)	Developed a monotonic intuitionistic fuzzy TOPSIS method using admissible distance measures to maintain natural partial orders in decision-making.
Mechanical and Control Engineering	
Kouikoglou et al. (2009)	Established conditions for monotonicity in hierarchical sum-product fuzzy systems, crucial for applications in queueing and sustainability assessments.
Azadeh et al. (2010)	Created a fuzzy inference system for diagnosing pump failures with monotonicity preserved through expert-derived linguistic rules and historical data.
He et al. (2011)	Proposed a fuzzy logic algorithm for multi-state control of wet friction clutches, ensuring monotonic pressure adjustments for smooth engagement.
Birle et al. (2013)	Reviewed fuzzy logic control applications in food processes, emphasizing monotonicity in process control to handle uncertainty and variability.
Zhu et al. (2018)	Developed a fuzzy neural-network inverse system for decoupling control in fermentation processes, maintaining monotonic control actions.
Zhang et al. (2013)	Ensured monotonicity in SIRMs-connected FLCs for overhead crane control by incorporating prior information into the design process.
Biomedical Engineering	
Wang and Winters (2005)	Implemented dynamic recurrent neuro-fuzzy systems with monotonic state transitions to ensure accurate context-aware rehabilitation.
Tsipouras et al. (2007)	Transformed crisp models into fuzzy models, ensuring monotonicity through optimization of model parameters using stochastic global optimization.
Seki et al. (2012)	Investigated monotonicity properties of SIRMs method with consequent fuzzy sets, ensuring monotonic inference results through specific rule conditions.
Seki et al. (2014)	Enhanced SIRMs-connected fuzzy inference models with functional weights to ensure monotonicity in high-dimensional domains.
Ding et al. (2022)	Applied a higher-order FIS with a C-loss function to ensure robust, monotonic identification of DNA N4-methylcytosine sites.

Reliability Engineering and risk assessment	
Tay and Lim (2008a; 2008b)	Introduced the fuzzy Failure Mode and Effect Analysis (FMEA) methodology with monotone FIS for calculating the FRPN, i.e., the monotone FIS-based RPN model. Designed to ensure both monotone output and output resolution properties.
Jong et al. (2013)	Applied the fuzzy FMEA methodology with monotone FIS-based RPN model to Edible Bird Nest (EBN) production using fuzzy rules and membership functions for risk prioritization.
Kerk et al. (2015)	Proposed an analytical interval FIS that transformed incomplete and nonmonotone rules into interval-valued rules. Applied to the monotone FIS-based RPN model in fuzzy FMEA for risk prioritization.
Kerk et al. (2019)	Introduced Monotone Interval FIS (MIFIS) with interval-valued fuzzy rules to handle uncertainties. Applied to the monotone FIS-based RPN model in fuzzy FMEA, demonstrating reliable and rational prioritization.
Pang et al. (2020)	Developed a Monotone Fuzzy Rule Relabeling (MFRR) framework for TSK FIS in FMEA, involving relabeling fuzzy rules to restore monotonicity. Applied to FMEA, achieving minimal computational cost while ensuring monotonicity of fuzzy rule bases.
Civil Engineering	
Chiew et al. (2018)	Implemented a SIRMs-connected FIS with a monotonicity index to maintain monotonic relationships between inputs and outputs in predicting concrete strength.

2.9 Monotonicity Test and Related Works

A monotonicity index is introduced by Tay and Lim (2011b) as an approximate indicator to measure the fulfilment of an FIS model to the monotonicity property. Integration monotonicity index and optimization method allows an FIS model to be constructed to satisfy the monotonicity property without knowing the exact mathematical conditions of the FIS model (Tay et al., 2012a).

The usefulness of the monotonicity index in FIS modelling has been shown in a few literatures. Tay and Lim (2011b) demonstrated the application of the monotonicity index to a real industry problem. Monotonicity index was used to measure the fulfilment of an FIS-based RPN model to the monotonicity property. Tay et al. (2012a) presented a genetic algorithm-based data-driven FIS model where the monotonicity index is used as a constraint

to guide the optimization process. Tay et al. (2012b) also proposed a synthesis of the monotonicity index and similarity reasoning optimization to build a monotonicity-preserving FIS model.

A conventional FIS model with n inputs, (i.e., $y = f(\bar{x})$) is considered where $\bar{x} = (x_1, x_2 \dots x_n) \in X_1 \times X_2 \dots \times X_n$. x_i is the i -th input where $1 \leq i \leq n$ and $x_i \in X_i$. Then, the monotonicity property between y and x_i is evaluated using the procedure as follows.

- (a) Determine the upper and lower limits of the universe discourse for the input(s), x_i and denote them as $\bar{x}_i$ and $\underline{x}_i$, respectively, i.e., $X_i \in [\underline{x}_i, \bar{x}_i]$.
- (b) Divide x_i into $(k_i + 1)$ division and obtain the gird size of x_i , i.e., $g_i = (\bar{x}_i - \underline{x}_i) / k_i$. $y = f(\bar{x})$ is denoted by $\prod_{i=1}^n (k_i + 1)$ matrix.
- (c) Compare $y_{x_i=\underline{x}_i+g_i \times (m_i), \bar{s}}$ and $y_{x_i=\underline{x}_i+g_i \times (m_i+1), \bar{s}}$, $m_i = 0, 1, 2 \dots k_i - 1$, with a function denoted by *monotone* $\left(y_{x_i=\underline{x}_i+g_i \times m_i, \bar{s}} \right)$. $\bar{s}$ is a subset of $\bar{x}$ where x_i is excluded (i.e. $\bar{s} \subset \bar{x}; x_i \notin \bar{s}$). The monotonic increasing or decreasing relationship between y and x_i are determinated using the Equation 2.15 or Equation 2.16, respectively.

$$\begin{aligned} & \text{monotone} \left(y_{x_i=\underline{x}_i+g_i \times m_i, \bar{s}} \right) \\ &= \begin{cases} 1 & y_{x_i=\underline{x}_i+g_i \times m_i, \bar{s}} \leq y_{x_i=\underline{x}_i+g_i \times (m_i+1), \bar{s}} \\ 0 & \text{else} \end{cases} \end{aligned} \quad \text{Equation 2.15}$$

$$\begin{aligned} & \text{monotone} \left(y_{x_i=\underline{x}_i+g_i \times m_i, \bar{s}} \right) \\ &= \begin{cases} 1 & y_{x_i=\underline{x}_i+g_i \times m_i, \bar{s}} \geq y_{x_i=\underline{x}_i+g_i \times (m_i+1), \bar{s}} \\ 0 & \text{else} \end{cases} \end{aligned} \quad \text{Equation 2.16}$$

- (d) Compute the *monotone index* between y and x_i using Equation 2.17 as follows.

Monotone Index (x_i)

$$= \frac{\sum_{m_1=0, m_1 \neq m_i}^{m_1=k_1} \cdots \sum_{m_n=0, m_n \neq m_i}^{m_n=k_n} \sum_{m_i=0}^{m_i=k_i-1} \left(\text{monotone} \left(y_{x_i=\underline{x_i} + g_i \times m_i, \bar{s}} \right) \right)}{\sum_{m_1=0, m_1 \neq m_i}^{m_1=k_1} \cdots \sum_{m_n=0, m_n \neq m_i}^{m_n=k_n} \sum_{m_i=0}^{m_i=k_i-1} (1)} \quad \text{Equation 2.17}$$

2.10 Summary

Chapter 2 begins with a comprehensive review of the fuzzy set theory, distinguishing between T1 FSs and T2 FSs. This sets the stage for a detailed exploration of FIS in Section 2.3, where the section focuses on the defining components of FIS such as fuzzy if-then rules and fuzzy reasoning. The chapter progresses to illustrate the paradigms of T1 FIS in Section 2.4, with an emphasis on the T1 Mamdani FIS and T1 TSK FIS models. Section 2.5 continues the narrative with the introduction of the general paradigm for IT2 FIS model. The core of the chapter is devoted to the Monotonicity Property of FIS models, elaborated in Section 2.6. It commences with a clarification of Monotone FIS and later navigates the application of the monotonicity property in T1 Mamdani FIS and T1 TSK FIS, referencing relevant works and research in these domains. The in-depth analysis of monotonicity further extends to the IT2 FIS model and associated research. Section 2.7 discusses the Monotonicity Test, providing an approach to measure the fulfilment of an FIS model to the monotonicity property. Section 2.8 reviews Monotone FIS and related work in engineering.

Moving forward, Chapter 3 presents parametric conditions for the formulation of monotone T1 FIS models. Subsequent chapters feature simulated examples to evaluate monotone T1 FIS models, offering a practical understanding of the concepts discussed.

CHAPTER 3

MONOTONICITY PROPERTY ON TYPE-1 FIS MODELLING

3.1 Overview

In this chapter, the monotonicity property of T1 TSK FIS is defined. The aim is to outline a set of sufficient conditions for TSK FIS to exhibit monotonicity, supported by a simulated example. The usefulness of these sufficient conditions is demonstrated through applications in image processing and TCEA, presented in Chapters 4 and 5, respectively. Extensions of the sufficient conditions to IT2 FIS, along with their applications, are discussed in Chapters 6 and 7. This chapter is organized as follows.

In Section 3.2, fundamental concepts encompassing monotonicity, monotonic functions, FIS, and the monotonicity attributes of monotone FIS models are elucidated. Subsequently, Section 3.3 sheds light on the sufficient condition for monotone T1 SISO and MISO Takagi-Sugeno-Kang (TSK) FIS models. To preserve the monotonicity property of the TSK FIS, works from Won et. al. (2002), Tay and Lim (2011a) and Won and Karray (2014) are simplified to Theorem 3.1 and Theorem 3.2 and adopted as the governing equations for formulation of TSK FIS. Section 3.4 provides an exposition on the design of T1 Trapezoidal FMF and T1 Gaussian FMF, governed by the principles articulated in Theorem 3.1 and Theorem 3.2. Further, Section 3.5 entails the application of Theorem 3.1 and Theorem 3.2 in devising various consequent functions of Fuzzy Rules for monotone TSK FIS, i.e., first-order and second-order polynomial function are introduced.

3.2 Background

In mathematics, monotonicity denotes the tendency of a function (i.e., Monotone function) to preserve or reverse the inherent order between its input and output variables (Royden and Fitzpatrick, 2010). Monotonic functions can be classified as either increasing or decreasing. In a monotonically increasing function, as the input value increases, the output value either increases or remains constant. Conversely, in a monotonically decreasing function, the output value decreases or remains constant as the input increases. Monotonic functions are integral to the convergence of sequences and series, and are crucial in the investigation of limits and continuity (Burkhill, 1924). Moreover, monotone functions are pivotal both in theoretical explorations and pragmatic modelling across a plethora of domains such as biology (Bhaskar and Farzana, 2002), chemistry (de Leenheer et al., 2007; Sontag, 2007; Smith, 2017) and economics (Royden and Fitzpatrick, 2010).

On other hands, FIS operates as a function approximation technique, adept at mapping input spaces to output spaces amidst uncertainty and vagueness, utilizing fuzzy set theory and fuzzy logic principles (Zadeh, 1965). Comprising elements such as a rule base, fuzzification interface, inference engine, and defuzzification interface, FIS employs fuzzy if-then rules and fuzzy reasoning as pivotal components (Abraham, 2005; Jang and Sun, 1996). The applications of FIS span across an extensive range of fields, encompassing vision, and has been particularly efficacious in addressing real-world challenges (Mendel, 1995; Jang and Sun, 1996). Moreover, interdisciplinary nature of FIS has led it to be known by various terminologies, including fuzzy-rule-based system, fuzzy expert system, fuzzy model, fuzzy associative memory, fuzzy logic controller, and fuzzy system (Jang and Sun, 1996).

Monotone FIS models specifically adhere to the monotonicity criteria between input and output variables and have garnered recognition for their efficacy in an assortment of applications such as feedback systems (Angeli and Sontag, 2004), a monotone control system (Angeli and Sontag, 2003), a biological system (Enciso and Sontag, 2006), and a risk priority model in Failure Mode and Effect Analysis (FMEA) (Tay and Lim, 2008a; 2008b). As such, preserve the monotonicity property of FIS models constitutes an important research question. Search of literatures reports various useful mathematical conditions to satisfy the monotonicity property for different FIS models, e.g., Mamdani FIS (Van Broekhoven and De Baets, 2009), TSK FIS (Won and Karray, 2014; Pang et al., 2016; Tay and Lim, 2011a), single-input rule module FIS (Seki et al., 2010), hierarchical FIS (Kouikoglou and Phillis, 2009), and IT2 FIS (Li et al., 2014). Besides that, studies in Tay and Lim (2011a), Kouikoglou and Phillis (2009) and Husek (2016) have demonstrated on adopt these mathematical conditions to facilitate the FIS modelling procedure. Other investigations on the monotonicity property of fuzzy relational inference (Mandal and Jayaram, 2016) and implicative-based fuzzy models (Štěpnička and De Baets, 2013) are also reported.

On this regards, mathematic analyses in Won et al. (2002), Pang et al. (2016), Tay and Lim (2011a) and Seki et al. (2010), indicate monotone fuzzy If-then rules offer an important building block to construct a monotone TSK FIS model. Inspired by the mathematical analyses in Won et al. (2002), Tay and Lim (2011a), and Won and Karray (2014), the mathematical conditions were adopted to facilitate the construction of a monotone T1 TSK FIS. The usefulness of these mathematical conditions was examined with simulated experiments in this section.

3.3 Monotone T1 FIS Model

SISO monotone T1 TSK FIS and MISO monotone TSK FIS are reviewed in this chapter. Contemplate an input space denoted as X mapped to an output space denoted as Y . A SISO monotone FIS model that adheres to the monotonicity property between the input and output variables satisfies Definition 3.1, delineated as follows:

Definition 3.1 (SISO monotone TSK FIS): SISO FIS model i.e., $y = f(x)$ with x is a vector with one numeral, in range $(\underline{x} \leq x \leq \bar{x}) \in X$ and y is the output in the range $(\underline{y} \leq y \leq \bar{y}) \in Y$. Then, $y = f(x)$ considerable to fulfil the monotonicity property between X and Y , if $\forall (x > x') \in X$ fulfil either solely $f(x) \geq f(x')$ or $f(x) \leq f(x')$.

Extending Definition 3.1 for MISO monotone TSK FIS has Definition 3.2 as shown below:

Definition 3.2 (MISO monotone TSK FIS): MISO FIS model i.e., $y = f(\mathbf{x})$ is considered, where $\mathbf{x} = (x_1, x_2, \dots, x_n)$ is the input in space $X = X_1 \times X_2 \times \dots \times X_n$ and $y \in Y$ is the output. Note that input x_i is in range $(\underline{x}_i \leq x_i \leq \bar{x}_i) \in X_i$ and output y is in the range $(\underline{y} \leq y \leq \bar{y}) \in Y$. Let denoted $\mathbf{x}' = (x_1, \dots, x'_i, \dots, x_n)$ where $\forall (x_i > x'_i) \in X_i$. $y = f(\mathbf{x})$ is considerable to fulfil the monotonicity property between X_i and Y , if $\forall (\mathbf{x} > \mathbf{x}') \in X$ fulfil either solely $f(\mathbf{x}) \geq f(\mathbf{x}')$ or $f(\mathbf{x}) \leq f(\mathbf{x}')$ for all for all possible combinations $i \in \{1, \dots, n\}$.

The monotone TSK FIS could be either monotonically increasing or monotonically decreasing. For easy explanation for following section, only the monotonically increasing case is considered.

3.3.1 Sufficient Conditions for SISO Monotone TSK FIS

An SISO monotone TSK FIS with x as sole input in $y = f(x)$ is considered. The fuzzy rule of the TSK FIS i.e., $R^j \in \{1, \dots, m\}$ as shown in Equation 3.1 is explained as mapping $R^j: A^j \rightarrow y = b^j(x)$ where A^j as fuzzy set from input space $x \in X$ to a consequent function $b^j(x)$ at output space $y \in Y$. The membership function of A^j is denoted as $\mu^j(x)$.

$$R^j: \text{If } x \text{ is } A^j \text{ then } y = b^j(x) \quad \text{Equation 3.1}$$

An antecedent of fuzzy rule at input space X is denoted as A^j and the consequent function i.e., $y = b^j(x)$, is the consequence of fuzzy rules. At the condition where $b^j(x)$ is a zero-order polynomial function, i.e., b^j as a constant, it is known as zero-order SISO TSK FIS model. A SISO monotone TSK FIS model is formulated as Equation 3.2, where firing strength of A^j is denoted as $\mu^j(x)$.

$$y = \frac{\sum_{j=1}^m (\mu^j(x) \times b^j(x))}{\sum_{j=1}^m \mu^j(x)} \quad \text{Equation 3.2}$$

Theorem 3.1: Theorem from Won et al. (2002) and Won and Karray (2014) is simplified, SISO monotone TSK FIS (as in Equation 3.2) is monotonically increasing w.r.t x as in Definition 3.1 if the following conditions are satisfied:

Condition 1: Lets denote $\mu'^j(x)$ as the first derivative of $\mu^j(x)$ w.r.t x . $\mu^p(x) \cdot \mu'^q(x) \geq \mu'^p(x) \cdot \mu^q(x)$ should meet for all $1 \leq p \leq q \leq m$ and $x \in X = [\underline{x}, \bar{x}]$.

Condition 2: Lets denoted $b'^j(x)$ as the first derivative of $b^j(x)$ w.r.t x . $b'^j(x) \geq 0$ is meet for $j \in \{1, \dots, m\}$

Condition 3: $b^q(x) \geq b^p(x)$ for all $1 \leq p \leq q \leq m$ and $x \in X = [\underline{x}, \bar{x}]$.

3.3.2 Sufficient Conditions for MISO Monotone TSK FIS

MISO monotone TSK FIS with input $\mathbf{x} = (x_1, x_2, \dots, x_n)$ is considered, i.e., $y = f(\mathbf{x})$. The fuzzy rule of the MISO Monotone TSK FIS i.e., $R^{\mathbf{j}} = (j_1, j_2, \dots, j_n)$ as shown in Equation 3.3 is explained as mapping $A^{\mathbf{j}}$ from input space $\mathbf{x} \in X_1 \times X_2 \dots \times X_n$ to a consequent function $b^{\mathbf{j}}(\mathbf{x})$ at output space $y \in Y$.

$$R^{\mathbf{j}} = (j_1, j_2, \dots, j_n): \text{ If } (x_1 \text{ is } A_1^{j_1}) \text{ and } (x_2 \text{ is } A_2^{j_2}) \dots (x_n \text{ is } A_n^{j_n}) \text{ then } y = b^{\mathbf{j}}(\mathbf{x}) \quad \text{Equation 3.3}$$

An antecedent of fuzzy rule at input space X is denoted as $A^{\mathbf{j}} = A_1^{j_1} \times A_2^{j_2} \dots A_n^{j_n}$, where $\mathbf{j} = (j_1, j_2, \dots, j_n)$ is a integer vector, $j_i \in \{1, \dots, m_i\}$ is an integer. The membership function of $A_i^{j_i}$ is denoted as $\mu_i^{j_i}(x_i)$. The consequent function i.e., $y = b^{\mathbf{j}}(\mathbf{x})$, is the consequence of fuzzy rules. At the condition where $b^{\mathbf{j}}(\mathbf{x})$ is a zero-order polynomial function, i.e., $b^{\mathbf{j}}$ as a constant, it is known as a zero-order TSK FIS model.

A TSK FIS model is formulated as Equation 3.4, where firing strength (also known as activation degree) of $A^{\mathbf{j}}$ at $\mathbf{x}$ is denoted as $\mu^{\mathbf{j}}(\mathbf{x}) = \prod_{i=1}^n \mu_i^{j_i}(x_i)$ and the total number of possible $A^{\mathbf{j}}$ is $\prod_{i=1}^n m_i$.

$$y = \frac{\sum_{j_n=1}^{j_n=m_n} \dots \sum_{j_1=1}^{j_1=m_1} (\mu^{\mathbf{j}}(\mathbf{x}) \times b^{\mathbf{j}}(\mathbf{x}))}{\sum_{j_n=1}^{j_n=m_n} \dots \sum_{j_1=1}^{j_1=m_1} \mu^{\mathbf{j}}(\mathbf{x})} \quad \text{Equation 3.4}$$

Theorem 3.2: Theorem from Won et al. (2002), Tay and Lim (2011a), Won and Karray (2014) are adopted, MISO monotone TSK FIS (as in Equation 3.4) is monotonically increasing w.r.t x_i as in Definition 3.2 if the following conditions are satisfied.

Condition 1: Lets denoted $\mu_i^{j_i}(x_i)$ as first derivative of $\mu_i^{j_i}(x_i)$ w.r.t x_i . Thus, $\mu_i^p(x_i) \cdot \mu_i^q(x_i) \geq \mu_i^p(x_i) \cdot \mu_i^q(x_i)$ for all $1 \leq p \leq q \leq m_i$ where $x_i \in X_i = [\underline{x_i}, \overline{x_i}]$ and $i = 1, \dots, n$.

Condition 2: The first derivative of $b^j(x)$ i.e., $d(b^j(x))/dx_i \geq 0$ is satisfied for all $i = (1, \dots, n)$, where $j = (j_1, j_2, \dots, j_n)$ is a integer vector, $j_i \in \{1, \dots, m_i\}$ is an integer.

Condition 3: $b^{j+e_i}(x) \geq b^j(x)$ is true for all j , such that $j_i < m_i$. Note that, $j + e_i$ denotes vector $(j_1, \dots, j_i + 1, \dots, j_n)$ and e_i denotes an n -dimensional unit vector in direction i .

3.4 Design of Membership Function for T2 TSK FIS

Condition 1 in Theorem 3.1 and Theorem 3.2 is applied to the monotone TSK FIS with different FMF, and the useful parametric condition is explored for the FIS model with trapezoidal and Gaussian T1 FMF.

3.4.1 T1 Trapezoidal FMF

A T1 trapezoidal FMF (see Figure 3.1) within the domain X , denoted as $A = (a, b, c, d)$, is articulated as per Equation 3.5. Should the condition $b = c$ be satisfied, A transforms into a T1 triangular FMF.

At the condition when stipulations $a = b$ and $c = d$ are met concurrently, A attains the characteristics of a T1 rectangular FMF. In addition, $a = b = c = d$ is fulfilled, A epitomizes a T1 singleton FMF. To facilitate a more streamlined explication in the subsequent chapters' applications, the T1 triangular FMF is denoted as $\mu_A(x; a, b, d)$ with considered $b = c$.

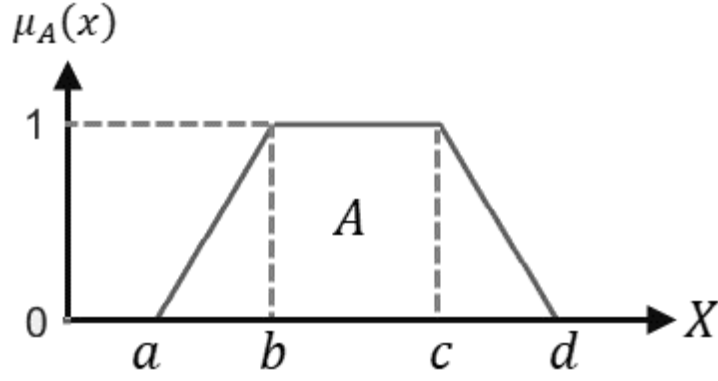


Figure 3.1: Trapezoidal fuzzy membership function.

$$\mu_A(x: a, b, c, d) = \begin{cases} (x - a)/(b - a), & a < x \leq b, \\ 1, & b < x \leq c, \\ (d - x)/(d - c), & c < x \leq d, \\ 0, & \text{Otherwise} \end{cases} \quad \text{Equation 3.5}$$

A SISO monotone TSK FIS as in Definition 3.1 with m T1 trapezoidal FMF space $x \in X$ is considered, the following remarks is derived.

Remark 3.1: (Won and Karray, 2014). T1 trapezoidal FMF i.e., $\mu^j(x: a^j, b^j, c^j, d^j)$ in input space X for $1 \leq j \leq m$ should satisfy *Condition 1* in Theorem 3.1 if $a^q \geq a^p$, $b^q \geq b^p$, $c^q \geq c^p$ and $d^q \geq d^p$ for all $1 \leq p \leq q \leq m$.

Extending Remark 3.1 for MISO Monotone TSK FIS the Remark 3.2 is derived.

Remark 3.2: (Won and Karray, 2014). T1 trapezoidal FMF i.e., $\mu_i^{j_i}(x_i: a_i^{j_i}, b_i^{j_i}, c_i^{j_i}, d_i^{j_i})$ in input space X_i for $1 \leq j_i \leq m_i$ should satisfy *Condition 1* in Theorem 3.2 if $a_i^q \geq a_i^p$, $b_i^q \geq b_i^p$, $c_i^q \geq c_i^p$ and $d_i^q \geq d_i^p$ for all $1 \leq p \leq q \leq m_i$.

Simulated experiment on SISO monotone TSK FIS is demonstrated in Section 3.5.1, wherein Remark 3.1 is employed as the governing condition for T1 trapezoidal FMFs design.

Meanwhile, in Section 3.6.1, Remark 3.2 is invoked as the governing condition for T1 trapezoidal FMFs design in MISO Monotone TSK FIS.

3.4.2 T1 Gaussian FMF

A T1 gaussian FMF as (see Figure 3.2) in the X domain i.e., $A = (c, \sigma)$, formulated as Equation 3.6, c and σ , are the centre and width of the FMF. At condition $\sigma = 0$, A is a singleton FMF.

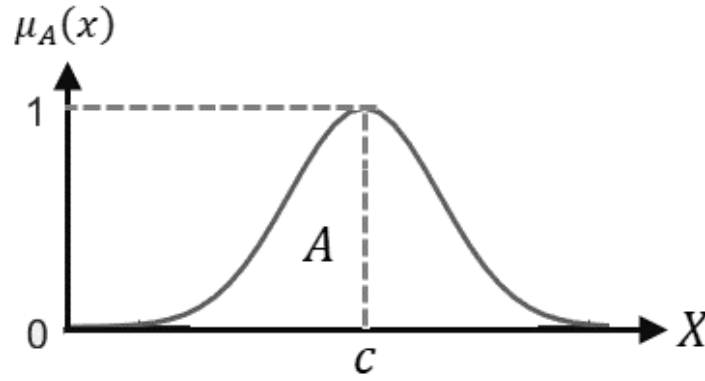


Figure 3.2: Gaussian fuzzy membership function.

$$\mu_A(x; c, \sigma) = \begin{cases} e^{-\frac{[x-\underline{c}]^2}{2\sigma^2}}, & x \leq \underline{c}, \\ 1, & \underline{c} < x \leq \bar{c}, \\ e^{-\frac{[x-\bar{c}]^2}{2\sigma^2}}, & x \geq \bar{c}, \end{cases} \quad \text{Equation 3.6}$$

A SISO monotone TSK FIS as defined in Definition 3.1 with m T1 trapezoidal FMF in space $x \in X$ is considered, and Remark 3.3 is derived. For a MISO monotone TSK FIS as defined in Definition 3.2 with T1 Gaussian FMF in space $x_i \in X_i$, Remark 3.4 is derived.

Remark 3.3: (Won et al., 2002; Won and Karray, 2014). T1 Gaussian FMF i.e., $\mu^j(x; c^j, \sigma^j)$ in input space X for $1 \leq j \leq m$ should satisfy *Condition 1* in Theorem 3.2 if $\sigma^q = \sigma^p$ and $c_i^q \geq c_i^p$ for all $1 \leq p \leq q \leq m$.

Remark 3.4: (Won et al., 2002; Won and Karray, 2014). T1 Gaussian FMF i.e., $\mu_i^{j_i}(x_i: c_i^{j_i}, \sigma_i^{j_i})$ in input space X_i for $1 \leq j_i \leq m_i$ should satisfy *Condition 1* in Theorem 3.2 if $\sigma_i^q = \sigma_i^p$ and $c_i^q \geq c_i^p$ for all $1 \leq p \leq q \leq m_i$.

Alternatively, Tay and Lim (2011a) introduced a method to project the T1 Gaussian FMF, thereby allowing *Condition 1* of Theorem 3.1 and *Condition 1* of Theorem 3.2 can be visualized. In this context, a SISO monotone TSK FIS, as outlined in Definition 3.1 and involving m T1 trapezoidal FMF space with $x \in X$, is considered. From this, Remark 3.5 is derived. It is noteworthy that the expression for projects a linear function onto the T1 Gaussian FMF, i.e., $\mu_A^j(x: c^j, \sigma^j)$, is given by $E^j(x) = (d\mu^j(x)/dx)/\mu^j(x) = (c^j/(\sigma^j)^2) - (1/(\sigma^j)^2)x$. As such, the inequality $\mu^p(x)(d\mu^q(x)/dx) \geq (d\mu^p(x)/dx)\mu^q(x) \Leftrightarrow E^q(x) \geq E^p(x)$ is satisfied, fulfilling the requirements of *Condition 1* of Theorem 3.1. For a MISO monotone TSK FIS as in Definition 3.2 with T1 Gaussian FMF space $x_i \in X_i$ is considered, the Remark 3.6 is derived.

Remark 3.5: (Tay and Lim, 2011a). T1 Gaussian FMF i.e., $\mu_A^j(x: c^j, \sigma^j)$ in input space X_i for $1 \leq j_i \leq m_i$ should satisfy *Condition 1* Theorem 3.1 in if $E^q(x) \geq E^p(x)$ for all $1 \leq p \leq q \leq m$.

Remark 3.6: (Tay and Lim, 2011a). T1 Gaussian FMF i.e., $\mu_A^{j_i}(x_i: c^{j_i}, \sigma^{j_i})$ in input space X_i for $1 \leq j_i \leq m_i$ should satisfy *Condition 1* in Theorem 3.2 if $E^q(x_i) \geq E^p(x_i)$ for all $1 \leq p \leq q \leq m_i$.

Noted that $E^{j_i}(x_i) = (d\mu_i^{j_i}(x_i)/dx_i)/\mu_i^{j_i}(x_i) = (c_i^{j_i}/(\sigma_i^{j_i})^2) - (1/(\sigma_i^{j_i})^2)x_i$ is a linear function projection the T1 Gaussian FMF i.e., $\mu_i^{j_i}(x_i: c^{j_i}, \sigma^{j_i})$. As such, the

inequality $\mu^p(x_i)(d\mu_i^q(x_i)/dx_i) \geq (d\mu_i^p(x_i)/dx_i)\mu^q(x_i) \Leftrightarrow E^q(x_i) \geq E^p(x_i)$ is satisfied, fulfilling the requirements of *Condition 1* of Theorem 3.2

3.5 Simulated Experiments in SISO Monotone TSK FIS

In this section, simulated experiments are conducted to examine SISO Monotone TSK FIS model, which encompasses singular input $x \in X = [0,10]$, and employs four FMF denoted as $A^{j \in \{1,2,3,4\}}$. The membership function of A^j is denoted as $\mu^j(x)$. Simulated experiments concentrating on trapezoidal and Gaussian FMF design in SISO Monotone TSK FIS are demonstrated. Specifically, Section 3.5.1 utilizes Remark 3.1 governing the design of T1 trapezoidal FMFs, while Section 3.5.2 adopts Remark 3.4 serves as the governing condition for T1 Gaussian FMF design.

Let's denoted the consequent of fuzzy rules R^j as constant b^j , and in this context, a zero-order SISO Monotone TSK FIS is evaluated. The zero-order SISO Monotone TSK FIS incorporates four fuzzy rules as delineated in Equation 3.7. As in Equation 3.7, the consequents are designated as $b^1 = 1$, $b^2 = 2$, $b^3 = 3$ and $b^4 = 4$. Given that the first derivative of b^j w.r.t x is identically zero for $j \in \{1, \dots, m\}$, Condition 2 of Theorem 3.1 is satisfied. Moreover, the experimental configuration ensures the fulfilment $b^1 \leq b^2 \leq b^3 \leq b^4$, which aligning with *Condition 3* in Theorem 3.1. The zero-order SISO monotone TSK FIS is formulated as in Equation 3.8 Surface plot is used to evaluated output of zero-order SISO monotone TSK FIS, as delineated Section 3.5.1 and 3.5.2.

$$\begin{aligned}
 R^1: & \text{ If } x \text{ is } A^1 \text{ then } y = 1 \\
 R^2: & \text{ If } x \text{ is } A^2 \text{ then } y = 2 \\
 R^3: & \text{ If } x \text{ is } A^3 \text{ then } y = 3 \\
 R^4: & \text{ If } x \text{ is } A^4 \text{ then } y = 4
 \end{aligned}
 \tag{Equation 3.7}$$

$$f(x) = \frac{1\mu^1(x) + 2\mu^2(x) + 3\mu^3(x) + 4\mu^4(x)}{\mu^1(x) + \mu^2(x) + \mu^3(x) + \mu^4(x)} \quad \text{Equation 3.8}$$

3.5.1 SISO Monotone TSK FIS with T1 Trapezoidal FMF

An zero-order SISO Monotone TSK FIS with T1 Trapezoidal FMF is evaluated in this example. As illustrated in Figure 3.3, the 4 T1 Trapezoidal FMFs are parameterized as, $A^1 = \mu^1(x: 0,0,0,2)$, $A^2 = \mu^2(x: 0,3,3,6)$, $A^3 = \mu^3(x: 4,6,7,9)$ and $A^4 = \mu^4(x: 7,9,10,10)$. The configuration of A^j , for $j \in (1,2,3,4)$, complies with Remark 3.1. It is noteworthy that that A^1 and A^2 , being Type-1 triangular FMFs, constitute a specific special case of the T1 trapezoidal FMF.

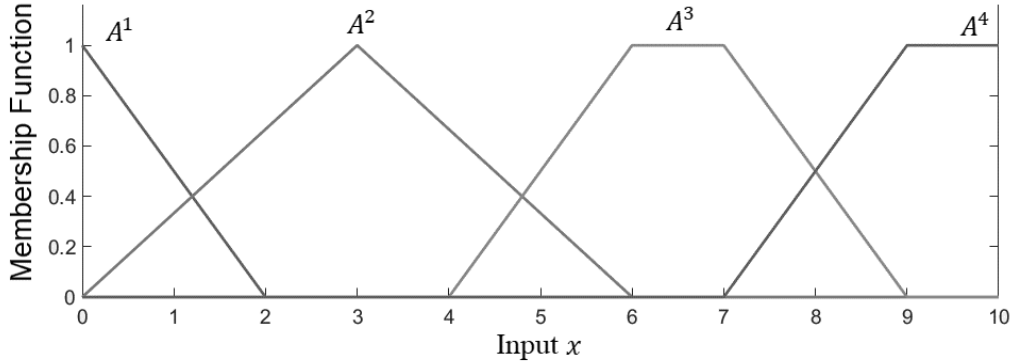


Figure 3.3: T1 Trapezoidal FMF design of the SISO Monotone TSK FIS

The output of the zero-order SISO Monotone TSK FIS, as articulated in Equation 3.8, is represented with the output y , as illustrated in Figure 3.4. As can be discerned from Figure 3.4, the output y exhibits a monotonically increasing relationship w.r.t x .

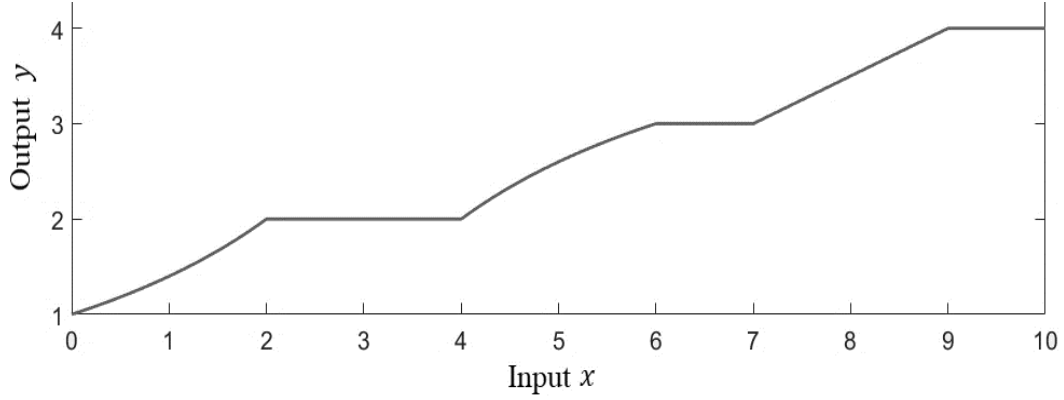


Figure 3.4: Output of zero-order SISO Monotone TSK FIS with T1 Trapezoidal FMF

3.5.2 SISO Monotone TSK FIS with T1 Gaussian FMF

In this section, a zero-order SISO Monotone TSK FIS employing T1 Gaussian FMFs is subjected to evaluation. Figure 3.5 delineates four T1 Gaussian FMFs parameterized as $A^1 = \mu^1(x; 0, 0.8)$, $A^2 = \mu^2(x; 3, 1.2)$, $A^3 = \mu^3(x; 6.5, 1.5)$ and $A^4 = \mu^4(x; 10, 1.7)$, whereby the design of $A^{j \in (1,2,3,4)}$ in domain X fulfil Remark 3.4. The linear projections of the T1 Gaussian FMFs, denoted as $E^1(x)$, $E^2(x)$, $E^3(x)$ and $E^4(x)$, are depicted in Figure 3.6.

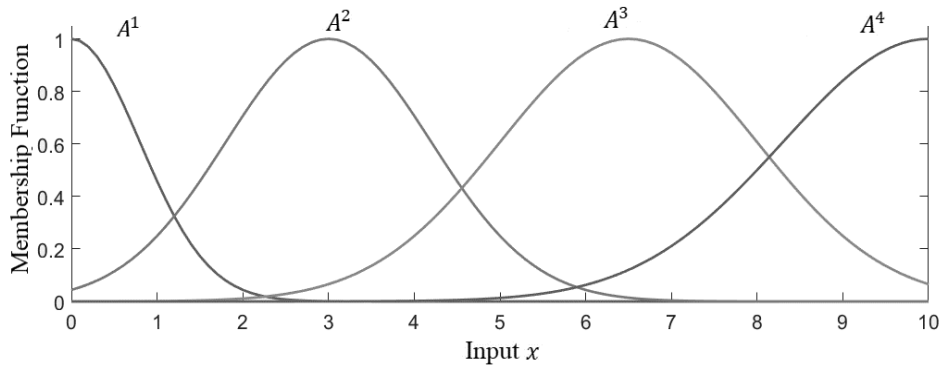


Figure 3.5: T1 Gaussian FMF design of the zero-order SISO Monotone TSK FIS

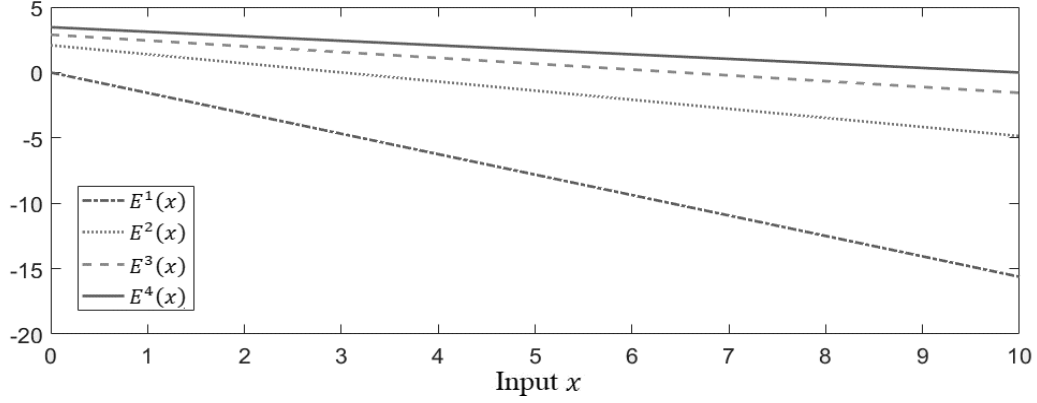


Figure 3.6: Linear projection of T1 Gaussian FMF

The output of the zero-order SISO Monotone TSK FIS, as articulated in Equation 3.8, is represented with the output y , as illustrated in Figure 3.7. As can be discerned from Figure 3.7, the output y exhibits a monotonically increasing relationship w.r.t x .

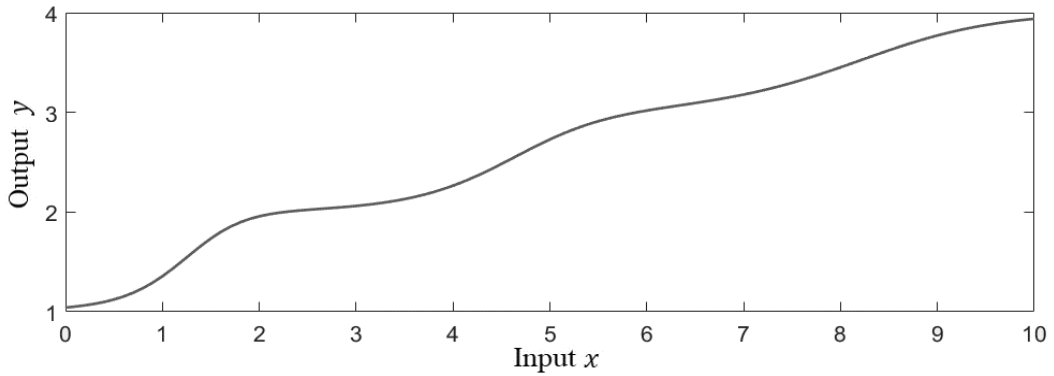


Figure 3.7: Output of zero-order SISO Monotone TSK FIS with T1 Gaussian FMF

3.6 Simulated Experiments in MISO Monotone TSK FIS

In this section, simulated experiments are conducted to examine MISO Monotone TSK FIS model, which comprises two inputs, articulated as $y = f(\mathbf{x})$ where $\mathbf{x} = (x_1, x_2) \in X_1 \times X_2 = [0,5] \times [0,5]$. Each input incorporates two T1 FMFs, denoted as $A_i^{j_i}$, with $j_i = \{1,2\}$ for $i = \{1,2\}$. The membership function of $A_i^{j_i}$ is denoted as $\mu_i^{j_i}(x_i)$. Simulated experiments concentrating on trapezoidal and Gaussian FMF design in MISO Monotone

TSK FIS are demonstrated. Specifically, Section 3.6.1 utilizes Remark 3.2 governing the design of T1 trapezoidal FMFs, while Section 3.6.2 adopts Remark 3.6 serves as the governing condition for T1 Gaussian FMF design.

Let's represent the consequent of fuzzy rules R^j as constant b^j , with $j = (j_1, j_2)$. A zero-order MISO Monotone TSK FIS is evaluated in this section. The zero-order MISO Monotone TSK FIS incorporates four fuzzy rules, given by $\prod_{i=1}^2 m_i = 2 \times 2 = 4$, as delineated in Equation 3.9. From Equation 3.9, the consequents denoted by $b^{(1,1)} = 1$, $b^{(1,2)} = 2$, $b^{(2,1)} = 3$ and $b^{(2,2)} = 4$. The consequent of fuzzy rules R^j is designed as such the *Condition 2* and *Condition 3* of Theorem 3.2 are satisfied. Given that the first derivative of b^j w.r.t x_i is identically zero for all $j_i \in \{1,2\}$ and $i = \{1,2\}$, *Condition 2* of Theorem 3.2 is satisfied. Moreover, the experimental design ensures that $b^{(1,1)} \leq b^{(1,2)} \leq b^{(2,1)} \leq b^{(2,2)}$ is fulfilled to align with *Condition 3* in Theorem 3.2. The firing strength of $R^{(j_1, j_2)}$ is denoted as $\mu^{(j_1, j_2)} = \mu_1^{j_1}(x_1) \times \mu_2^{j_2}(x_2)$. The output of the zero-order MISO monotone TSK FIS is outlined in Equation 3.10. Surface plot is used to evaluated output of zero-order MISO monotone TSK FIS, as delineated Section 3.6.1 and 3.6.2.

$$\begin{aligned}
R^{(1,1)}: & \text{ If } x_1 \text{ is } A_1^1 \text{ and } x_2 \text{ is } A_2^1 \text{ then } y = b^{(1,1)} = 1 \\
R^{(1,2)}: & \text{ If } x_1 \text{ is } A_1^1 \text{ and } x_2 \text{ is } A_2^2 \text{ then } y = b^{(1,2)} = 2 \\
R^{(2,1)}: & \text{ If } x_1 \text{ is } A_1^2 \text{ and } x_2 \text{ is } A_2^1 \text{ then } y = b^{(2,1)} = 3 \\
R^{(2,2)}: & \text{ If } x_1 \text{ is } A_1^2 \text{ and } x_2 \text{ is } A_2^2 \text{ then } y = b^{(2,2)} = 4
\end{aligned}
\tag{Equation 3.9}$$

$$f(x_1, x_2) = \frac{\sum_{j_1=1}^2 \sum_{j_2=1}^2 (\mu^{(j_1, j_2)}(\mathbf{x}) \times b^{(j_1, j_2)})}{\sum_{j_1=1}^2 \sum_{j_2=1}^2 \mu^{(j_1, j_2)}(\mathbf{x})}
\tag{Equation 3.10}$$

3.6.1 MISO Monotone TSK FIS with T1 Trapezoidal FMF

An zero-order MISO Monotone TSK FIS with T1 Trapezoidal FMF is evaluated in this example. The T1 Trapezoidal FMFs for the x_1 are parameterized as $A_1^1 = \mu_1^1(x_1:0,0,0,3)$, $A_1^2 = \mu_1^2(x_1:2,4,5,5)$, as illustrated in Figure 3.8. Concurrently, T1 Trapezoidal FMFs for input x_2 are parameterized as $A_2^1 = \mu_2^1(x_2:0,0,2,4)$ and $A_2^2 = \mu_2^2(x_2:1,5,5,5)$, as shown in in Figure 3.9. The design of $A_i^{j_i}$ adhere to Remark 3.2. Note that the T1 triangular FMF, i.e., A_1^1 and A_2^2 , are special case of the T1 trapezoidal FMF.

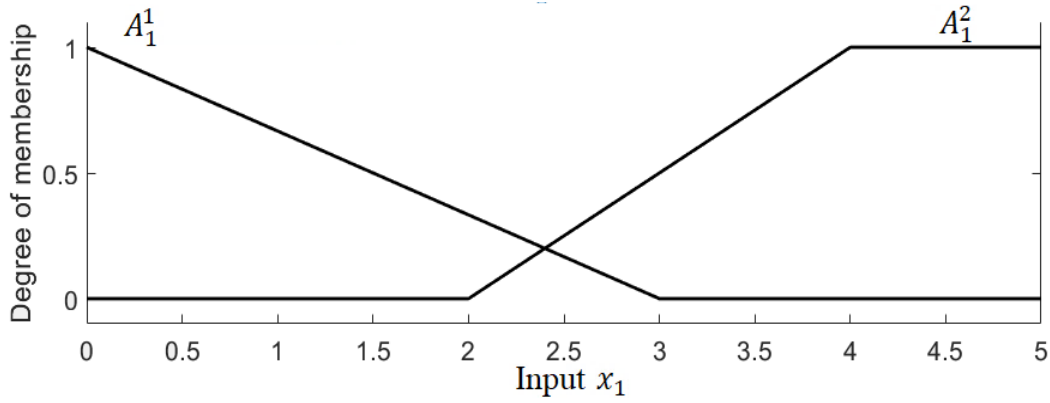


Figure 3.8: Input x_1 of zero-order MISO Monotone TSK FIS with T1 Trapezoidal FMF

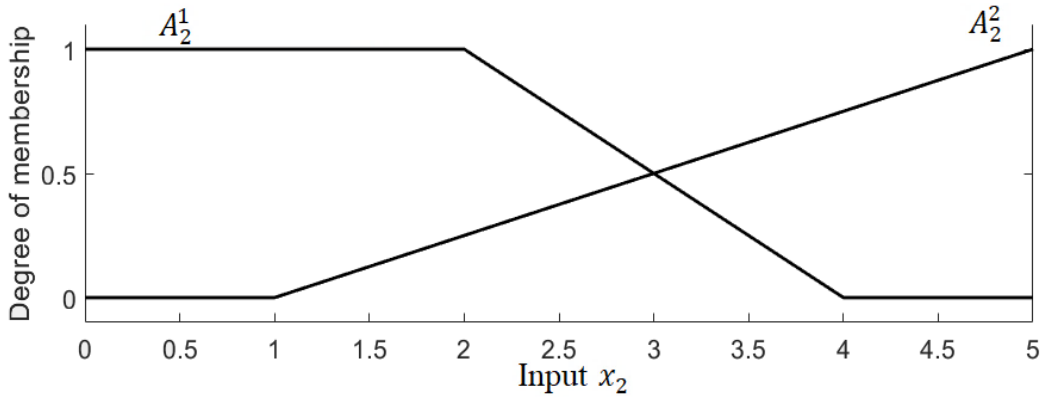


Figure 3.9: Input x_2 of zero-order MISO Monotone TSK FIS with T1 Trapezoidal FMF

The output of the zero-order MISO Monotone TSK FIS, as articulated in Equation 3.10, is represented with the output y , as illustrated in Figure 3.10. As can be discerned from

Figure 3.10, the output y is monotonically increasing w.r.t $\forall x_{i \in \{1,2\}}$ for all dimensional unit vector in direction $i = 1,2$

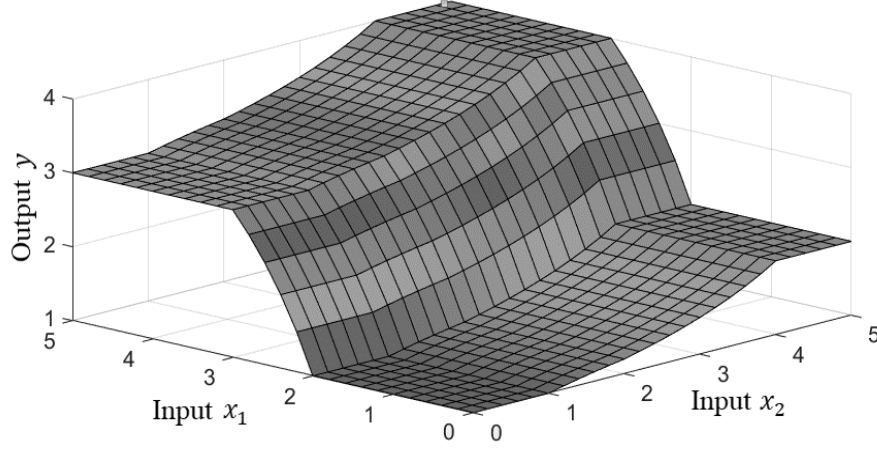


Figure 3.10: Output of zero-order MISO Monotone TSK FIS

3.6.2 MISO Monotone TSK FIS with T1 Gaussian FMF

In this section, a zero-order MISO Monotone TSK FIS employing T1 Gaussian FMFs is subjected to evaluation. The T1 Gaussian FMFs for the x_1 are parameterized as $A_1^1 = \mu_1^1(x_1; 0,1)$, $A_1^2 = \mu_1^2(x_1; 5,1)$, as illustrated in Figure 3.11. Concurrently, T1 Trapezoidal FMFs for input x_2 are parameterized as $A_2^1 = \mu_2^1(x_2; 0,2)$ and $A_2^2 = \mu_2^2(x_2; 5,2)$, as shown in in Figure 3.12. The design of A_i^j adhere to Remark 3.6.

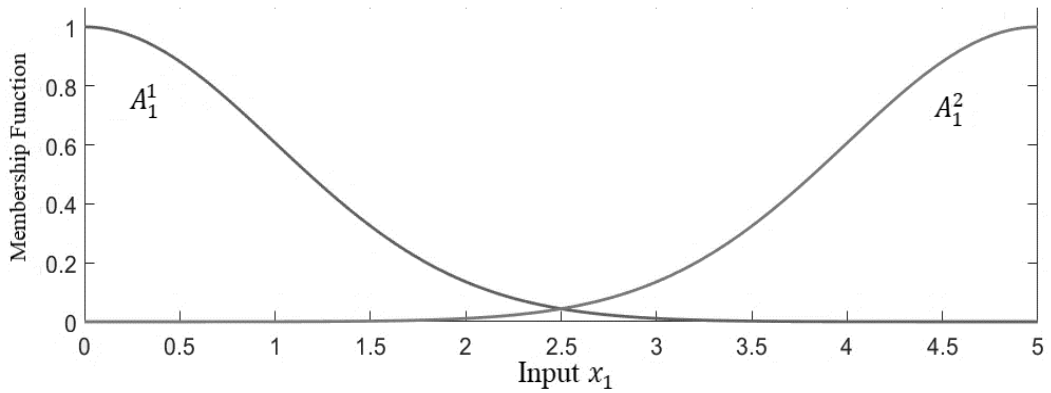


Figure 3.11: Input x_1 of zero-order MISO Monotone TSK FIS with T1 Gaussian FMF

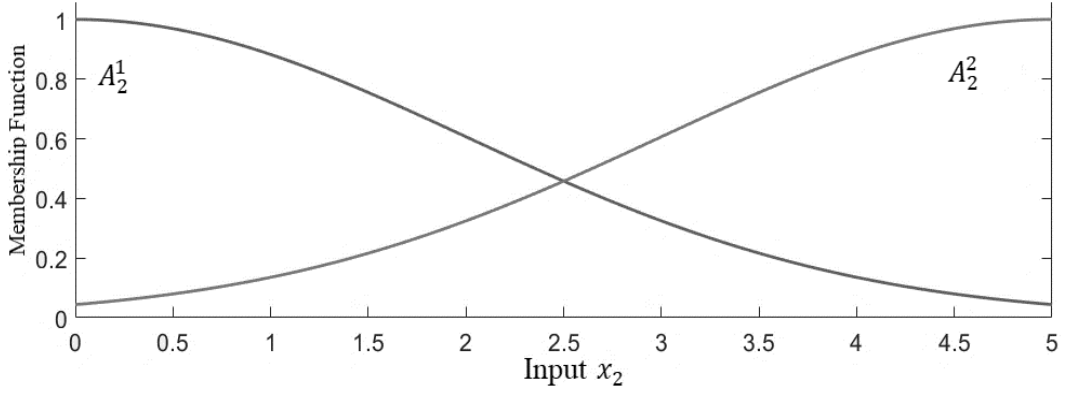


Figure 3.12: Input x_2 of zero-order MISO Monotone TSK FIS T1 Gaussian FMF

The output of the zero-order MISO Monotone TSK FIS, as articulated in Equation 3.10, is represented with the output y , as illustrated in Figure 3.13. As can be discerned from Figure 3.13, the output y is monotonically increasing w.r.t $\forall x_{i \in \{1,2\}}$ for all dimensional unit vector in direction $i = 1,2$

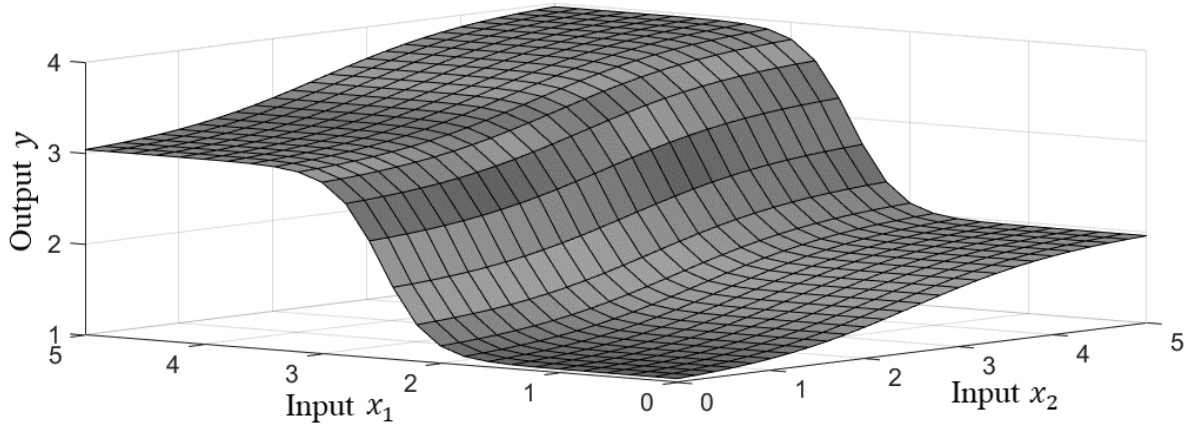


Figure 3.13: Output of zero-order MISO Monotone TSK FIS with T1 Gaussian FMF

3.7 Consequent Functions of Fuzzy Rules for Monotone TSK FIS

The *Condition 2* and *Condition 3* of Theorem 3.1 and Theorem 3.2 are evaluated in this section. First-order and other higher-order polynomial function types of consequent function are examined in this section. Pertaining to the MISO TSK FIS as delineated section 3.7.2 and 3.7.3, the output of TSK FIS is evaluated through two approaches, i.e., (i) visual

inspection of surface plot of the model and (ii) monotonicity test approach as propounded in the works of Tay and Lim (2011b), Tay et al. (2012a, 2012b) and Teh et al. (2018).

3.7.1 SISO Monotone TSK FIS with Consequent as First-Order Polynomial Function

Simulated experiment is structured to ensure that *Condition 1* in Theorem 3.1 are fulfilled. In this experiment, $x \in X = [0,10]$ operates as the singular input for the the first-order SISO TSK FIS. Input x employs three T1 Trapezoidal FMFs, parameterized as $A^1 = \mu^1(x: 0,0,0,4)$, $A^2 = \mu^2(x: 0,3,6,8)$ and $A^3 = \mu^3(x: 6,8,10,10)$, which represented in Figure 3.14. $A^{j \in (1,2,3)}$ is designed as such the Remark 3.1 is fulfilled.

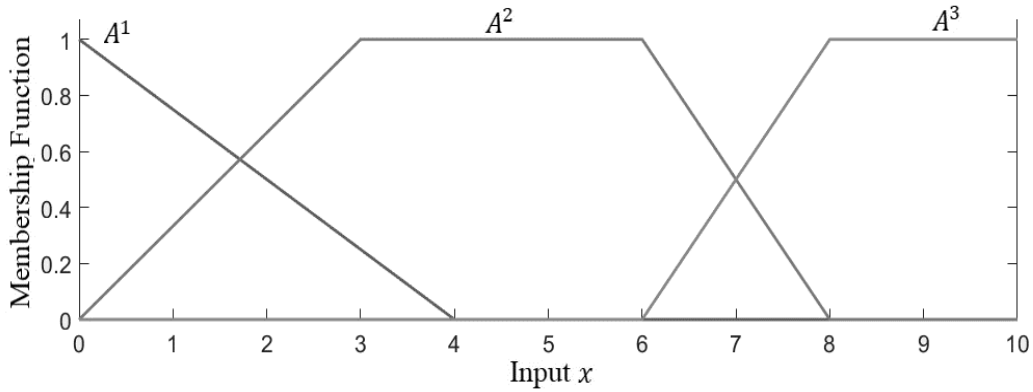


Figure 3.14: T1 Trapezoidal FMF design of the first order SISO TSK FIS

The first-order SISO TSK FIS comprised 3 fuzzy rules as in Equation 3.11. As in Equation 3.11, the consequent of fuzzy rules i.e., $b^j(x) = b^{(0,j)} + b^{(1,j)} \cdot x^1$ is a first order polynomial function of x . The first order SISO TSK FIS is formulated as in Equation 3.12.

$$\begin{aligned}
 R^1: & \text{ If } x \text{ is } A^1 \text{ theny} = b^1(x) \\
 R^2: & \text{ If } x \text{ is } A^2 \text{ theny} = b^2(x) \\
 R^3: & \text{ If } x \text{ is } A^3 \text{ theny} = b^3(x)
 \end{aligned}
 \tag{Equation 3.11}$$

$$f(x) = \frac{\mu^1(x) \cdot b^1(x) + \mu^2(x) \cdot b^2(x) + \mu^3(x) \cdot b^3(x)}{\mu^1(x) + \mu^2(x) + \mu^3(x)}
 \tag{Equation 3.12}$$

The formulation of the consequent functions is governed by the implementation of the *Condition 2* and *Condition 3* as posited in Theorem 3.1. Employing *Condition 2* from Theorem 3.1, the first-derivative of $b^j(x)$ w.r.t x is derived, as in Equation 3.13, for $j = \{1,2,3\}$. Three simulated experiments, denoted as Experiment (A), Experiment (B), and Experiment (C), are executed. In experiment (A), both *Condition 2* and *Condition 3* of Theorem 3.1 are satisfied, ensuring such $0 \leq \frac{d}{dx}(b^j(x)) = b^{(1,j)}$ and $b^q(x) \geq b^p(x)$ for $x \in X = [\underline{x}, \bar{x}]$. Experiment (B) satisfies only *Condition 2* of Theorem 3.1, ensuring $0 \leq \frac{d}{dx}(b^j(x)) = b^{(1,j)}$. In experiment (C), adheres solely *Condition 3* of Theorem 3.1, ensuring $b^q(x) \geq b^p(x)$.

$$0 \leq \frac{d}{dx}(b^j(x)) = b^{(1,j)} \quad \text{Equation 3.13}$$

In experiment(A), consequents of fuzzy rules are $b^1(x) = 1 + x$, $b^2(x) = 2 + x$ and $b^3(x) = 3 + x$, with $0 \leq \frac{d}{dx}(b^j(x)) = b^{(1,j)}$ is fulfilled for $\forall j = (1,2,3)$ and $b^3(x) \geq b^2(x) \geq b^1(x)$ as depicted in Figure 3.15. As presented in Figure 3.16, the output of the SISO TSK FIS i.e., y is monotonically increasing w.r.t x .

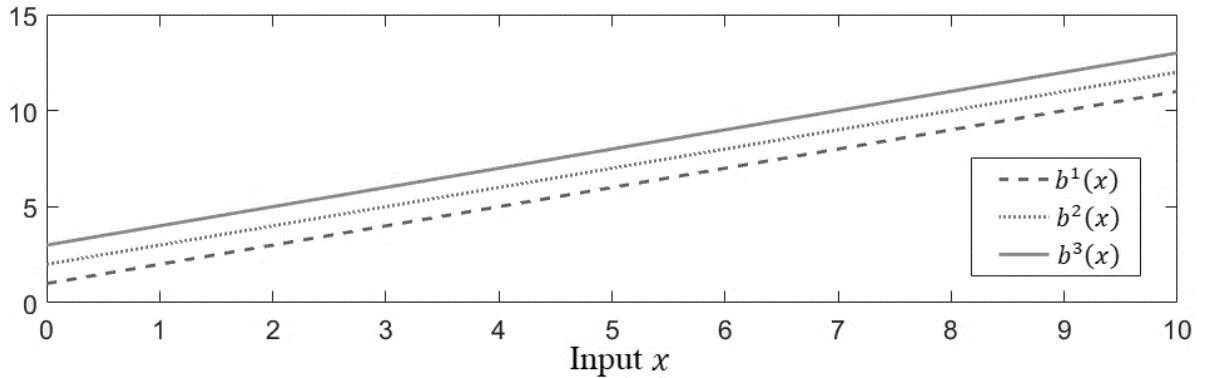


Figure 3.15: Output of $b^3(x) \geq b^2(x) \geq b^1(x)$ w.r.t x for experiment (A)

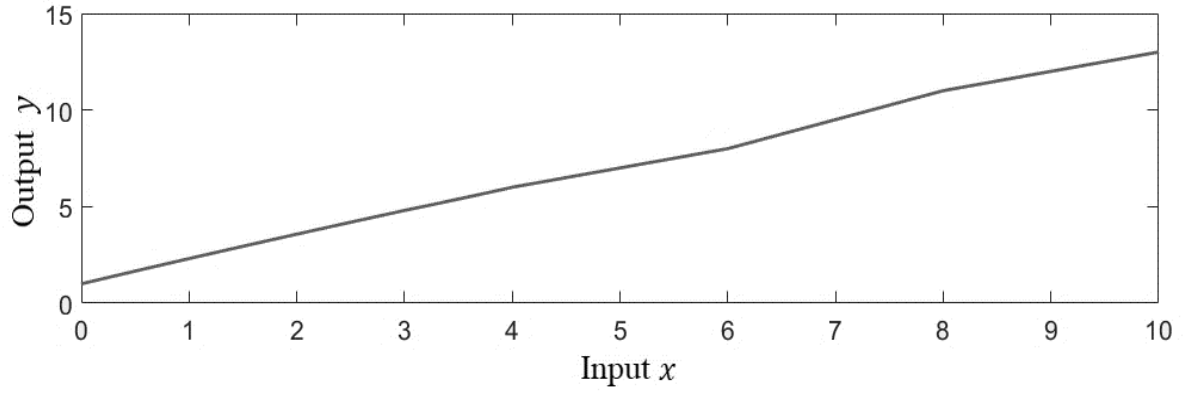


Figure 3.16: Output of first-order SISO TSK FIS as in experiment (A)

In experiment (B), consequents of fuzzy rules are $b^1(x) = 1 - x$, $b^2(x) = 2 - x$ and $b^3(x) = 3 - x$ where the consequents $b^j(x)$ is designed in such, $b^3(x) \geq b^2(x) \geq b^1(x)$ remain hold as depicted in Figure 3.17. However, consequents $b^j(x)$ is not fulfilled the condition as in Equation 3.13, as such $0 \geq b^{(1,j)}$ for $\forall j = (1,2,3)$. As presented in Figure 3.18, the output of the SISO TSK FIS i.e., y is monotonically decreasing w.r.t x .

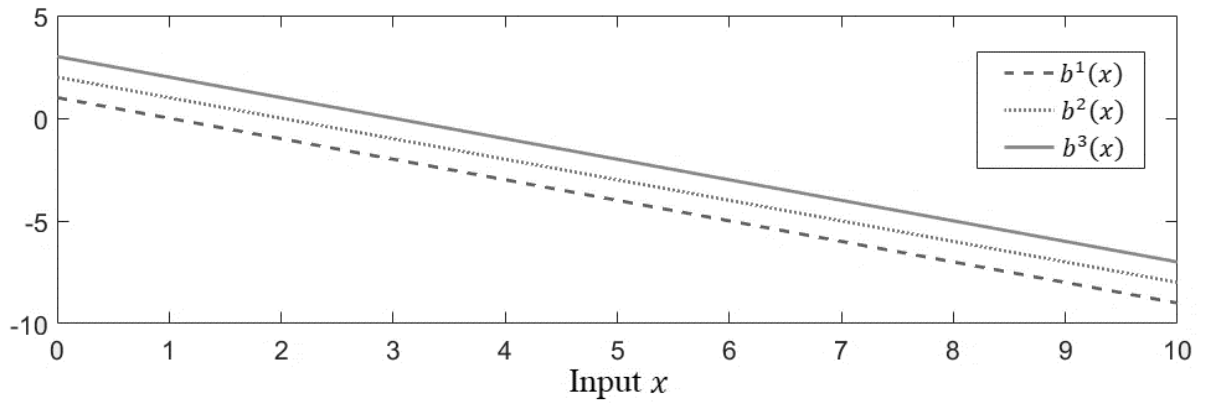


Figure 3.17: Output of $b^3(x) \geq b^2(x) \geq b^1(x)$ w.r.t x for experiment (B)

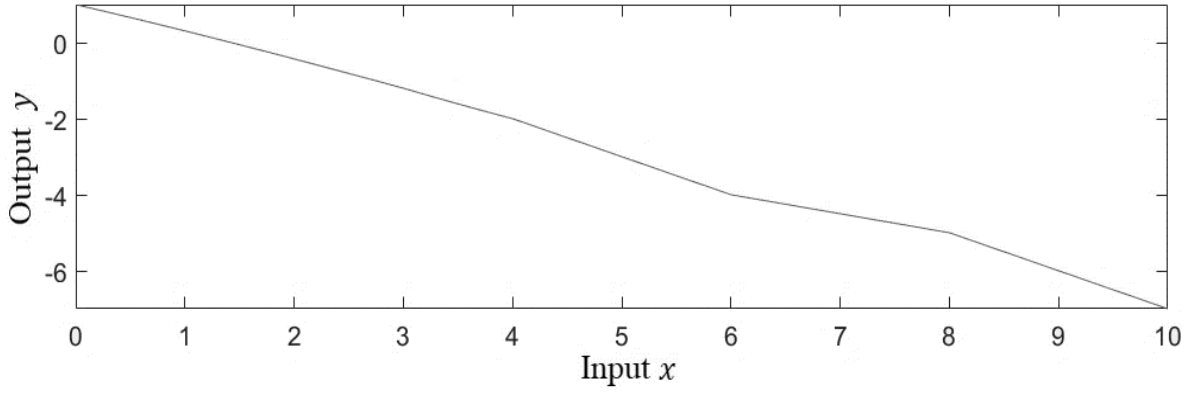


Figure 3.18: Output of first-order SISO TSK FIS as in experiment (B)

In experiment (C), consequents of fuzzy rules are $b^1(x) = 1 + x$, $b^2(x) = 2 + x$ and $b^3(x) = 3 - x$ where the consequents $b^j(x)$ is designed in such, $b^3(x) \geq b^2(x) \geq b^1(x)$ remain hold as depicted in Figure 3.17. However, consequents $b^j(x)$ is not fulfilled the condition as in Equation 3.13, as such $0 \leq b^{(1,j \in \{1,2\})}$ and $0 \geq b^{(1,3)}$. As presented in Figure 3.20, the non-monotone output y w.r.t x is obtained of the SISO TSK FIS.

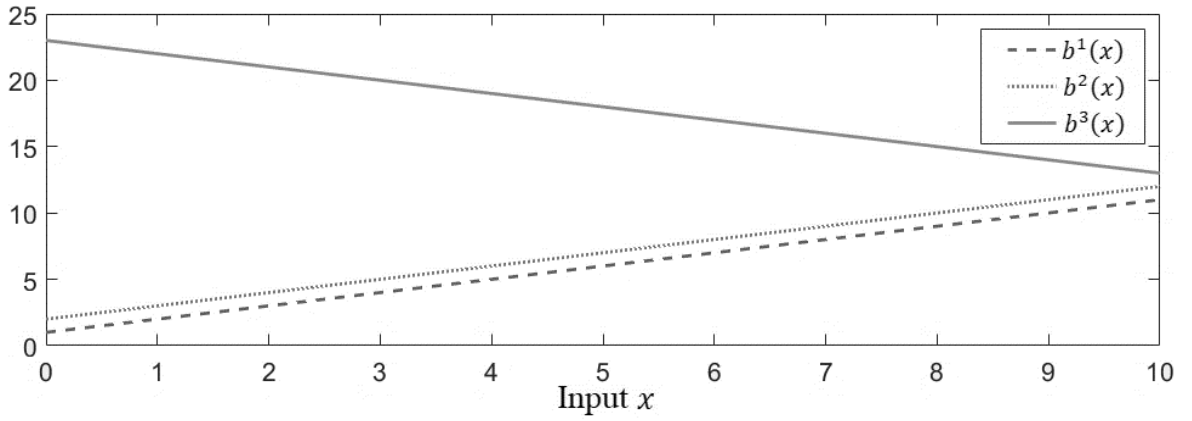


Figure 3.19: Output of $b^3(x) \geq b^2(x) \geq b^1(x)$ w.r.t x for experiment (C)

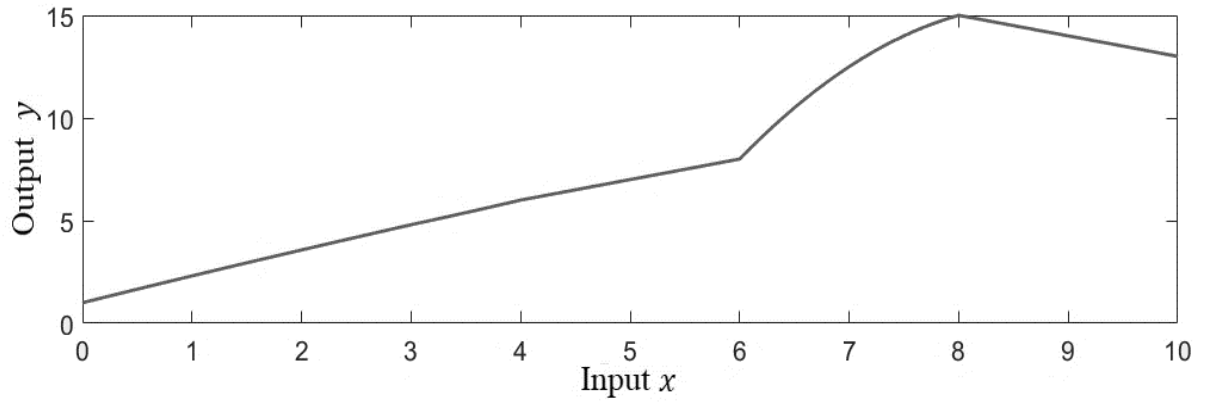


Figure 3.20: Output of first-order SISO TSK FIS as in experiment (C)

As the results shown in experiment (A), experiment (B) and experiment (C), noted that it is important to that consequent function of fuzzy rules need to meet both *Condition 2* and *Condition 3* of Theorem 3.1 for ensure TSK FIS i.e., y is monotonically increasing w.r.t x .

3.7.2 MISO Monotone TSK FIS with Consequent as First-Order Polynomial Function

In this section, simulated experiment is structured to ensure that *Condition 1* in Theorem 3.2 are fulfilled. The simulated experiment contemplates a first-order MISO TSK FIS featuring two inputs, represented as $y = f(x)$, where $x = (x_1, x_2) \in X = [0,5] \times [0,5]$. The T1 Trapezoidal FMF of input x_1 and x_2 are parameterized as $A_1^1 = A_2^1 = \mu_{i \in \{1,2\}}^1(x_i; 0,0,1,4)$, and $A_1^2 = A_2^2 = \mu_{i \in \{1,2\}}^2(x_i; 1,4,5,5)$, such the design of $A_i^{j_i}$ in X compliant with Remark 3.2. Noted that T1 Trapezoidal FMF design are the same for both input x_1 and x_2 as depicted in Figure 3.21. The TSK FIS in this experiment necessitates a total of $\prod_{i=1}^n m_i = 2 \times 2 = 4$ fuzzy rules, articulated in Equation 3.14. The output of the MISO Monotone TSK FIS is computed utilizing Equation 3.15.

$R^{(1,1)}$: If x_1 is A_1^1 and x_2 is A_2^1 then $y = b^{(1,1)}(\mathbf{x})$

$R^{(1,2)}$: If x_1 is A_1^1 and x_2 is A_2^2 then $y = b^{(1,2)}(\mathbf{x})$

$R^{(2,1)}$: If x_1 is A_1^2 and x_2 is A_2^1 then $y = b^{(2,1)}(\mathbf{x})$

$R^{(2,2)}$: If x_1 is A_1^2 and x_2 is A_2^2 then $y = b^{(2,2)}(\mathbf{x})$

Equation 3.14

$$f(x_1, x_2) = \frac{\sum_{j_1=1}^2 \sum_{j_2=1}^2 (\mu^{(j_1, j_2)}(\mathbf{x}) \times b^{(j_1, j_2)}(\mathbf{x}))}{\sum_{j_1=1}^2 \sum_{j_2=1}^2 \mu^{(j_1, j_2)}(\mathbf{x})}$$

Equation 3.15

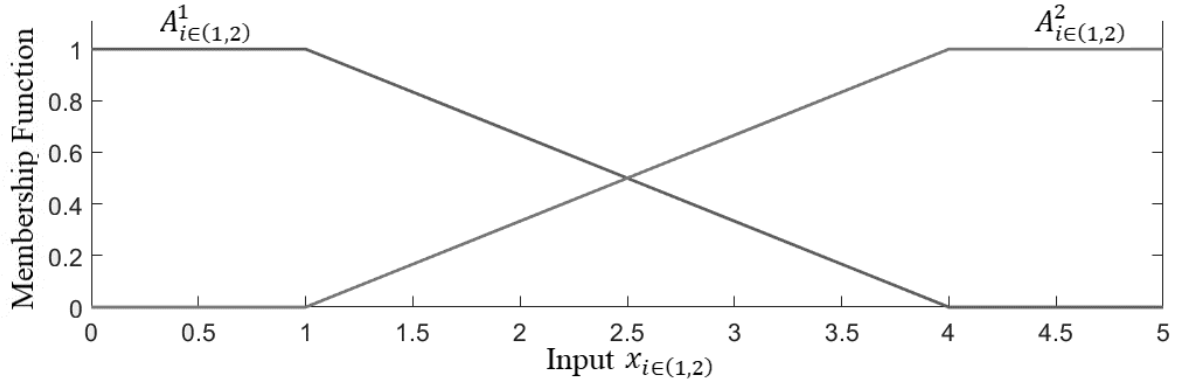


Figure 3.21: Input $x_{i \in \{1,2\}}$ of first-order MISO TSK FIS with T1 Trapezoidal FMF

As in Equation 3.14 the consequent of fuzzy rules i.e., $b^j(\mathbf{x}) = b^{(0,j)} + b_1^{(1,j)} \cdot x_1^1 + b_2^{(1,j)} \cdot x_2^1$ is a first order multivariate polynomial function of $\mathbf{x} = (x_1, x_2)$. The formulation of the consequent functions is governed by the implementation of the *Condition 2* and *Condition 3* as posited in Theorem 3.2. The simulated experiment is designed as such. Let's denote $b^j(\mathbf{x}) \in \{c^1(\mathbf{x}), c^2(\mathbf{x}), c^3(\mathbf{x}), c^4(\mathbf{x})\}$ with $c^{t=(1,2,3,4)}(\mathbf{x})$ as listed in Equation 3.16 to Equation 3.19. Noted that $c^{t=(1,2,3,4)}(\mathbf{x})$ fulfill an order i.e., $c^4(\mathbf{x}) \geq c^3(\mathbf{x}) \geq c^2(\mathbf{x}) \geq c^1(\mathbf{x})$ in range $\mathbf{x} = (x_1, x_2) \in X = [0,5] \times [0,5]$.

$$c^1(\mathbf{x}) = 1 + x_1 + x_2 \quad \text{Equation 3.16}$$

$$c^2(\mathbf{x}) = 2 + x_1 + x_2 \quad \text{Equation 3.17}$$

$$c^3(\mathbf{x}) = 3 + x_1 + x_2 \quad \text{Equation 3.18}$$

$$c^4(\mathbf{x}) = 15 + x_1 - x_2 \quad \text{Equation 3.19}$$

Consider that $c^3(\mathbf{x}) \geq c^2(\mathbf{x}) \geq c^1(\mathbf{x})$ should hold for the valid range $\mathbf{x} = (x_1, x_2) \in X = [0,5] \times [0,5]$. The evaluation focuses on validating $c^4(\mathbf{x}) \geq c^1(\mathbf{x})$, $c^4(\mathbf{x}) \geq c^2(\mathbf{x})$, and $c^4(\mathbf{x}) \geq c^3(\mathbf{x})$. As experiment $c^4(\mathbf{x}) \geq c^1(\mathbf{x})$, resolve inequalities $15 + x_1 - x_2 \geq 1 + x_1 + x_2$ should obtain result $x_2 \leq 7$, indicate $c^4(\mathbf{x}) \geq c^1(\mathbf{x})$ is valid in range $\mathbf{x} = (x_1, x_2) \in X = [0,5] \times [0,5]$. Apply same technique resolve inequalities $c^4(\mathbf{x}) \geq c^2(\mathbf{x})$ shall obtain result $x_2 \leq 6.5$ which indicate $c^4(\mathbf{x}) \geq c^2(\mathbf{x})$ is valid in range $\mathbf{x} = (x_1, x_2) \in X = [0,5] \times [0,5]$. Also, resolve inequalities $c^4(\mathbf{x}) \geq c^3(\mathbf{x})$ shall obtain result $x_2 \leq 6$ which indicate $c^4(\mathbf{x}) \geq c^3(\mathbf{x})$ is valid in range $\mathbf{x} = (x_1, x_2) \in X = [0,5] \times [0,5]$.

As in *Condition 2* in Theorem 3.2, first-derivative of $b^j(\mathbf{x}) = b^{(0,j)} + b_1^{(1,j)} \cdot x_1^1 + b_2^{(1,j)} \cdot x_2^1$ w.r.t x_i is derived as in Equation 3.20. Noted that Equation 3.16 to Equation 3.18 fulfilled the requirement as in Equation 3.20, i.e., $b_i^{(1,j)} \geq 0$ for $\forall x_{i \in \{1,2\}}$. However, first-derivative of $c^4(\mathbf{x})$ w.r.t to x_2 doesn't fulfill the requirement as in Equation 3.16 i.e., $d(c^4(\mathbf{x}))/dx_2 = d(15 + x_1 - x_2)/dx_2 = -1$ and $0 \geq d(c^4(\mathbf{x}))/dx_2$.

$$0 \leq \frac{d}{dx_i} (b^j(\mathbf{x})) = b_i^{(1,j)} \quad \text{Equation 3.20}$$

(A) Surface Plot

2 simulated experiments are demonstrated in this section, denoted as Experiment (A) and Experiment (B). In experiment (A), consequent function of $b^j(\mathbf{x})$ are structured such that both *Condition 2* and *Condition 3* in Theorem 3.2 are satisfied, i.e., $b_i^{(1,j)} \geq 0$ for $\forall x_{i \in \{1,2\}}$ and $b^{(2,2)}(\mathbf{x}) \geq b^{(2,1)}(\mathbf{x}) \geq b^{(1,2)}(\mathbf{x}) \geq b^{(1,1)}(\mathbf{x})$. Consequently $b^{(1,1)}(\mathbf{x}) =$

$c^1(\mathbf{x})$, $b^{(1,2)}(\mathbf{x}) = b^{(2,1)}(\mathbf{x}) = c^2(\mathbf{x})$ and $b^{(2,2)}(\mathbf{x}) = c^3(\mathbf{x})$ are selected as the consequent function of $b^j(\mathbf{x})$. As shown in Figure 3.22, a monotone output y w.r.t $\forall x_{i \in \{1,2\}}$ is obtained of the MISO TSK FIS.

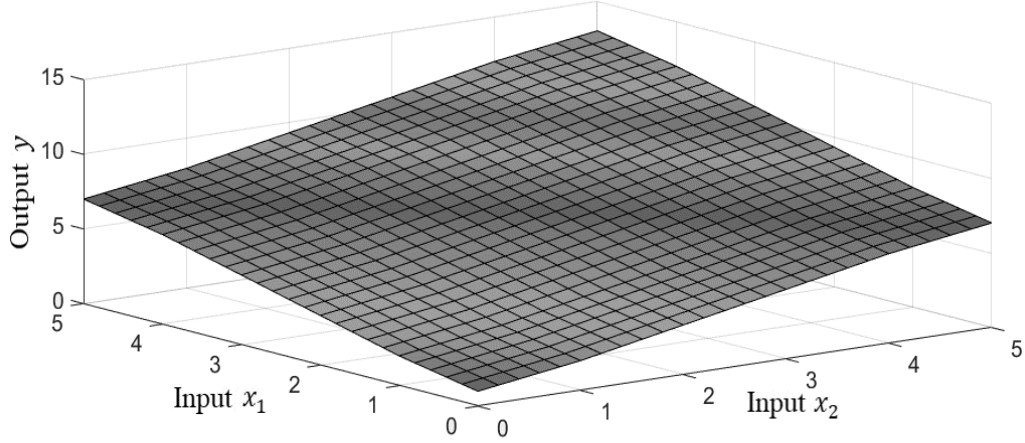


Figure 3.22: Output of first-order MISO TSK FIS with as in experiment (A)

In experiment (B), consequent function of $b^j(\mathbf{x})$ are designed in such only *Condition 2* Theorem 3.2 is met, whilst *Condition 3* is contravened. Specifically, $b^{(2,2)}(\mathbf{x}) \geq b^{(2,1)}(\mathbf{x}) \geq b^{(1,2)}(\mathbf{x}) \geq b^{(1,1)}(\mathbf{x})$ holds, but $b_i^{(1,j)} \geq 0$ is violate for $\forall x_{i \in \{1,2\}}$. Consequently, $b^{(1,1)}(\mathbf{x}) = c^1(\mathbf{x})$, $b^{(1,2)}(\mathbf{x}) = b^{(2,1)}(\mathbf{x}) = c^2(\mathbf{x})$ and $b^{(2,2)}(\mathbf{x}) = c^4(\mathbf{x})$ are adopted as the consequent functions of $b^j(\mathbf{x})$. As region highlighted in the circle in Figure 3.23, the non-monotone output y w.r.t $x_{i \in \{1,2\}}$ is obtained for the first-order MISO TSK FIS.

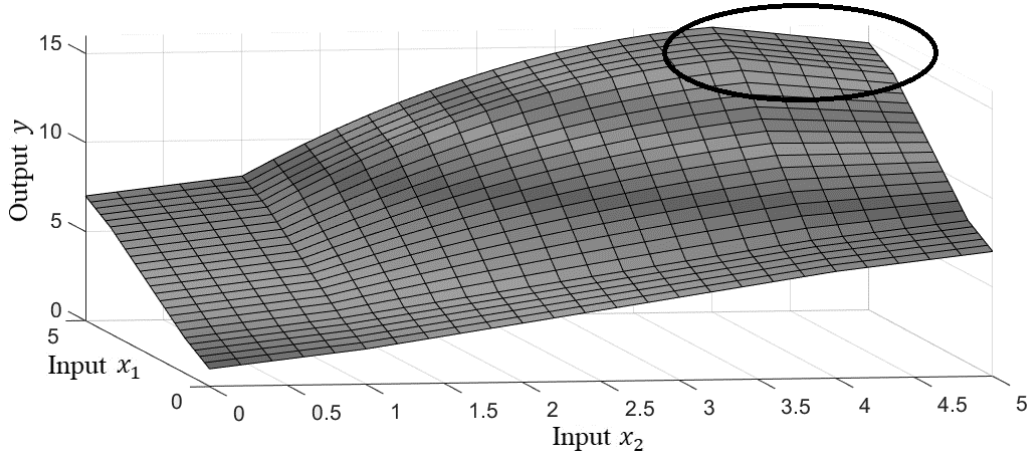


Figure 3.23: Output of first-order MISO TSK FIS with as in experiment (B)

(B) Monotonicity Test

A monotonicity test approach adopted from Tay and Lim (2011b), Tay et al. (2012a, 2012b) and Teh et al. (2018) is used to evaluate the output of MISO TSK FIS as in experiment (A) and experiment (B). The procedure is as follows.

Step 1: Divide input range of $X_{\forall i \in \{1,2\}} = [0, 5]$ into $n = 200$ subinterval. As such, fixed

step size as denoted as $\delta_{\forall i \in \{1,2\}} = \frac{(5-0)}{200} = 0.025$ form total spaced points

$\{x_{i,0}, \dots x_{i,m_i}, \dots x_{i,200}\} \in [0, 5]$ where $x_{i,m_i} = 0 + 0.025 \times m_i$.

Step 2: Let's denote $j = \{1,2\}$ and $j \neq i$, evaluate any pair $f(x_j, x_i)$ and $f(x_j, x'_i)$ as in

Equation 3.21, where $[x_i, x'_i] \in \{x_{i,0}, \dots x_{i,m_i}, \dots x_{i,200}\}$. For a selected $x_i = x_{i,m_i}$,

$x' = x_{i,m_i+1}$ should always true where $m_i < 200$.

$$\text{Monotone} \left(f(x_j, x_i), f(x_j, x'_i) \right) = \begin{cases} 1 & \text{if } f(x_j, x'_i) \geq f(x_j, x_i) \\ 0 & \text{otherwise} \end{cases} \quad \text{Equation 3.21}$$

Step 3: Evaluate $f(x)$ for all combination of grid points for $\forall i = \{1,2\}$ as in as in Equation

3.21. Compute the Monotone Index between y and inputs x_i as in Equation 3.22

Monotone Index (x_i)

$$= \frac{\sum_{m_{j \neq i}=0}^{m_{j \neq i}=n} \sum_{m_i=0}^{m_i=n-1} \left(\text{monotone} \left(f(x_j, x_i), f(x_j, x'_i) \right) \right)}{n(n+1)} \quad \text{Equation 3.22}$$

Based on the setting as in **Step 1** to **Step 3**, totally of $n(n+1) = 200(200+1) = 40200$ of compare-able pairs i.e., $f(x_j, x_i)$ and $f(x_j, x'_i)$ for each input x_i . Result of monotonicity test for MISO TSK FIS as in experiment (A) and experiment (B) are shown in Table 3.1 and Table 3.2, respectively. In experiment (A), monotone index of 1 is obtained for MISO TSK FIS as for both input x_1 and x_2 as shown Table 3.1. However, result in Table 3.2 shown MISO TSK FIS as in experiment (B) fail to obtain monotone index of 1 w.r.t input x_1 . The result consistent with non-monotone output y w.r.t $x_{i \in \{1,2\}}$ for MISO TSK FIS as shown in Figure 3.23.

Table 3.1: Results of experiment (A) with the monotonicity test

Input x_i	Number of compare-able pairs	Number of monotone compare-able pairs	Monotone Index (x_i)
x_1	40200	40200	$\frac{40200}{40200} = 1$
x_2	40200	40200	$\frac{40200}{40200} = 1$

Table 3.2: Results of experiment (B) with the monotonicity test

Input x_i	Number of compare-able pairs	Number of monotone compare-able pairs	Monotone Index (x_i)
x_1	40200	36200	$\frac{36200}{40200} = 0.9005$
x_2	40200	40200	$\frac{40200}{40200} = 1$

3.7.3 MISO Monotone TSK FIS with consequent as second-order polynomial function

In this section, a second-order MISO TSK FIS encompassing two inputs is examined, i.e., $y = f(\mathbf{x})$ where $\mathbf{x} = (x_1, x_2) \in X = [0,5] \times [0,5]$. The experiment adopted the T1 Trapezoidal FMF designs (as depicted in Figure 3.21) for (x_1, x_2) and fuzzy rules (as in Equation 3.14) in Section 3.7.2. The output of second-order MISO TSK FIS is denoted in Equation 3.10.

In this section, the consequent of fuzzy rules i.e., $b^j(\mathbf{x}) = b^{(0,j)} + \sum_{p=1}^2 b_1^{(p,j)} \cdot x_1^p + b_2^{(p,j)} \cdot x_2^p$ is a second order multivariate polynomial function of $\mathbf{x} = (x_1, x_2)$. Let's denote $b^j(\mathbf{x}) \in \{c^1(\mathbf{x}), c^2(\mathbf{x}), c^3(\mathbf{x})\}$ with $c^{t=(1,2,3)}(\mathbf{x})$ as listed in Equation 3.23 to Equation 3.25. Noted that, $c^t(\mathbf{x})$ is selected in such condition $0 \leq d(c^t(\mathbf{x}))/dx_i$ for $(x_1, x_2) = [0,5] \times [0,5]$. For $c^1(\mathbf{x})$, $d(x_1^2 + x_2^2)/dx_1 = 2x_1$ and $d(x_1^2 + x_2^2)/dx_2 = 2x_2$ where $0 \leq d(c^1(\mathbf{x}))/dx_i$ is hold for $(x_1, x_2) = [0,5] \times [0,5]$. In case $c^2(\mathbf{x})$, $d(c^2(\mathbf{x}))/dx_1 = 2x_1 + 2$ and $0 \leq d(c^2(\mathbf{x}))/dx_1$ is true for $x_1 \in [0,5]$ since $-1 \leq x_1$. Meanwhile, $d(c^2(\mathbf{x}))/dx_2 = 2x_1$ and $0 \leq d(c^2(\mathbf{x}))/dx_2$ is hold for $x_2 \in [0,5]$. For $c^3(\mathbf{x})$, consider that $d(c^3(\mathbf{x}))/dx_i = 0$ is always true for $\forall i = \{1,2\}$ and thus, $0 \leq d(c^3(\mathbf{x}))/dx_i$ should hold for both x_1 and x_2 .

$$c^1(\mathbf{x}) = x_1^2 + x_2^2 \quad \text{Equation 3.23}$$

$$c^2(\mathbf{x}) = 2 + x_1^2 + 2x_1 + x_2^2 \quad \text{Equation 3.24}$$

$$c^3(\mathbf{x}) = 80 \quad \text{Equation 3.25}$$

$c^{t=(1,2,3)}(\mathbf{x})$ fulfill an order i.e., $c^3(\mathbf{x}) \geq c^2(\mathbf{x}) \geq c^1(\mathbf{x})$ in range $(x_1, x_2) = [0,5] \times [0,5]$, which can be verified by solving the inequalities $c^2(\mathbf{x}) \geq c^1(\mathbf{x})$, $c^3(\mathbf{x}) \geq c^1(\mathbf{x})$ and $c^3(\mathbf{x}) \geq c^2(\mathbf{x})$ as follows. For $c^2(\mathbf{x}) \geq c^1(\mathbf{x})$, resolve the inequalities $2 + x_1^2 + 2x_1 + x_2^2 \geq x_1^2 + x_2^2$ should obtain $x_1 \geq -1$ and thus, $c^2(\mathbf{x}) \geq c^1(\mathbf{x})$ is true for $(x_1, x_2) = [0,5] \times [0,5]$. For $c^3(\mathbf{x}) \geq c^1(\mathbf{x})$. Consider $c^3(\mathbf{x}) \geq c^1(\mathbf{x})$ i.e., $80 \geq x_1^2 + x_2^2$, noted that $\sup(x_1^2 + x_2^2) = 5^2 + 5^2 = 50$ for $(x_1, x_2) = [0,5] \times [0,5]$. Thus, $c^3(\mathbf{x}) \geq c^1(\mathbf{x})$ should hold for $(x_1, x_2) = [0,5] \times [0,5]$. For $c^3(\mathbf{x}) \geq c^2(\mathbf{x})$, solve the inequalities $80 \geq 2 + x_1^2 + 2x_1 + x_2^2$, as such, $78 \geq x_1^2 + 2x_1 + x_2^2$ is obtained. Noted that, $\sup(x_1^2 + 2x_1 + x_2^2) = 5^2 + 2 \times 5 + 5^2 = 60$ for $(x_1, x_2) = [0,5] \times [0,5]$. Thus, $c^3(\mathbf{x}) \geq c^2(\mathbf{x})$ should hold for $(x_1, x_2) = [0,5] \times [0,5]$

(A) Surface Plot

Simulated experiments are demonstrated in this section. Consequent function of $b^j(\mathbf{x})$ are structured such that both *Condition 2* and *Condition 3* in Theorem 3.2 are satisfied, i.e., $0 \leq d(b^j(\mathbf{x}))/dx_i$ for $\forall x_{i \in \{1,2\}}$ and $b^{(2,2)}(\mathbf{x}) \geq b^{(2,1)}(\mathbf{x}) \geq b^{(1,2)}(\mathbf{x}) \geq b^{(1,1)}(\mathbf{x})$. Consequently, $b^{(1,1)}(\mathbf{x}) = c^1(\mathbf{x})$, $b^{(1,2)}(\mathbf{x}) = b^{(2,1)}(\mathbf{x}) = c^2(\mathbf{x})$ and $b^{(2,2)}(\mathbf{x}) = c^3(\mathbf{x})$ are selected as the consequent function of $b^j(\mathbf{x})$. As shown in Figure 3.24, a monotone output y w.r.t $\forall x_{i \in \{1,2\}}$ is obtained of the second-order MISO TSK FIS.

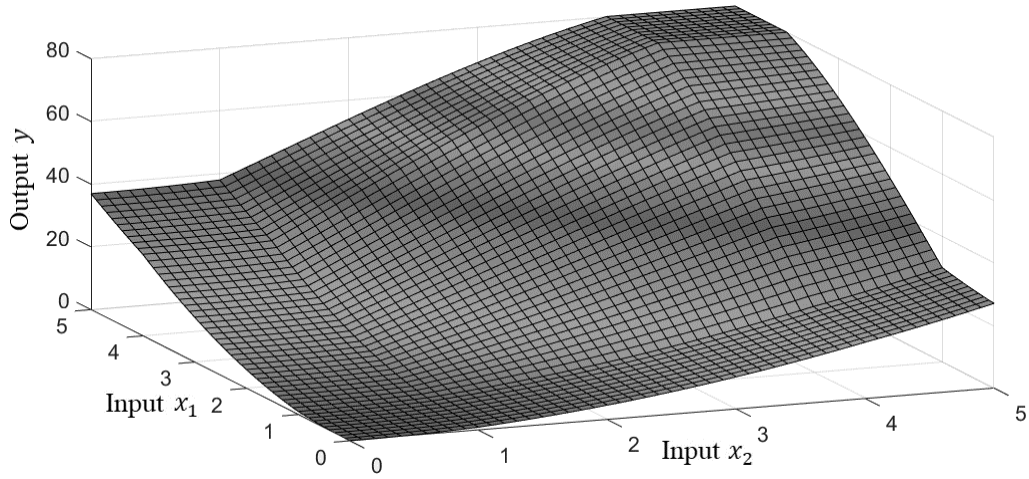


Figure 3.24: Output of the second-order MISO TSK FIS

(B) Monotonicity Test

The monotonicity test approach as in Section 3.7.2 adopted to examine the MISO TSK FIS. As shown in the result in Table 3.3, monotone index of 1 is obtained for MISO TSK FIS as for both input x_1 and x_2 .

Table 3.3: Results of the monotonicity test for the second-order MISO TSK FIS

Input x_i	Number of compare-able pairs	Number of monotone compare-able pairs	Monotone Index (x_i)
x_1	40200	40200	$\frac{40200}{40200} = 1$
x_2	40200	40200	$\frac{40200}{40200} = 1$

3.8 Summary

In Section 3.3, the sufficient conditions for monotone T1 SISO and MISO TSK FIS models are introduced. To ensure the preservation of the monotonicity property in TSK FIS, the works of Won et al. (2002), Tay and Lim (2011a), and Won and Karray (2014) have

been simplified into Theorem 3.1 and Theorem 3.2. These theorems are adopted as the governing equations for constructing monotone TSK FIS models. Section 3.4 demonstrates the design of T1 Trapezoidal FMF and T1 Gaussian FMF using the governing equations outlined in Theorem 3.1 and Theorem 3.2, providing practical examples of their application. In Section 3.5, these theorems are applied to formulate different consequent functions for fuzzy rules in monotone TSK FIS models, including first-order and second-order polynomial functions.

These sections collectively contribute to the achievement of RO1, which aims to analyze the parametric conditions required for TSK FIS to exhibit monotonicity, with a focus on functional consequents. Section 3.3 identifies and simplifies the necessary conditions (Theorem 3.1 and Theorem 3.2) to ensure monotonicity in TSK FIS models. Section 3.4 offers practical demonstrations of designing fuzzy membership functions (T1 Trapezoidal and T1 Gaussian FMF) that meet these conditions. Section 3.5 extends the application of the theorems to various consequent functions, showcasing their robustness and adaptability. In subsequent chapters, these monotonicity conditions are applied to a T1 TSK FIS-based RPN model, illustrating the effectiveness of using Theorems 3.1 and 3.2 to ensure monotonicity in practical applications.

The subsequent chapter introduces the application of monotonicity conditions to a T1 TSK FIS-based RPN model. The chapter illustrates the effectiveness of utilizing the mathematical conditions encapsulated in Theorems 3.1 and 3.2 to ensure monotonicity within a T1 TSK FIS-based RPN model.

CHAPTER 4

MONOTONE TYPE-1 FIS-BASED RPN MODELLING IN TRANSMISSION CAUSE AND EFFECT ANALYSIS (TCEA) FOR COMBATING COVID-19

4.1 Introduction

In the previous chapter, monotone T1 TSK FIS is outlined. In this chapter, a new Transmission Cause and Effect Analysis (TCEA) procedure is proposed. The use of monotone T1 TSK FIS as a part of TCEA procedure to combat COVID-19 is experimented. This chapter is organized as follows.

In Section 4.2, challenges and aims TCEA are discussed. Section 4.3 focuses on the representation of transmission risks within TCEA. The TCEA Procedure is presented in Section 4.4, providing an overview of the steps involved. In Section 4.5, the design of a monotone T1 FIS-based RPN is presented. Moving on to Section 4.6, a real-world case study is conducted to evaluate the usefulness of TCEA, using information collected from an IT company. The results and discussion of this case study are then presented in Section 4.7. Finally, Section 4.8 summarizes the chapter, highlighting the key points and conclusions that have been discussed throughout.

4.2 Background, Challenges and Aims

COVID-19 is a highly infectious respiratory illness caused by the SARS-CoV-2 virus with first cases was reported in Wuhan, China in late 2019 and has since become a global pandemic (Ciotti et al., 2020). It spreads mainly through respiratory droplets and close contact with infected individuals, and its symptoms range from mild to severe, including fever, cough, and difficulty breathing. A variety of preventive measures that aim to reduce

or prevent transmission of the coronavirus, such as social distancing and isolation (World Health Organization, 2021; Tomar and Gupta, 2020), wearing mask and/or personal protective equipment (PPE) (WHO, 2021), maintaining personal hygiene (World Health Organization, 2021), and lockdown (Tomar and Gupta, 2020) have been practiced worldwide. In this study, the *transmission risk* of an infectious disease is perceived as a combination of the presence of a harmful pathogen (i.e., a microorganism, e.g., a SARS-CoV-2) with the requisite factors that lead to infection of a human (e.g., COVID-19) and the *aftermath* of infection (Peterson, 2014) (e.g., health and socio-economy impact). Factors such as human population, population activities, and population immunity are considered to be modifiers of the basic distribution of the pathogen leading to disease transmission (Peterson, 2014).

The section discusses the challenges of understanding and reducing the transmission risk of infectious diseases like COVID-19 in a real-world environment by utilizing systems engineering tools and methods. To address these challenges, a systematic approach called Transmission Cause and Effect Analysis (TCEA) is proposed. TCEA involves utilizing a causal map of possible transmission potentials (TPs), transmission causes (TCs), transmission effects (TEs), and evaluating the severity, occurrence, and detection ratings of preventive and control actions. A monotone T1 FIS-based RPN model is used to prioritize preventive and control actions based on their effectiveness. Literature search (Jee et al., 2015; Kerk et al., 2015; Pang et al., 2016; Kerk et al., 2019) reports T1 FIS-based RPN model should satisfy the monotonicity property to ensure rationality of the produced fuzzy RPN (FRPN) score. On this regard, Theorem 3.2 from Chapter 3 are adopted for facilitating the Monotone T1 FIS-based RPN modelling in TCEA.

4.3 Representation of Transmission Risks

An TCEA has goals pertaining to reducing the transmission risk of infectious diseases (i.e., COVID-19) in a real-world environment. In formulating an effective solution for undertaking the challenge, a proposed causal map is adopted to represent the possible transmission scenarios within a real-world environment in a form of transmission causes (TCs), possible transmission potentials (TPs), transmission effects (TEs), preventive and control actions as well as their associated severity (Sev), occurrence (Occ), and detection (Det) ratings. The Sev, Occ, and Det scale tables are defined for evaluating the severity of TEs, the likelihood of TCs, and the effectiveness of relevant prevention and control actions, respectively.

4.3.1 A Proposed Causal Map of Disease Transmission

A causal map is a concept representation to capture causality or influence relationships (Huff 1990). It has been widely employed in structuring intervention related problems (Scavarda et al., 2006; Chaib-draa, 2002), serving as an effective tool to represent subjective knowledge about a phenomenon, a discourse on perceived causes and effects (Scavarda et al., 2006).

The proposed causal map is depicted in Figure 4.1. In practice, it is important to consider the possible transmission modes (Nelson and Williams, 2014) during the brainstorming session for TPs. This is particularly crucial when it is difficult to identify the transmission modes of an infectious disease (Han et al., 2009; Tellier et al., 2019). In addition, the transmission modes of an infectious disease can be dynamic, due to evolutionary mechanism of the disease (Antonovics et al, 2017). The same arguments apply to TCs.

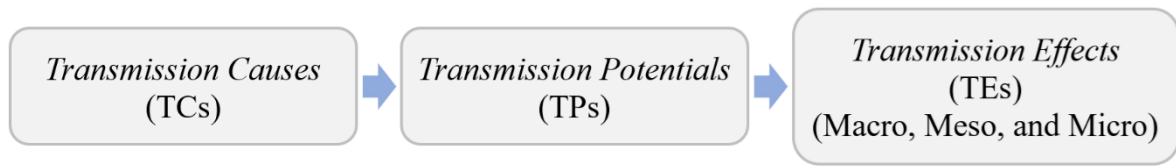


Figure 4.1: The proposed causal map of disease transmission

Three types of TEs are available, i.e., micro (direct), meso (indirect), and macro (indirect and global). In this study, the micro TEs covers the direct effects of disease transmission to human health. Meso TEs are indirect effects of disease transmission to the operations of an organization, while macro TEs cover potential global effects to the social, economic, political and other concerns.

The use of causal representations together with reasoning and inferencing methods in infection diseases modelling is not new. As an example, a causal diagram (Pearl, 2000; Pearl and Mackenzie, 2018) was constructed to represent the phenomenon of small-pox and its vaccination. Instead of representing a scientific phenomenon, in this study, a causal map is employed to represent qualitative and subjective knowledge for allowing TCs, TPs, and TEs to be identified by a taskforce involved in making decision.

In this respect, the use of the Ishikawa diagram or fishbone diagram together with the principles of 5Ms, 8Ps, and 4Ss to identify TCs constitutes an interesting direction. The 5Ms framework includes Manpower, Methods, Materials, Machines, and Measurement, focusing on human factors, processes, raw materials, equipment, and data. The 8Ps framework covers Product, Price, Place, Promotion, People, Process, Physical evidence, and Performance, addressing various aspects of marketing and operational factors. The 4Ss framework consists of Surroundings, Suppliers, Systems, and Skills, highlighting environmental conditions, external providers, organizational systems, and competencies.

Utilizing these comprehensive frameworks allows for a detailed analysis of potential causes, aiding in the identification and mitigation of transmission risks.

4.3.2 The Scale Table of Severity (Sev), Occurrence (Occ), and Detection (Det)

Table 4.1 to Table 4.3 show an example of the Sev, Occ, Det scales for our case study, respectively. Sev is a rating of how severe a TE is; Occ is a rating of the occurrence of a TC; while Det is a rating of how effective a corrective/preventive action is. Each scale table consists of three columns, i.e., “Ranking”, “Linguistic term”, and “Descriptions”, while the score intervals (from 1 to 10) are shown in “Ranking”.

The Sev scale table is designed to rate the TEs pertaining to a susceptible host. For simplicity, Table 4.1 is designed based on a risk stratification method with respect to vital health indicators, lifestyle, and medical history of interest. In accordance with Wolff et al. (2021) the possible risk factors of morbidity and mortality caused by COVID-19 can be divided into five categories, i.e., lifestyle factors, demographic factors, pre-existing comorbidities, developed comorbidities, and clinical factors.

Table 4.1: Scale for Severity (Sev)

Ranking	Linguistic term	Descriptions
1-3	Low	• Young workers maintain health practices and standard precautions ➤ Non-smoker, no excessive alcohol intake ➤ Regular contact with healthcare providers, maintain healthy routines ➤ Good public hygiene • Young workers clean and disinfect frequently touched objects and surfaces
4-7	Medium	• Workers without health issues • Potential objects susceptible to contamination
8-9	High	• Senior age group (e.g., >60 years old), regardless of any medical conditions • Pregnant women • Worker with health issues (e.g., diabetic, hypertension, cardiovascular diseases, chronic lung diseases, cerebrovascular disease, chronic liver diseases, chronic kidney disease, malignancy)
10	Very High	• Senior age group with health conditions issues • Pregnant women with health issues

The scale table of Occ is developed to rate the likelihood of TC occurrences. Table 4.2 is designed with two considerations: (i) the confidence level to prevent humans/objects/surfaces infected or contaminated by the COVID-19 virus from entering to a company premise; and (ii) the judgement of the likelihood for humans/objects to transmit the COVID-19 virus to other humans/objects, considering the nature of daily work activities and social interactions.

Table 4.2: Scale for Occurrence (Occ)

Ranking	Linguistic term	Descriptions
1-3	Low	• High confidence that all humans are not infected • High confidence that objects /surfaces are not contaminated • Close communication can be avoided.
4-6	Moderate	• High confidence that all humans are not infected • High confidence that objects /surfaces are not contaminated • Close communication can be hardly avoided.
7-8	High	• Low confidence that all humans are not infected. • Objects/surfaces contamination can hardly be avoided • Close communication can hardly be avoided
9-10	Very High	• Low confidence that all humans are not infected • Objects/surfaces contamination can hardly be avoided • Confined spaces, crowded places, and close communication

Table 4.3 is formulated to evaluate the effectiveness of the proposed strategy. Factors that influence the effectiveness are (i) screening protocols of risks and symptoms (Gostic *et al*, 2020), (ii) cleaning and disinfection protocols (Fathizadeh et al., 2020), and (iii) PPE (Ranney et al., 2020).

Table 4.3: Scale for Detection (Det)

Ranking	Linguistic term	Descriptions
1	Very High	Detection is very good • Single-use objects/surfaces • Traveling to high risk geographical location can be totally avoided. • Face-to-face interaction and communication can be totally avoided.
2-3	High	Detection is good • Objects/ Surfaces which can be effectively cleaned, disposed, or replaced without special handling • Travelling staff members aware of exposure risk, within Sarawak. • Face-to-face interaction and communication cannot be totally avoided, but social distancing with preventive protocols (e.g. face shield/mask) can be implemented.
4-7	Moderate	Manageable detection • Disposable objects that need special handling (e.g., hand gloves) • Travelling staff members aware of exposure risk, within the nation
8-9	Low	Hard to detect. • Non-disposable objects that cannot be disinfected • International travelling staff members aware of exposure risk.
10	Very low	Detection is almost impossible. • Non-disposable objects that cannot be disinfected, special handling needed. • International travelling staff with regular employee travel members aware of exposure risk.

4.4 The TCEA Procedure

The TCEA procedure is presented in Figure 4.2. This procedure can be adapted for various processes, designs, or services to align with the specific needs of different industries and businesses. The detailed steps, from Step 1 to Step 15, provide a comprehensive guide for implementation and customization.

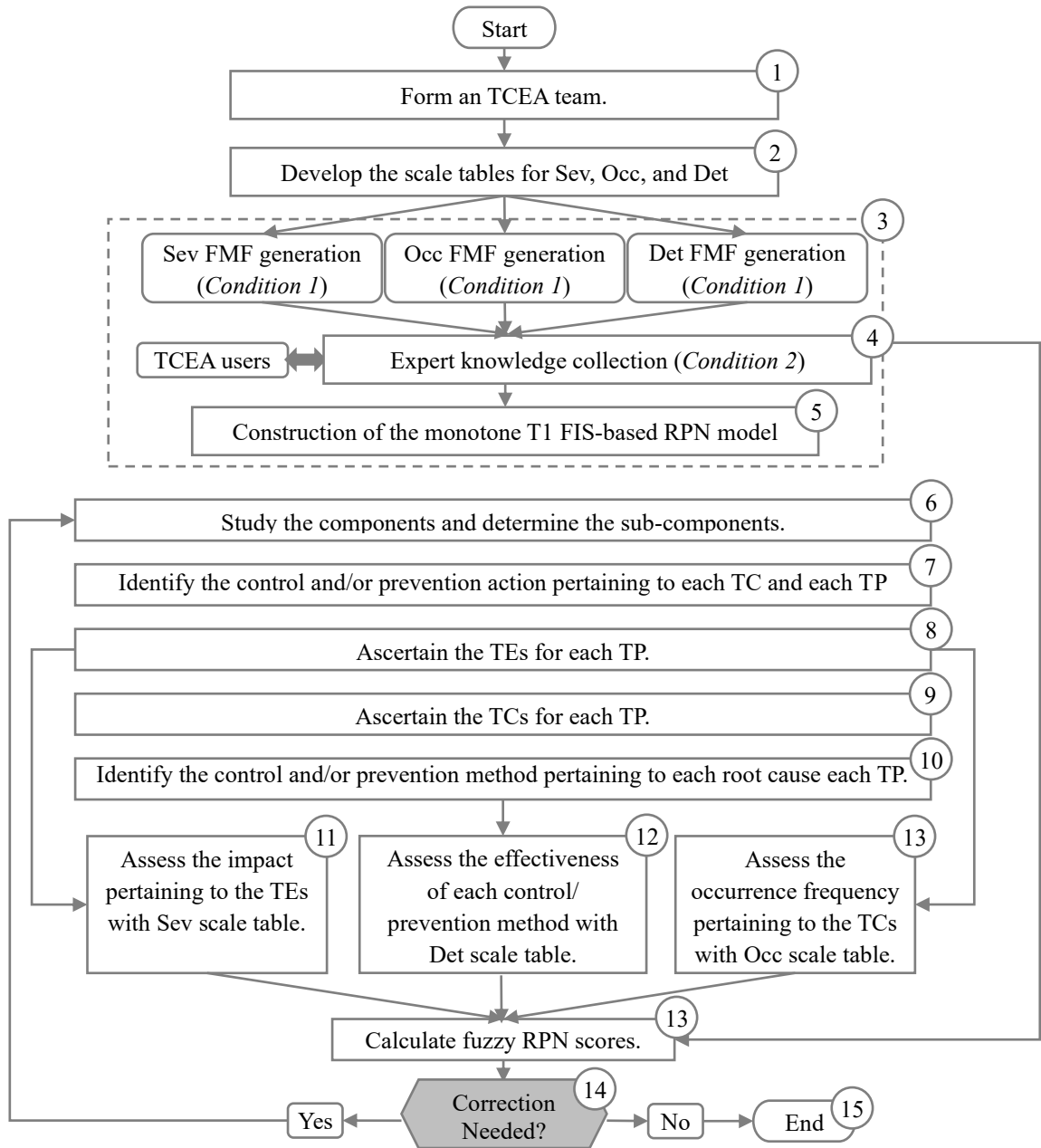


Figure 4.2: The TCEA procedure

Step-1: Form a team to perform TCEA activities.

Step-2: Develop the scale tables for Sev, Occ, and Det. In this step, the general descriptions are assigned to each item in the scale tables of Sev, Occ, and Det.

Step-3: Construct the FMF for each input factor (i.e., Sev, Occ and Det). *Condition 1* (Refer to Theorem 3.2 in Chapter 3) is used as the governing equation (as detailed in Section 4.5.2).

Step-4: Gather expert knowledge to construct the fuzzy rule base. (Refer to Theorem 3.2 in Section 3.3.2) is imposed in the construction phase (as detailed in Section 4.5.3).

Step-5: Construct the monotone T1 FIS-based RPN model (as detailed in Section 4.5.1).

Step-6: Study the components and determine the sub-components.

Step-7: Ascertain the TPs pertaining to the sub-components.

Step-8: Ascertain the TEs for each TP.

Step-9: Ascertain the TCs for each TP.

Step-10: Identify the control and/or prevention action pertaining to each TC and each TP.

Step-11-12: Assess the TPs with the Sev, Occ, and Det scale tables.

Step 13: Calculate the FRPN scores for each TP is computed with monotone T1 FIS-based RPN model

Step-14: Determine the corrective actions. If any corrective action is needed, go to Step 3.

Step-15: End.

4.5 The Design of Monotone T1 FIS-based RPN

In this section, the proposed TCEA procedure (as shown in Figure 4.2) is applied. A few important steps in the TCEA procedure are detailed.

4.5.1 An Monotone T1 FIS-based RPN Model

In this chapter, the monotone T1 FIS-based RPN model is adopted as a quality and transmission risk assessment tool in the proposed TCEA procedure. Let's $\mathbf{x} = (\text{Sev}, \text{Occ}, \text{Det})$, the output of monotone T1 FIS-based RPN model is a FRPN score denoted by $\text{FRPN} = f(\mathbf{x})$. Each input is assigned with m_i fuzzy sets, i.e., $A^1, A^{j_i}, \dots, A^{m_i}$ with $i \in \{\text{Sev}, \text{Occ}, \text{Det}\}$ and the membership function of A^{j_i} is denoted by $\mu^{j_i}(x_i)$. The fuzzy rule R^J is explained as a mapping from to c , i.e., $R^J: A^J \rightarrow b^J$ where $J = (j_{\text{Sev}}, j_{\text{Occ}}, j_{\text{Det}})$, firing strength of A^J denoted by $\mu^J = \mu^{j_{\text{Sev}}} \times \mu^{j_{\text{Occ}}} \times \mu^{j_{\text{Det}}}$ and b^J is fuzzy singletons serve as the consequent of fuzzy rules R^J .

The relationship between the FRPN score and Sev, Occ and Det is represented by a set of If-Then fuzzy rules as shown in Figure 4.3. The total number of fuzzy rules required is denoted by $\prod_i^n m_i$ where $n \in \{\text{Sev}, \text{Occ}, \text{Det}\}$. The FRPN score is calculate using Equation 4.1

$R^J: \text{If } (\text{Sev is } A^{j_{\text{Sev}}}) \text{ and } (\text{Occ is } A^{j_{\text{Occ}}}) \text{ and } (\text{Det is } A^{j_{\text{Det}}}) \text{ then FRPN is } b^J$

Figure 4.3: If-Then fuzzy rules of a monotone T1 FIS-Based RPN model

$$\text{FRPN} = \frac{\sum_{j_{\text{Sev}}=1}^{m_{\text{Sev}}} \sum_{j_{\text{Occ}}=1}^{m_{\text{Occ}}} \sum_{j_{\text{Det}}=1}^{m_{\text{Det}}} (\mu^J \times b^J)}{\sum_{j_{\text{Sev}}=1}^{m_{\text{Sev}}} \sum_{j_{\text{Occ}}=1}^{m_{\text{Occ}}} \sum_{j_{\text{Det}}=1}^{m_{\text{Det}}} \mu^J} \quad \text{Equation 4.1}$$

4.5.2 The Fuzzy Membership Function Design

The scale tables for Sev, Occ, and Det in Table 4.1, Table 4.2, and Table 4.3 respectively, are being referenced. The FMF design for Sev, Occ and Det adhering to Theorem 3.2 in Section 3.3.2, can be seen in Figure 4.4, Figure 4.5 and Figure 4.6, respectively.

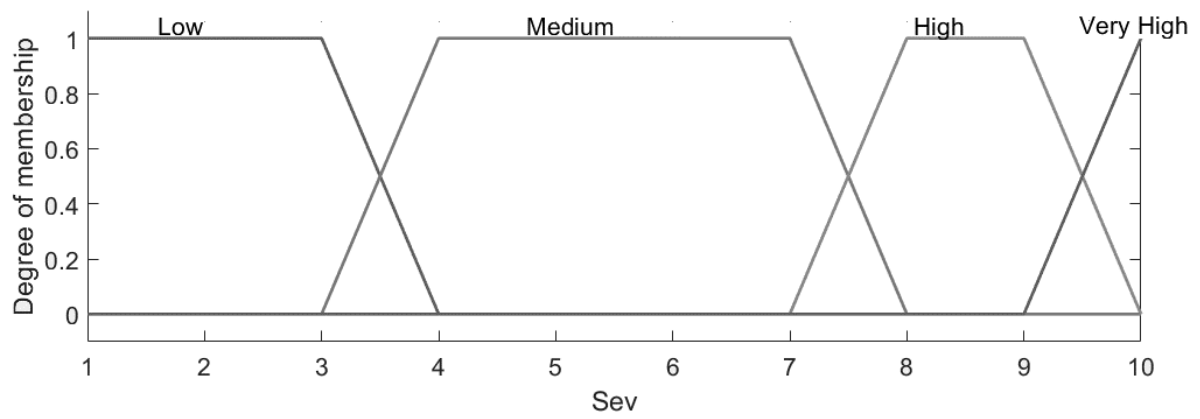


Figure 4.4: FMF design of Sev

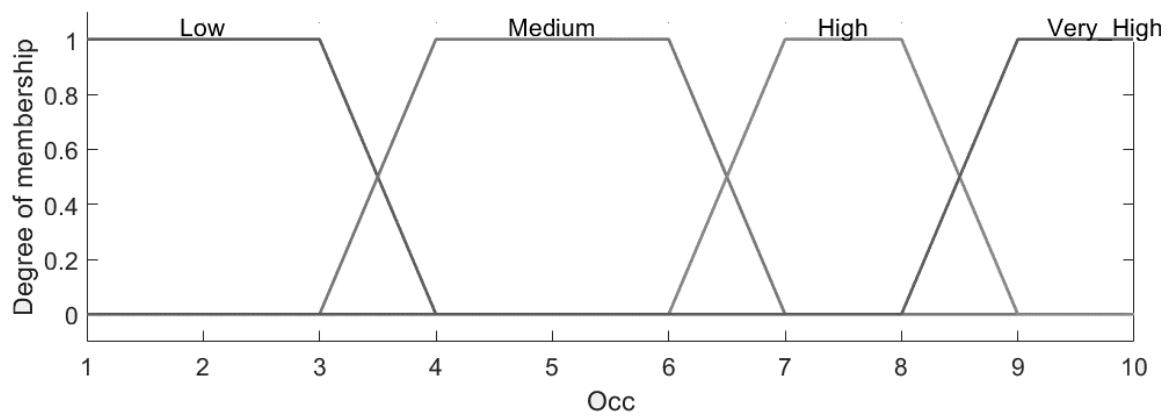


Figure 4.5: FMF design of Occ

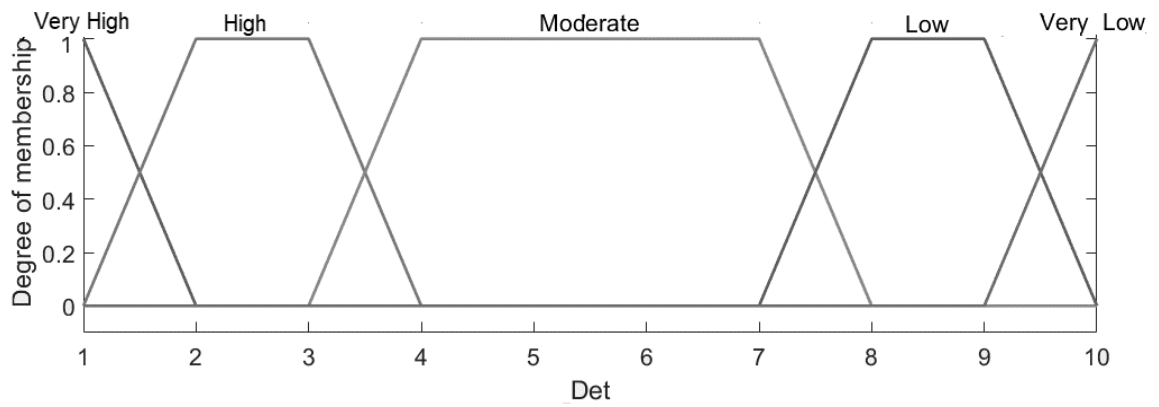


Figure 4.6: FMF design of Det

4.5.3 Fuzzy Rule Gathering

In this chapter, the FRPN score (i.e., $FRPN \in [1, 1000]$) for the monotone T1 FIS-based RPN model is divided into 7 equal partitions. Each partition is represented with a triangular FMF, namely, *Extremely Low*, *Very Low*, *Low*, *Medium*, *High*, *Very High* and *Extremely High*, respectively, as depicted in Figure 4.7. For simplicity, each FMF is represented by a fuzzy singleton i.e., 1.0, 167.5, 334.0, 500.5, 667.0, 833.5 and 1000.0.

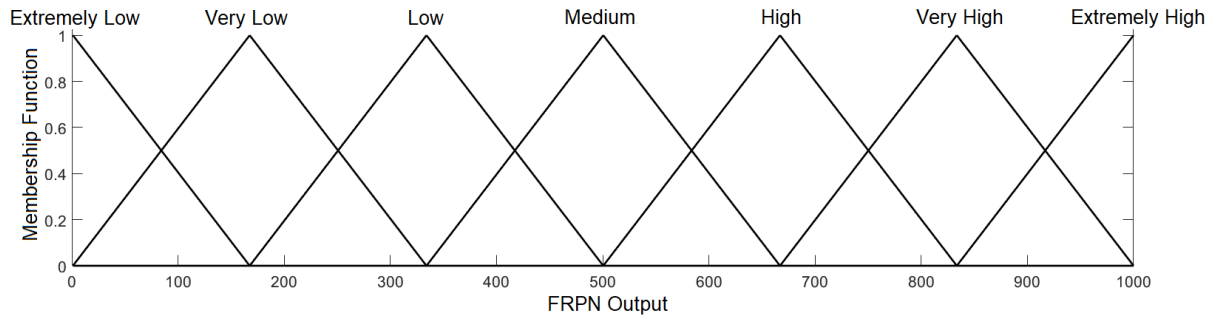


Figure 4.7: The fuzzy sets for the RPN output

The fuzzy rules are designed in such a way that *Condition 2* of Theorem 3.2 (refer to in Section 3.3.2) is satisfied. To evaluate the effectiveness of the proposed approach, a total of 80 fuzzy rules (i.e., $\prod_i^n m_i = 4 \times 4 \times 5$) are gathered from a TCEA expert. Figure 4.8 presents 20 If-Then fuzzy rules extracted from the original set of 80 fuzzy rules at the condition "Sev is Low" is met.

Fuzzy Rules

- 1 If Sev is Low and Occ is Low and Det is Very High Then FRPN is Extremely Low
- 2 If Sev is Low and Occ is Low and Det is High Then FRPN is Very Low
- 3 If Sev is Low and Occ is Low and Det is Moderate Then FRPN is Low
- 4 If Sev is Low and Occ is Low and Det is Low Then FRPN is Low
- 5 If Sev is Low and Occ is Low and Det is Very Low Then FRPN is Low
- 6 If Sev is Low and Occ is Medium and Det is Very High Then FRPN is Very Low
- 7 If Sev is Low and Occ is Medium and Det is High Then FRPN is Very Low
- 8 If Sev is Low and Occ is Medium and Det is Moderate Then FRPN is Low
- 9 If Sev is Low and Occ is Medium and Det is Low Then FRPN is Low
- 10 If Sev is Low and Occ is Medium and Det is Very Low Then FRPN is Medium

- 11 If Sev is Low and Occ is High and Det is Very High Then FRPN is Low
- 12 If Sev is Low and Occ is High and Det is High Then FRPN is Low
- 13 If Sev is Low and Occ is High and Det is Moderate Then FRPN is Low
- 14 If Sev is Low and Occ is High and Det is Low Then FRPN is Medium
- 15 If Sev is Low and Occ is High and Det is Very Low Then FRPN is High
- 16 If Sev is Low and Occ is Very High and Det is Very High Then FRPN is Low
- 17 If Sev is Low and Occ is Very High and Det is High Then FRPN is Low
- 18 If Sev is Low and Occ is Very High and Det is Moderate Then FRPN is Medium
- 19 If Sev is Low and Occ is Very High and Det is Low Then FRPN is High
- 20 If Sev is Low and Occ is Very High and Det is Very Low Then FRPN is High

Figure 4.8: Extracted 20 Fuzzy rules at the Sev is Low

4.5.4 Surface Plots

Figure 4.9 presents the surface plot for the FRPN score versus Occ and Det at Sev is 10. Figure 4.10 shows the surface plot for the FRPN score versus Sev and Det with Occ is 10. Figure 4.11 further depicts the surface plot for the FRPN scores versus Sev and Occ with Det is 10. As can be seen in Figure 4.9, Figure 4.10 and Figure 4.11, the surface plots shown the T1 FIS-Based RPN model demonstrate monotonic output (i.e., FRPN score) w.r.t the inputs Sev, Occ and Det at specified conditions.

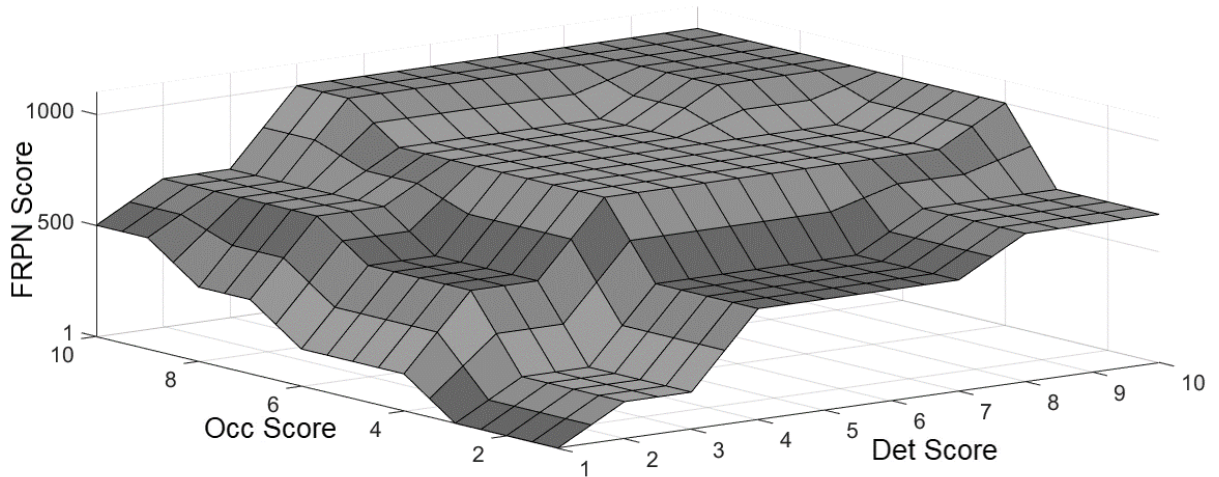


Figure 4.9: Surface plot for FRPN versus Occ and Det t with at Sev is 10.

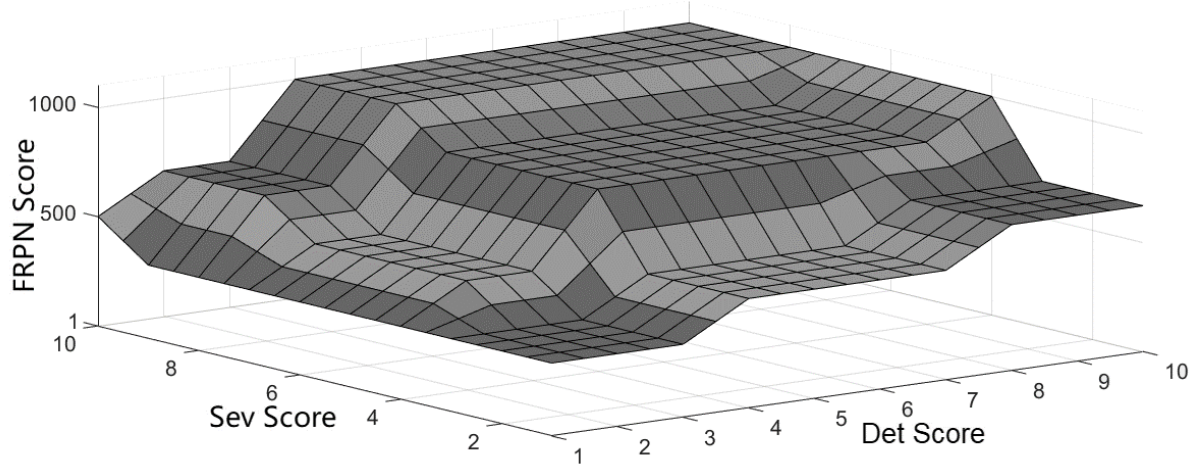


Figure 4.10: Surface plot for FRPN versus Sev and Det with Occ is 10

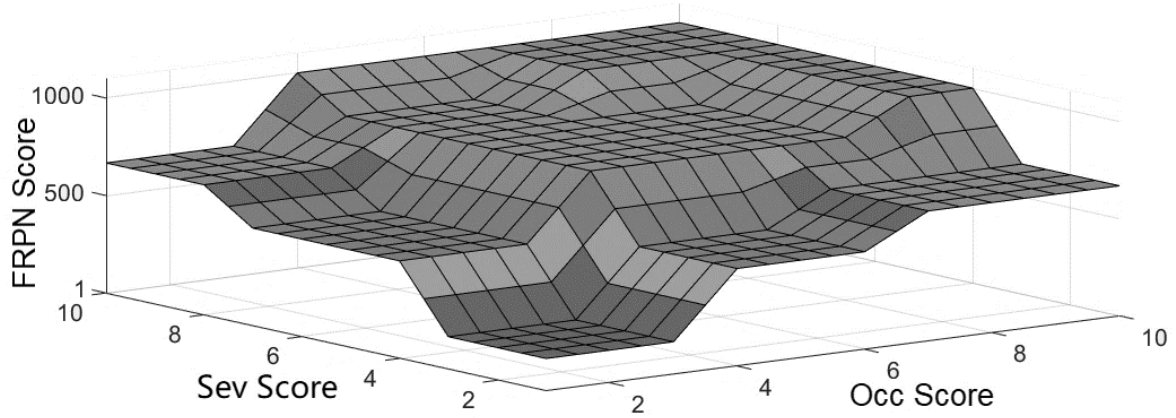


Figure 4.11: Surface plot for FRPN versus Sev and Det with Occ is 10

4.5.5 A Monotonicity Test

In this section, monotonicity test approach adopted from Tay and Lim (2011), Tay et al (2012a, 2012b) and Teh et al. (2018) is used to evaluate/test the degree of fulfilment of the monotonicity property of the monotone T1 FIS-based RPN denoted $FRPN = f(\mathbf{x})$ where $\mathbf{x} = (\text{Sev}, \text{Occ}, \text{Det})$. The procedure is as follows.

Step 1: Divide input range of $X_{\forall i \in \{\text{Sev}, \text{Occ}, \text{Det}\}} = [1, 10]$ into $n = 900$ subinterval. As such,

fixed step size as denoted as $\delta_{\forall i \in \{\text{Sev}, \text{Occ}, \text{Det}\}} = (10 - 1)/900 = 0.01$ form total spaced points $\{x_{i,0}, \dots x_{i,m_i}, \dots x_{i,900}\} \in [1, 10]$ where $x_{i,m_i} = 1 + 0.025 \times m_i$.

Step 2: Let's denote $i = \{\text{Sev}, \text{Occ}, \text{Det}\}$ and $\mathbf{s}$ as subset of $\mathbf{x}$ where x_i is excluded from $\mathbf{s}$ (i.e. $\mathbf{s} \subset \mathbf{x}; x_i \notin \mathbf{s}$). Evaluate any pair $f(\mathbf{s}, x_i)$ and $f(\mathbf{s}, x'_i)$ as in Equation 4.2, where $[x_i, x'_i] \in \{x_{i,0}, \dots x_{i,m_i}, \dots x_{i,900}\}$. For a selected $x_i = x_{i,m_i}$, $x' = x_{i,m_i+1}$ should always true where $m_i < 900$.

$$\text{monotone}(f(x_j, x_i), f(x_j, x'_i)) = \begin{cases} 1 & \text{if } f(\mathbf{s}, x'_i) \geq f(\mathbf{s}, x_i) \\ 0 & \text{otherwise} \end{cases} \quad \text{Equation 4.2}$$

Step 3: Evaluate $f(\mathbf{x})$ for all combination of grid points for $\forall i = \{\text{Sev}, \text{Occ}, \text{Det}\}$ as in as in Equation 3.21. Compute the Monotone Index between y and inputs x_i as in Equation 4.3. Noted that $\{i, j, k\} \in \{\text{Sev}, \text{Occ}, \text{Det}\}$ where $j \neq i$ and $k \neq i$

$$\begin{aligned} & \text{Monotone Index } (x_i) \\ &= \frac{\sum_{m_k=0}^{m_k=n} \sum_{m_j=0}^{m_j=n} \sum_{m_i=0}^{m_i=n-1} (\text{monotone}(f(\mathbf{s}, x'_i) \geq f(\mathbf{s}, x_i)))}{n(n+1)^2} \end{aligned} \quad \text{Equation 4.3}$$

Based on the setting as in Step 1 to Step 3, totally of $n(n+1)^2 = 900(900+1)^2 = 730620900$ of compare-able pairs i.e., $f(\mathbf{s}, x'_i)$ and $f(\mathbf{s}, x_i)$ for each input x_i . Table 4.4 presents the results of the monotonicity test for $x_{i \in \{\text{Sev}, \text{Occ}, \text{Det}\}}$. The results show that Sev, Occ, and Det have a monotone index of 1. For instance, there are 730620900 compare-able pairs available for x_{Sev} , and all of them satisfy the monotonicity property. Therefore, the monotonicity index of 1 is obtained, which is equal to $730620900/730620900 = 1$.

Table 4.4: Results with the monotonicity test

x_i	Number of compare-able pairs	Number of monotone compare-able pairs	Monotone Index (x_i)
x_{Sev}	730620900	730620900	$\frac{730620900}{730620900} = 1$
x_{Occ}	730620900	730620900	$\frac{730620900}{730620900} = 1$
x_{Det}	730620900	730620900	$\frac{730620900}{730620900} = 1$

4.6 Case Study

To evaluate the usefulness of TCEA, a real-world case study with information collected from an IT (information technology) company is conducted. The company employs a total of 10 staff members, i.e., 9 male and 1 female, ranging from 21 to 33 years old. The company office occupies a total of 1920ft². As depicted in Figure 4.12, it consists of a computer lab (Lab 1), a system integration lab (Lab 2), office of the Chief Technical Officer (CTO) (i.e., CTO Office), a discussion room, a server room, a pantry, a restroom, and a document room.

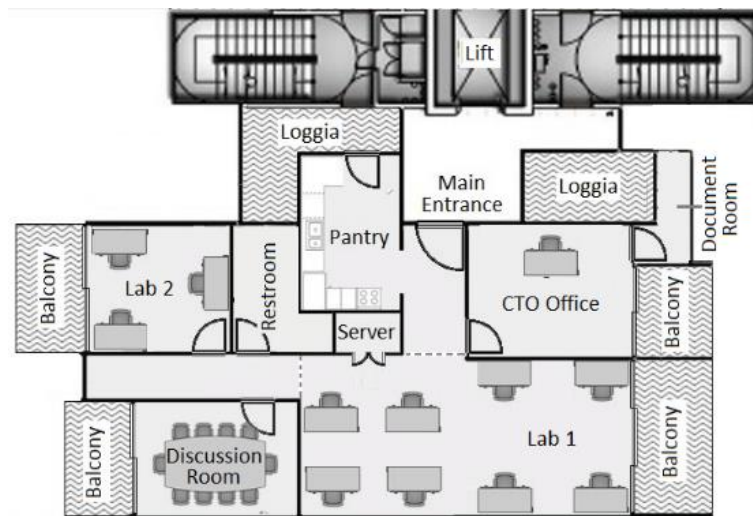


Figure 4.12: The floorplan of the ICT company

In this case study, four possible transmission modes are identified for COVID-19 prevention, i.e., contact, droplet, and fomite, as well as, airborne, transmission modes. are the scale Table 4.1, Table 4.2 and Table 4.3 for Sev, Occ, and Det ratings. The TCEA spreadsheet consists of 11 columns, i.e., “Area/Room”, “Function and Description”, “ID”, “Transmissions Potentials (TPs)”, “Transmission Effects (TEs)”, “Transmission Causes (TCs)”, “Control/prevention action”, and Sev, Occ, Det ratings as well as RPN and FRPN score. To have a better understanding, the TCEA spreadsheet is presented in two tables, i.e., Table 4.5 and Table 4.6. Table 4.5 covers the key workspaces, i.e., Lab 1/ Lab 2, Discussion Room, Server Room and CTO Office, while Table 4.6 covers the lift, main entrance, pantry, restroom, and document room.

Table 4.5: TCEA spreadsheet for workplace

Area/ Room	Functions and description	ID	Transmission Potentials (TPs)	Transmission Effects (TEs)	Sev	Transmission Causes (TCs)	Occ	Control/prevention strategy (s)	Det	RPN	FRPN
Lab 1/ Lab 2	Computer Lab with workstations	TP1.1	Social interaction in close distance, which involve 2 and more staff members	Infected, limited to the young and low risk group	2	- Close distance conversation without wearing a mask. - Physical social interaction (e.g., handshake)	4	- Restrict unnecessary conversation among staff members without wearing a mask. - Each staff member has a personal office desk with a partition board installed. - Strictly enforce no social interaction in workplace. - Work from home; rotating office hours routine to keep minimum workforce in the office; shorten the working hours in office. - Daily body temperature scan to ensure the visitors have no symptom of fever (body temperature > 37.5°C)	3	24	167.0
		TP1.2	Sharing of office stationery	Infected, limited to the young and low risk group	1	Contamination of stationery by infected staff members	2	- Restrict stationary sharing among staff members - Clean and sanitize stationery after use	1	2	1.0
		TP1.3	Passing papers and documents among staff members.	Infected, limited to the young and low risk group	1	Contamination of paper and document by infected staff members	2	- Implement a paperless office - Assign tasks through emails and social media, information sharing through electronic documents only (words or PDF)	1	2	1.0
		TP1.4	Room, office furniture and workstations contamination with infectious agents (e.g., office tables, chairs, computers)	Infected, limited to the young and low risk group	1	Contamination caused by infected staff members	5	- Each staff member has a personal office desk (with a partition board installed) and chair; no sharing allowed. - Advise staff members clean their seat at least once a day.	2	10	167.0
		TP1.5	Necessary entry of contactors to the premise for facilities and utilities maintenance	Infected, limited to the young and low risk group	3	Failure of basic utilities that lead to necessary entry of contractors for repair or maintenance	8	- Provide masks to the contactors - Body temperature scan to ensure the visitors have no symptom of fever (body temperature > 37.5°C)	5	120	334.0

			• Air-conditional repair and maintenance • Power utility and electric wiring • Telecommunication services					• Clean the area after the repair work.			
Discussion Room	To support small group (max 4 staff members) face-to-face meeting	TP1.6	Social interaction in close distance, which involve 2 and more staff members	Infected, limited to the young and low risk group	2	• Close distance conversation • Physical social interaction (e.g., handshake)	4	• Strictly enforce social distancing >1.5m • Strictly enforce mask wearing in the meeting room. • Restricted access by non-employees. • Strictly enforce no physical social interaction in workplace. • Restrict the meeting duration within 30 minutes • Use of video conferencing facilities instead • Daily body temperature scan to ensure the visitors have no symptom of fever (body temperature > 37.5°C)	1	8	167.0
		TP1.7	Sharing of office stationery	Infected, limited to the young and low risk group	1	• Contamination of stationery by infected staff members	2	• Restrict stationery sharing among staff members • Clean and sanitize e stationery after use	1	2	1.0
		TP1.8	Passing papers and documents among staff members	Infected, limited to the young and low risk group	1	• Contamination of paper and document by infected staff members	2	• Avoid hardcopy document sharing during discussion. • Implement a paperless office e.g., information sharing through electronic documents only (words or PDF), use only PPT presentation in the discussion room.	1	2	1.0
		TP1.9	Room and office furniture contamination with infectious agents (e.g., door handles, floor, meeting tables and chairs)	Infected, limited to the young and low risk group	1	• Contamination caused by potential infected staff members	5	• Clean the room at least twice a week. • Clean and sanitize office furniture after use	2	10	167.0
		TP1.10	Necessary entry of contractors to the premise for facilities	Infected, limited to the young and low risk group	3	• Failure of basic utilities that lead to necessary entry of contractors for repair or maintenance	8	• Provide masks to the contractors • Body temperature scan to ensure the visitor has no	5	120	334.0

			and utilities maintenance • Air-conditional repair and maintenance • Power utility and electric wiring • Telecommunication services					• symptom of fever (body temperature > 37.5°C) • Clean the area after the repair work.			
CTO Office	Personnel office of CTO	TP1.11	Social interaction in close distance, involve 2 and more staff members	Infected, limited to the young and low risk group	2	• Close distance conversation without wearing a mask • Physical social interaction (e.g., handshake)	4	• Restrict unnecessary conversation among staff members without wearing a mask. • Strictly enforce no physical social interaction in workplace.	2	16	167.0
		TP1.12	Passing papers and documents among staff members	Infected, limited to the young and low risk group	1	• Contamination of papers and documents by infected staff members	2	• Implement a paperless office • Assign tasks through emails and social media, information sharing through electronic documents only (words or PDF)	1	2	1.0
		TP1.13	Room, office furniture and computer contamination with infectious agents (e.g., door handles, office tables and chairs)	Infected, limited to the young and low risk group	1	• Contamination caused by potential infected staff members	5	• Clean the room at least twice a week.	2	10	167.0
		TP1.14	Necessary entry of contractors to the premise for facilities and utilities maintenance • Air-conditional repair and maintenance • Power utility and electric wiring • Telecommunication services	Infected, limited to the young and low risk group	3	• Failure of basic utilities that lead to necessary entry of contractors for repair or maintenance	8	• Provide masks to the contractors • Body temperature scan to ensure the visitors have no symptom of fever (body temperature > 37.5°C) • Clean the area after the repair work.	5	120	334.0
Server Room	Mainframe server room for research support	TP1.15	Potential of server room contamination with infectious agents.	Infected, limited to the young and low risk group (technicians)	1	• Potential infected staff members	1	• Staff members need to use hand gloves when conducting physical operations in the Server room	1	1	1.0

		TP1.16	Potential of repair tools and equipment contamination with infectious agents.	Infected, limited to the young and low risk group(technicians)	1	Potential infected staff members	1	Clean and sanitize the repair tools and equipment after use	1	1	1.0
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Table 4.6: TCEA spreadsheet for other facilities

Area/ Room	Functions and description	ID	Transmission Potentials (TPs)	Transmission Effects (TEs)	Sev	Transmission Causes (TCs)	Occ	Control/prevention strategy (s)	Det	RPN	FRPN
Shared-Lift	Shared facility of the building used to transport both people and goods from one floor to another	TP2.1	Infected non employees /outsiders using the shared lift	Infected, limited to the young and low risk group	3	Sharing lift with non-employees	9	- Restrict unnecessary conversation in lift without wearing a mask. - Allow maximum 2 staff members only to access the lift at any one time - Strictly enforce no physical social interaction in lift. - Sanitize both hands and the lift button after use.	3	81	334.0
Main entrance	Main entrance of the office	TP2.2	Social interaction in close distance, which involve 2 and more staff members	Infected, limited to the young and low risk group	2	- Close distance conversation without wearing a mask. - Physical social interaction (e.g., handshake)	4	- Restrict unnecessary conversation at the entrance without wearing a mask. - Avoid overcrowding at the entrance by strictly enforcing social distancing >1m	1	8	167.0
		TP2.3	Contamination of the area with infectious agents (door handle of the entrance, floor, etc.)	Infected, limited to the young and low risk group	1	Potential infected staff members	3	- Clean and sanitize both hands before entering to office/workplace. - Clean the area at least once a week.	1	3	1.0
Pantry	Dining area for staff, equipped with kitchenware and basic appliances	TP2.4	Potential of contamination of the tableware and kitchenware with infectious agents	Infected, limited to the young and low risk group	1	- Sharing of tableware with infected staff members - Sharing of kitchenware with infected staff members	1	- Only allow one staff member enter the pantry each time. - Restrict sharing of tableware - Clean and sanitize kitchenware after use.	1	1	1.0
		TP2.5	Room and kitchen furniture contamination with infectious agents (e.g., dining tables and chairs)	Infected, limited to the young and low risk group	1	Contamination caused by potential infected staff members	6	- Clean after use. - Clean the area at least twice a week.	3	18	167.0

		TP2.6	Necessary entry of contractors to the premise for facilities and utilities maintenance - Power utility and electric wiring - Water supply and piping service	Infected, limited to the young and low risk group	3	Failure of basic utilities that lead to necessary entry of contractors for repair or maintenance	8	- Provide masks to the contractors - Body temperature scan to ensure the visitors have no symptom of fever (body temperature > 37.5°C) - Clean the area after the repair work.	5	120	334.0
Restroom	Toilet facilities	TP2.7	Room contamination (e.g., toilet, faucet handle and door handle) with infectious agents	Infected, limited to the young and low risk group	1	Potential infected staff members	4	- Clean the restroom twice a day - Clean the toilet and faucet handle after use - Clean and sanitize door handle, toilet and sink twice a day - Only allow one staff member enter the restroom each time.	2	8	167.0
		TP2.8	Necessary entry of contractors to the premise for facilities and utilities maintenance - Power utility and electric wiring - Water supply and piping service	Infected, limited to the young and low risk group	3	Failure of basic utilities that lead to necessary entry of contractors for repair or maintenance	8	- Provide masks to the contractors - Body temperature scan to ensure the visitors have no symptom of fever (body temperature > 37.5°C) - Clean the area after the repair work.	5	120	334.0
Document room	Room for storing important office document.	TP2.9	Potential of contamination of the documents and files with infectious agents	Infected, limited to the young and low risk group	1	Contamination of documents and files by infected staff members	1	- Restrict access to the document room - Implement a paperless office; backup important documents at a local server for intranet access of staff members	1	1	1.0
		TP2.10	Potential of room contamination (e.g., door handle, wall, floor) with infectious agents.	Infected, limited to the young and low risk group	1	Potential infected staff members	1	- Restrict access to the document room - Implement a paperless office; backup important documents at a local server for intranet access of staff members	1	1	1.0

4.6.1 The TCEA Spreadsheet

A total of 26 TPs are identified, together with the corresponding TEs (tagged with Sev ratings), TCs (tagged with Occ ratings), and control or prevention actions (tagged with Det ratings). Figure 4.13 summarizes the Sev, Occ, and Det ratings for all TPs. From Figure 4.13, there are 16 TEs with Sev rating of 1, 4 TEs with Sev rating of 2, and 6 TEs with Sev rating of 3. The Sev ratings are low (from 1 to 3) because all staff members are from the low-risk group, i.e., young and non-smokers.

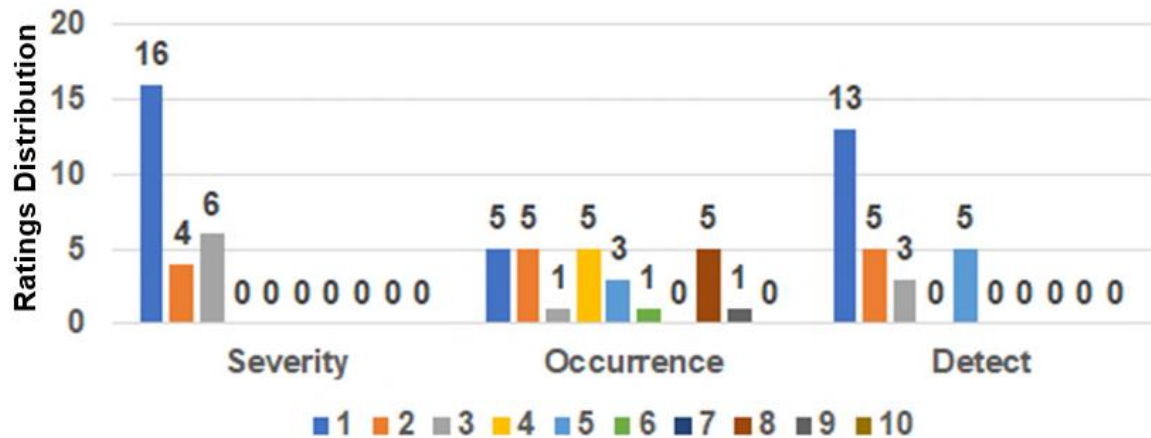


Figure 4.13: Distributions of Sev, Occ, and Det ratings

Besides that, the most frequent Occ ratings are 1, 2, 4 and 8, each with 5 TCs. This is followed by Occ ratings of 5 for 3 TCs, and of 3, 6 and 9 (each with 1 TC). High Occ ratings indicate that not all TCs for COVID-19 can be totally prevented with the standard measures if the company is operating during the pandemic period. As such, additional transmission-based control and preventive actions are necessary. From Figure 4.13, a total of 21 control and preventive actions are associated with Det ratings from 1 to 3. However, there are 5 control and prevention actions associated with high Det rating of 8.

4.6.2 Activities of High Occ Ratings

From Table 4.7, Occ ratings of 8 to 10 reflect a low confidence that humans, objects, or surfaces are not infected or not contaminated by COVID-19 virus. Table 4.7 summaries a list of TPs with Occ ratings that fall between 8 and 10. Two identified TCs with high Occ ratings are “sharing lift with non-employees” and “failure of basic utilities and facilities that lead to necessary entry of contractors for repair or maintenance”. Both TCs involve non-employees. The lift, which is a small and confined space, is shared with non-employees or contractors of other companies; hence the confidence level is low. The second TC indicates necessary entry of contractors of other companies to the premise for maintenance services; hence the confidence level is low too.

Table 4.7: Potential Root causes with High Occ ratings

TPs	Occ	TCs	Main activity area
TP2.2	9	Sharing lift with non-employees	Shared Lift
TP1.5 TP1.10 TP1.15 TP2.7 TP2.9	8	Failure of basic utilities and facilities that lead to necessary entry of contractors for repair or maintenance	Lab 1/ Lab 2, Discussion, Room, CTO Office, Pantry, Restroom

4.6.3 Analysis of Transmission Risks

Figure 4.14 and Table 4.8 depict qualitative analysis of transmission risks for all TPs pertaining to Sev and Occ. A matrix of Sev versus Occ is established, which can be divided into five categories, i.e., very low risk (light green), low risk (light blue), acceptable risk (yellow), high risk (orange), and very high risk (light red). There are total of 19 TPs associated with the risk level of “Very Low”, 1 TP with the risk level of “Low”, and 6 TPs

with the risk level of “Acceptable”. This analysis indicates that TP with the highest risk (which is still acceptable) appears at the shared lift, which is used by outsiders.

Occ \ Sev	1	2	3	4	5	6	7	8	9	10	Color	Risk Level
1	5	5	1	1	3	1	-	-	-	-		Very Low
2	-	-	-	4	-	-	-	-	-	-		
3	-	-	-	-	-	-	-	5	1	-		Low
4	-	-	-	-	-	-	-	-	-	-		
5	-	-	-	-	-	-	-	-	-	-		Acceptable
6	-	-	-	-	-	-	-	-	-	-		
7	-	-	-	-	-	-	-	-	-	-		High
8	-	-	-	-	-	-	-	-	-	-		
9	-	-	-	-	-	-	-	-	-	-		Very High
10	-	-	-	-	-	-	-	-	-	-		

Figure 4.14: A risk matrix of Sev and Occ ratings

Table 4.8: Summaries of the risk levels for all TPs

Sev	Occ	Risk Level	List of TPs
1	1	Very low	• Server Room : TP1.15, TP1.16 • Pantry : TP2.4 • Document room : TP2.9, TP2.10
1	2	Very low	• Lab 1/ Lab 2 : TP1.2, TP1.3 • Discussion Room : TP1.7, TP1.8 • CTO Office : TP1.12
1	3	Very low	• Main entrance : TP2.3
1	4	Very low	• Restroom : TP2.7
1	5	Very low	• Lab 1/ Lab 2 : TP1.4 • Discussion Room : TP1.9 • CTO Office : TP1.13
1	6	Very low	• Pantry : TP2.5
2	4	Very low	• Lab 1/ Lab 2 : TP1.1 • Discussion Room : TP1.6 • CTO Office : TP1.11

			• Main entrance : TP2.2
3	8	Acceptable	• Lab 1/ Lab 2 : TP1.5 • Discussion Room : TP1.10 • CTO Office : TP1.14 • Pantry : TP2.6 • Restroom : TP2.8
3	9	Acceptable	• Lift : TP2.1

4.6.4 Prioritization of TPs Using RPN and FRPN Scores

Prioritization of TPs in the TCEA spreadsheet (i.e., Table 4.5 and Table 4.6) is summarized, as presented in Table 4.9 and Table 4.10, respectively, in a descending order. Column “Transmission Potentials” explains TPs; column “Area/Room” explains the location in which TPs occur. The ratings for Sev, Occ, Det, RPN scores, and FRPN scores are provided in the columns labelled as “Sev”, “Occ”, “Det”, "RPN", and "FRPN", respectively.

From Table 4.9, TP1.5, TP1.10 and TP1.14 are related to “necessary entry of contactors to the premise for facilities and utilities maintenance”. They have the highest RPN score and FRPN score (i.e., RPN = 120 and FRPN = 334.0), which cover Labs 1 and 2, CTO Office, and the Discussion room. One solution is to conduct the maintenance work after office hours or weekend to prevent or minimize contacts between staff members and contactors. This is followed by TP1.1, TP1.11, TP1.4, TP1.9 and TP1.13, with the RPN scores ranging from 8 and 24, and an identical FRPN score of 167. The remaining TPs are associated with low RPN scores of less than 8, and an identical FRPN score of 1. TPs associated with the Server room have the lowest RPN, as it is a restricted area, and only

authorized technical staff can access the Server room. In addition, server maintenance tasks are usually conducted via a local area network.

In Table 4.10, TP2.6 and TP2.8 are related to “necessary entry of contactors to the premise for facilities and utilities maintenance”, and they have the highest RPN scores (i.e., RPN=120). This is followed by TP2.1 “Infected non-employees/personnel’s using shared lift”, which is associated with an RPN of 81. This is because the lift is shared with outsiders (non-employees). Interestingly, all three of these TPs, TP2.7, TP2.9, and TP2.1, share the same FRPN score of 334.0. The next group of TPs exhibit RPN scores between 8 to 18, with a consistent FRPN score of 167. TP2.5 have low RPN score and FRPN score because staff bring their own tableware to prevent sharing. TP2.7 has a low RPN score and FRPN score as a single-user restroom is administered. TP2.3, TP2.4, TP2.9, and TP2.10 exhibit RPN scores ranging from 1 to 3, and an identical FRPN score of 1. TP2.9 and TP2.10 have very low RPN scores and FRPN scores because most of the documents are in softcopies.

Table 4.9: Summary of the TCEA spreadsheet Table 4.5

ID	<i>Transmission Potentials (TPs)</i>	<i>Area/Room</i>	Sev	Occ	Det	RPN	FRPN
TP1.5	Necessary entry of contactors to the premise for facilities and utilities maintenance	Lab 1/ Lab 2	3	8	5	120	334.0
TP1.10	Necessary entry of contactors to the premise for facilities and utilities maintenance	Discussion Room	3	8	5	120	334.0
TP1.14	Necessary entry of contactors to the premise for facilities and utilities maintenance	CTO Office	3	8	5	120	334.0
TP1.1	Close distance social interaction which involves 2 and more staff.	Lab 1/ Lab 2	2	4	3	24	167.0
TP1.11	Close distance social interaction which involves 2 and more staff.	CTO Office	2	4	2	16	167.0
TP1.4	Room and office furniture contamination with infectious agents (e.g., office table and chair)	Lab 1/ Lab 2	1	5	2	10	167.0
TP1.9	Room and office furniture contamination with infectious agents (E.g., door handle, floor, meeting table and chair)	Discussion Room	1	5	2	10	167.0
TP1.13	Room and office furniture contamination with infectious agents (E.g., door handle office desk and chair)	CTO Office	1	5	2	10	167.0
TP1.6	Close distance social interaction which involve 2 and more staff.	Discussion Room	2	4	1	8	167.0

TP1.2	Sharing of office stationery	Lab 1/ Lab 2	1	2	1	2	1.0
TP1.3	Passing papers and document between staffs	Lab 1/ Lab 2	1	2	1	2	1.0
TP1.7	Sharing of office stationery	Discussion Room	1	2	1	2	1.0
TP1.8	Passing paper and document between staff	Discussion Room	1	2	1	2	1.0
TP1.12	Passing paper and document between staff	CTO Office	1	2	1	2	1.0
TP1.15	Potential of server room contamination with infectious agents.	Server Room	1	1	1	1	1.0
TP1.16	Potential of repair tools and equipment contamination with infectious agents.	Server Room	1	1	1	1	1.0

Table 4.10: A summary of the TCEA spreadsheet

ID	<i>Transmission Potentials (TPs)</i>	<i>Area/Room</i>	Sev	Occ	Det	RPN	FRPN
TP2.6	Necessary entry of contactors to the premise for facilities and utilities maintenance	Pantry	3	8	5	120	334.0
TP2.8	Necessary entry of contactors to the premise for facilities and utilities maintenance	Restroom	3	8	5	120	334.0
TP2.1	Infected non-employees/personnel's using shared lift	Shared-Lift	3	9	3	81	334.0
TP2.5	Room and kitchen furniture contamination with infectious agents (e.g., dining table and chair)	Pantry	1	6	3	18	167.0
TP2.2	Close distance social interaction which involve 2 and more staff.	Main entrance	2	4	1	8	167.0
TP2.7	Room contamination (e.g., toilet, faucet handle and door hander) with infectious agents	Restroom	1	4	2	8	167.0
TP2.3	Contamination of the area with infectious agents (door hander of the entrance, floor, etc.)	Main entrance	1	3	1	3	1.0
TP2.4	Potential of contamination of the tableware and kitchenware with infectious agents	Pantry	1	1	1	1	1.0
TP2.9	Potential of contamination of the document and files with infectious agents	Document room	1	1	1	1	1.0
TP2.10	Potential of room contamination (e.g., door hander, wall, floor) with infectious agents.	Document room	1	1	1	1	1.0

4.7 Discussion

As demonstrated in the case study, TCEA considers TPs for risk assessment and decision making of COVID-19 management. The study is important, as quoted from Morawska et al., (2020) “During the rapid rise in COVID-19 illnesses and deaths globally, and not withstanding recommended precautions, questions are voiced about routes of transmission for this pandemic disease”. All qualitative and subjective knowledge and information related to COVID-19 prevention are successfully modelled through TPs, TCs and TEs with the casual map. The Sev, Occ, and Det scales, which correspond to the TCs,

TEs and control and/or prevention actions, respectively, can be rated and compared using a scheme from 1 to 10. Then, the RPN model allows all TPs to be prioritized for decision-making purposes. The outcomes indicate that TCEA serves as a useful risk analysis and management tool for prevention of COVID-19 (Aven and Zio, 2021).

The case study highlights important potential risks for COVID-19 transmission in real-world environments. As an example, “Necessary entry of contactors to the premise for facilities and utilities maintenance” due to “Failure of basic utilities” appears to be an important TP for the ICT company. The TCEA outcome is in line with the suggestions on high exposure risks associated with workplaces (World Health Organization, 2020), as quoted: “Jobs that may fall under this category include domestic workers, social care workers, personal transport and home delivery providers and home repair technicians (plumbers, electricians) who have to provide services in the homes of people with COVID-19.”

Using the proposed TCEA approach, the respective control and preventive actions can be established. Importantly, TCEA is relatively simple to use, which requires minimal time and training, facilitating its implementation in real-world environments.

4.8 Summary

In this chapter, a new TCEA methodology is proposed for better management of transmission risks of infectious diseases (e.g., COVID-19) in real-world environments. Inspired by FMEA, TCEA adopts a systematic and easy-to-implement procedure that incorporates a causal map, a set of Sev, Occ, and Det rating and their scales, and a monotone T1 FIS-based RPN model. This methodology is intended for risk analysis and management

of infectious diseases in dynamic situations. A case study for COVID-19 management in a real-world environment has been conducted, demonstrating the effectiveness of the proposed TCEA methodology in understanding and analyzing transmission risks, thereby achieving the RO2 of applying the parametric conditions of monotone T1 TSK FIS in a systematic approach for COVID-19 prevention. The results indicate that the methodology effectively meets this objective by providing a structured approach to assess and mitigate transmission risks.

The subsequent chapter demonstrates the application of the theoretical concepts of monotonicity in T1 TSK FIS into actionable insights and applications within image processing, specifically edge detection in analyzing digitized mammograms.

CHAPTER 5

MONOTONE T1 FIS MODELLING FOR EDGE DETECTION

5.1 Introduction

In the preceding chapter, an application of monotone T1 TSK FIS to TCEA is outlined. In this chapter, the use of monotone T1 TSK FIS to an image processing application is presented. The focus is on the edge detection framework of Bezdek et al. (1998). The edge detection framework with T1 monotone FIS is examined with digitized mammogram of a normal patient obtained from the database of the Mammographic Image Analysis Society (MIAS) and transforms the digitized mammogram to edge images. This chapter is organized as follows.

Section 5.2 introduces the fundamentals and background of object edge detection. The edge detection framework proposed by Bezdek et al. (1998) is detailed in Section 5.3. Section 5.4 delves into the utilization of waterfall blending functions as a monotonic function. In Section 5.5, the monotonic T1 TSK FIS is applied within the context of fuzzy blending functions. Finally, Section 5.6 showcases the application of the edge detection framework to digitized mammograms.

5.2 Background

Extracting the object edges in the images one of the most important steps in object identification (Bustince et al., 2010). Search in the literature in report of use of object edges detection in the problem of automatic object identification on images (Juszczyk et al., 2015; Benninghoff and Garcke, 2016; Ferraz et al., 2016; Malof et al., 2016; Karaali et al., 2017).

Among many edge detection methods to extract the object edges in an image, the gradient vector to represent intensity jumps between pixels, as for example the well-known approaches by Sobel and Feldman (1968), Prewitt (1970) and Canny (1986), being considered important operators in extracting the object edges in an image. In this chapter, demonstration adopted edge detection framework of Bezdek et al. (1998) and the edge detection framework is detailed in next section.

5.3 Edge Detection Framework (Bezdek et al., 1998)

Let's denote $IJ = \{(i, j): i = 1, 2, \dots, m; j = 1, 2, \dots, n\} \in \mathfrak{R}$ as a spatial location (i.e., pixel address) in space of rectangular array with size $m \times n$, i.e., $\mathfrak{R}$. Here, (i, j) is represented as ij for easy explanation. Only digitized grayscale raw input image is considered in this chapter i.e., P_{IJ} with quantization grayscale level i.e., $Q \in [0, 255] \in \mathbb{Z}^+$, where $\mathbb{Z}^+$ is a positive integer number. The pixel ij in P_{IJ} is denoted as p_{ij} . Edge detection framework of Bezdek et al. (1998) composed of sequence of operations as denoted as (i) conditioning (c), (ii) feature extracting (f), (iii) blending (b) and (iv) scaling (s).

5.3.1 Conditioning Function

The conditioning function i.e., $c[P_{IJ}] = C_{IJ}$, enhancement input image P_{IJ} and remapping into space $\mathfrak{R}$, as such, $c: IJ \times Q \mapsto IJ \times \mathfrak{R}$, where C_{IJ} is a conditioned output image. Such procedures are significantly important when encountering input image P_{IJ} which is unduly dark, bright, noisy, etc. Thus, conditioning function improves input image P_{IJ} quality for utility as data in edge detection operations. In this regard, Bezdek et al. (1998) proposes the histogram equalization as the condition operation for edge detection. Histogram equalization normalizes image intensities so that has a linear cumulative histogram over Q .

Here, image histogram $\mathcal{H}(q)$ is denoted as function representing number of pixels at grayscale intensity value $q \in Q$ found in image P_{IJ} . The cumulative histogram functions of P_{IJ} is denoted as $\mathcal{C}(q) = \mathcal{C}(q - 1) + \mathcal{H}(q)$ and $\mathcal{C}(0) = \mathcal{H}(0)$. Thus, the equalized intensity at pixel of ij in C_{IJ} i.e., c_{ij} is defined as in Equation 5.1. Noted that $c_{ij} = 0$ whenever $p_{ij} = 0$

$$c_{ij} = \text{floor} \left[\frac{\mathcal{C}(q) \times (255 - 10^{-6})}{m \times n} \right] \text{ for } q > 0 \text{ at } p_{ij} \quad \text{Equation 5.1}$$

5.3.2 Feature Extraction Function

The feature extraction function i.e., $\mathbf{f}[C_{IJ}] = \mathbf{F}_{IJ}$ apply 3x3 image kernels to compute the gradient vector of c_{ij} in conditioned image C_{IJ} i.e., $\mathbf{f}_{ij}$, and mapping into space $\mathfrak{R}^P = \mathfrak{R}^1 \times \mathfrak{R}^2$, as such, $f: IJ \times \mathfrak{R} \mapsto IJ \times \mathfrak{R}^P$. F_{IJ} is a two bands (layers) feature extracted image denoted by $F_{IJ}^1 \in \mathfrak{R}^1$ and $F_{IJ}^2 \in \mathfrak{R}^1$. $\mathbf{f}_{ij} = [f_{ij}^1, f_{ij}^2]$ is the intensity of pixel of spatial location ij at F_{IJ} . where $f_{ij}^1 \in F_{IJ}^1$ and $f_{ij}^2 \in F_{IJ}^2$.

In this chapter, Sobel operator (Sobel and Feldman, 1968) and Prewitt operator (Prewitt 1970) are studied and applied for spatial location gradient vector measurement at given c_{ij} . To ease the explanation, Sobel operator and Prewitt operator are denoted as $\mathbf{S}$ and $\mathbf{P}$, respectively. Both of operator $\mathbf{S}$ and $\mathbf{P}$ compose two 3×3 image kernels (i.e., convolution matrix) i.e., horizontal kernel (h) and vertical kernel (v). As the example, horizontal kernel and vertical kernel of Sobel operator are denoted in $\mathbf{S}h$ and $\mathbf{S}v$, respective. The $\mathbf{S}h$ and $\mathbf{S}v$ kernels are shown in Equation 5.2. The horizontal kernel ($\mathbf{P}h$) and vertical kernel ($\mathbf{P}v$) of Prewitt operator are shown in Equation 5.3. Kernel $\mathbf{S}h$ and $\mathbf{P}h$ is used to

compute the gradient vector of c_{ij} at horizontal spatial directions, whereas $\mathbf{S}v$ and $\mathbf{P}v$ is used to compute the gradient vector of c_{ij} at vertical spatial directions.

$$\mathbf{S}h = \begin{bmatrix} -1 & -2 & -1 \\ 0 & 0 & 0 \\ 1 & 2 & 1 \end{bmatrix} \text{ and } \mathbf{S}v = \begin{bmatrix} -1 & 0 & 1 \\ -2 & 0 & 2 \\ -1 & 0 & 1 \end{bmatrix} \quad \text{Equation 5.2}$$

$$\mathbf{P}h = \begin{bmatrix} -1 & -1 & -1 \\ 0 & 0 & 0 \\ 1 & 1 & 1 \end{bmatrix} \text{ and } \mathbf{P}v = \begin{bmatrix} -1 & 0 & 1 \\ -1 & 0 & 1 \\ -1 & 0 & 1 \end{bmatrix} \quad \text{Equation 5.3}$$

Let's denoted 3×3 windows vector centered c_{ij} is as $\mathbf{w}_{ij}$ i.e., as shown in Equation 5.4. To ease explanation, $\mathbf{w}$ is used instead to represent $\mathbf{w} = \mathbf{w}_{ij}$, as shown in Equation 5.5. A Sobel feature extractor function for a given c_{ij} is denoted as $\mathbf{f}(\mathbf{w})$ as shown in Equation 5.6. Noted that $f_{\mathbf{S}h}(\mathbf{w})$ is the convolution result of $\mathbf{w}$ and $\mathbf{S}h$ (as shown in Equation 5.7), whereas $f_{\mathbf{S}v}(\mathbf{w})$ is the convolution result of $\mathbf{w}$ and $\mathbf{S}v$ (as shown in Equation 5.8).

$$\mathbf{w}_{ij} = \begin{bmatrix} c_{(i-1, j-1)} & c_{(i, j-1)} & c_{(i+1, j-1)} \\ c_{(i-1, j)} & c_{ij} & c_{(i+1, j)} \\ c_{(i-1, j+1)} & c_{(i, j+1)} & c_{(i+1, j+1)} \end{bmatrix} \quad \text{Equation 5.4}$$

$$\mathbf{w} = \begin{bmatrix} w_1 & w_2 & w_3 \\ w_4 & w_5 & w_6 \\ w_7 & w_8 & w_9 \end{bmatrix} = \mathbf{w}_{ij} \quad \text{Equation 5.5}$$

$$\mathbf{f}(\mathbf{w}) = [f_{\mathbf{S}h}(\mathbf{w}_{ij}), f_{\mathbf{S}v}(\mathbf{w}_{ij})] \in \mathbb{R}^1 \times \mathbb{R}^2 \quad \text{Equation 5.6}$$

$$\begin{aligned} f_{\mathbf{S}h}(\mathbf{w}) &= \mathbf{w} * \mathbf{S}h \\ &= (w_7 + 2w_8 + w_9) - (w_1 + 2w_2 + w_3) \end{aligned} \quad \text{Equation 5.7}$$

$$\begin{aligned} f_{\mathbf{S}v}(\mathbf{w}) &= \mathbf{w} * \mathbf{S}v \\ &= (w_3 + 2w_6 + w_9) - (w_1 + 2w_4 + w_7) \end{aligned} \quad \text{Equation 5.8}$$

A Prewitt feature extractor function for a given c_{ij} is denoted as $\mathbf{f}(\mathbf{w})$ as shown in Equation 5.9. Noted that $f_{\mathbf{P}h}(\mathbf{w})$ is the convolution result of $\mathbf{w}$ and $\mathbf{P}h$ (as shown in Equation 5.10), whereas $f_{\mathbf{P}v}(\mathbf{w})$ is the convolution result of $\mathbf{w}$ and $\mathbf{P}v$ (as shown in Equation 5.11).

$$\mathbf{f}(\mathbf{w}) = [f_{\mathbf{P}h}(\mathbf{w}), f_{\mathbf{P}v}(\mathbf{w})] \in \mathfrak{R}^1 \times \mathfrak{R}^2 \quad \text{Equation 5.9}$$

$$\begin{aligned} f_{\mathbf{P}h}(\mathbf{w}) &= \mathbf{w} * \mathbf{P}h \\ &= (w_7 + w_8 + w_9) - (w_1 + w_2 + w_3) \end{aligned} \quad \text{Equation 5.10}$$

$$\begin{aligned} f_{\mathbf{P}v}(\mathbf{w}) &= \mathbf{w} * \mathbf{P}v \\ &= (w_3 + w_6 + w_9) - (w_1 + w_4 + w_7) \end{aligned} \quad \text{Equation 5.11}$$

5.3.3 Blending Function

A Blending function i.e., $b[|\mathbf{F}_{IJ}|] = B_{IJ}$ aggregates the information about edges possessed by $|\mathbf{f}(\mathbf{w})|$ i.e., $\mathbf{f}_{ij} \in \mathfrak{R}^1 \times \mathfrak{R}^2$ and mapping into space $\mathfrak{R}$, as such, $b: IJ \times \mathfrak{R}^P \mapsto IJ \times \mathfrak{R}$. B_{IJ} is the blended image output and b_{ij} is the intensity of pixel of spatial location ij at B_{IJ} . Consider the input spaces of a blending function are $[0,4] \times [0,4]$ space. Thus, remapping of $\mathfrak{R}^1 \times \mathfrak{R}^2 \mapsto [0,4] \times [0,4]$ is necessary, depend on which type of the feature extractor function being used.

Consider $f_{ij}^1 = |f_{\mathbf{S}h}(\mathbf{w})| \in \mathfrak{R}^1$ and $f_{ij}^2 = |f_{\mathbf{S}v}(\mathbf{w})| \in \mathfrak{R}^2$, the range of space $\mathfrak{R}^1 \times \mathfrak{R}^2$ for a Sobel feature extractor function is determined by Equation 5.12. For this case, divide $|f_{\mathbf{S}h}(\mathbf{w})|$ and $|f_{\mathbf{S}v}(\mathbf{w})|$ by 255 to remap the $\mathfrak{R}^1 \times \mathfrak{R}^2 \mapsto [0,4] \times [0,4]$. For Prewitt feature extractor function with $|f_{\mathbf{P}h}(\mathbf{w})| \in \mathfrak{R}^1$ and $|f_{\mathbf{P}v}(\mathbf{w})| \in \mathfrak{R}^2$, the range of space $\mathfrak{R}^1 \times \mathfrak{R}^2$ is determined by Equation 5.13. Remap the $\mathfrak{R}^1 \times \mathfrak{R}^2 \mapsto [0,4] \times [0,4]$ by multiplying the outputs of the $|f_{\mathbf{P}h}(\mathbf{w})|$ and $|f_{\mathbf{P}v}(\mathbf{w})|$ with 4/765.

$$\begin{aligned} \mathfrak{R}^1 \times \mathfrak{R}^2 &= [\inf(|f_{\mathbf{S}h}(\mathbf{w}: w_i = q)|), \sup(|f_{\mathbf{S}h}(\mathbf{w}: w_i = q)|)] \\ &\quad \times [\inf(|f_{\mathbf{S}v}(\mathbf{w}: w_i = q)|), \sup(|f_{\mathbf{S}v}(\mathbf{w}: w_i = q)|)] \text{ for } \forall q \in Q \end{aligned} \quad \text{Equation 5.12}$$

$$\mathfrak{R}^1 \times \mathfrak{R}^2 = [0,1020] \times [0,1020]$$

$$\begin{aligned} \mathfrak{R}^1 \times \mathfrak{R}^2 &= [\inf(|f_{\mathbf{P}h}(\mathbf{w}: w_i = q)|), \sup(|f_{\mathbf{P}h}(\mathbf{w}: w_i = q)|)] \\ &\quad \times [\inf(|f_{\mathbf{P}v}(\mathbf{w}: w_i = q)|), \sup(|f_{\mathbf{P}v}(\mathbf{w}: w_i = q)|)] \text{ for } \forall q \in Q \end{aligned} \quad \text{Equation 5.13}$$

$$\mathfrak{R}^1 \times \mathfrak{R}^2 = [0,765] \times [0,765]$$

Two blending functions are focused on this chapter i.e., (i) Waterfall Blending Functions and (ii) T1 TSK FIS Blending Functions. To ease explanation, the inputs of blending function are denoted as $\mathbf{x} = [x_1, x_2] \in |\mathbf{f}(\mathbf{w})| = [0,4] \times [0,4]$.

(A) Waterfall Blending Functions

A waterfall functions (also known as Generalized logistic functions) as introduced in Bezdek et al. (1998), is a blending functions that is nearly concave (instead of convex) in the neighborhood of the origin in feature space $\mathfrak{R}^1 \times \mathfrak{R}^2$, in analogy with the shape possessed by a profile view of a typical waterfall (i.e., “shed water”). A waterfall functions i.e., $b(\mathbf{x}: |\mathbf{x}|_t, \lambda, \beta)$ is denoted as in Equation 5.14 and Equation 5.15, where $\{t > 0, \lambda, \beta\} \in \mathbb{R}^+$ is a real positive constants $\mathbb{R}^+$. Example of waterflow function output of $b(\mathbf{x}: |\mathbf{x}|_2, 5, 1)$ with input space $\mathfrak{R}^1 \times \mathfrak{R}^2 = [0,4] \times [0,4]$ is depicted in Figure 5.1.

$$b(\mathbf{x}: |\mathbf{x}|_t, \lambda, \beta) = \frac{1}{1 + e^{-\lambda(|\mathbf{x}|_t - \beta)}} \quad \text{Equation 5.14}$$

where

$$|\mathbf{x}|_t = (|x_1^t| + |x_2^t|)^{\frac{1}{t}} \quad \text{Equation 5.15}$$

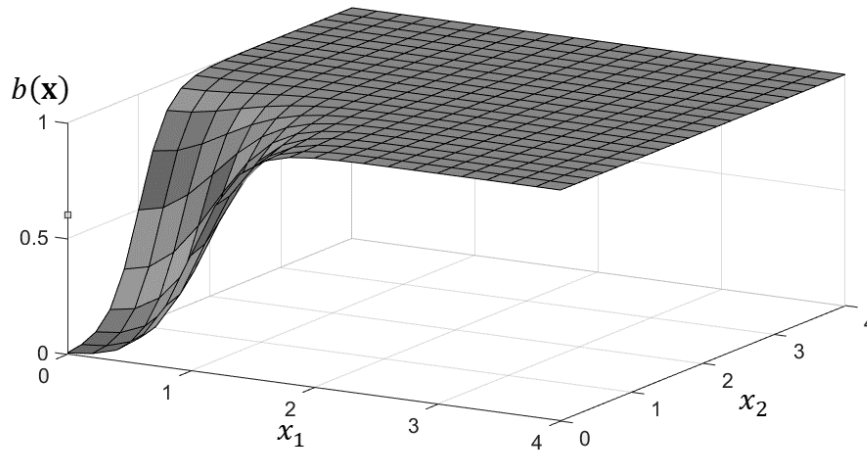


Figure 5.1: Waterflow function output of $b(\mathbf{x}: |\mathbf{x}|_2, 5, 1)$ with input space $[0,4] \times [0,4]$

(B) T1 TSK FIS Blending Function

Research in Bezdek et. al (1998) demonstrates a simple T1 TSK FIS as an approximate function to model waterfall blending functions. The T1 TSK FIS Blending Function i.e., $b(\mathbf{x})$, consists of with 2 inputs $\mathbf{x} = [x_1, x_2]$ and 4 fuzzy rules (i.e., $R^{(j_1, j_2)}$) as shown in Equation 5.16. Each of input i.e., $x_{i \in \{1,2\}}$ consists of two T1 Triangular FMF i.e., $A_i^{j_i}$ with membership functions $\mu_i^{j_i}$ where $j_i \in \{1,2\}$ represents the j_i -th FMF in input x_i . The consequent of fuzzy rule $R^{(j_1, j_2)}$ is denoted as $\mathcal{B}^{(j_1, j_2)}$. As shown in Figure 5.2, the T1 Triangular FMF of input x_1 and x_2 are parameterized as $A_1^1 = A_2^1 = \mu_{i \in \{1,2\}}^1(x_i; 0, 4, 4)$, and $A_1^2 = A_2^2 = \mu_{i \in \{1,2\}}^2(x_i; 0, 0, 4)$. The T1 TSK FIS Blending Function is formulated as in Equation 5.17 where firing strength of $R^{(j_1, j_2)}$ is denoted as $\mu^{(j_1, j_2)} = \mu_1^{j_1}(x_1) \times \mu_2^{j_2}(x_2)$.

$$\begin{aligned}
 R^{(1,1)}: & \text{ If } x_1 \text{ is } A_1^1 \text{ and } x_2 \text{ is } A_2^1 \text{ then } \mathcal{B}^{(1,1)}(\mathbf{x}) = x_1^t + x_2^t \\
 R^{(1,2)}: & \text{ If } x_1 \text{ is } A_1^1 \text{ and } x_2 \text{ is } A_2^2 \text{ then } \mathcal{B}^{(1,2)}(\mathbf{x}) = \chi \\
 R^{(2,1)}: & \text{ If } x_1 \text{ is } A_1^2 \text{ and } x_2 \text{ is } A_2^1 \text{ then } \mathcal{B}^{(2,1)}(\mathbf{x}) = \gamma \\
 R^{(2,2)}: & \text{ If } x_1 \text{ is } A_1^2 \text{ and } x_2 \text{ is } A_2^2 \text{ then } \mathcal{B}^{(2,2)}(\mathbf{x}) = \omega
 \end{aligned}
 \tag{Equation 5.16}$$

$$b(\mathbf{x}) = \frac{\sum_{j_1=1}^2 \sum_{j_2=1}^2 \left(\mu^{(j_1, j_2)}(\mathbf{x}) \times \mathcal{B}^{(j_1, j_2)}(\mathbf{x}) \right)}{\sum_{j_1=1}^2 \sum_{j_2=1}^2 \mu^{(j_1, j_2)}(\mathbf{x})}
 \tag{Equation 5.17}$$

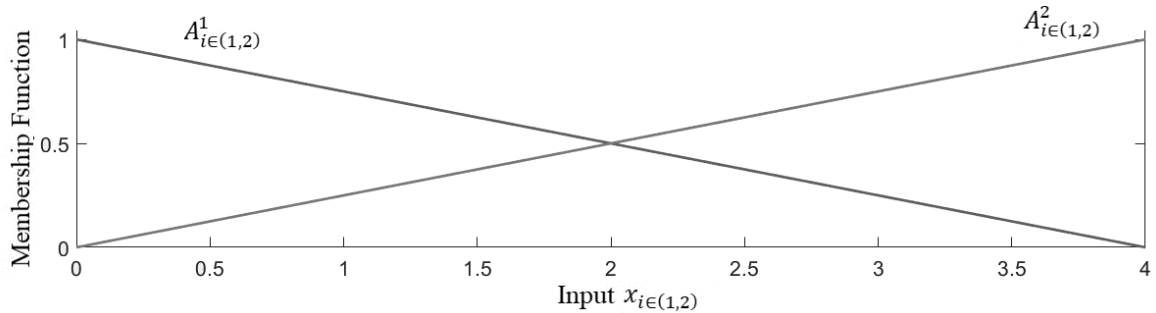


Figure 5.2: T1 Triangular FMF of input $x_{i \in \{1,2\}}$

5.3.4 Scaling Function

A scaling function i.e., $s[B_{IJ}] = E_{IJ}$ scale the intensities b_{ij} in B_{IJ} to grayscale levels $Q \in [0, 255] \in \mathbb{Z}^+$, as such, $s: IJ \times \mathfrak{R} \mapsto IJ \times Q$. E_{IJ} is scaled output edge image and e_{ij} intensity of pixel of spatial location ij at E_{IJ} . Consider that $b_{ij} = B_{IJ} \in [b_{\min}, b_{\max}]$, the scaling of b_{ij} to e_{ij} is shown in Equation 5.18

$$e_{ij} = \text{floor} \left[\frac{255 \times (b_{ij} - b_{\min})}{(b_{\max} - b_{\min})} \right] \quad \text{Equation 5.18}$$

5.3.5 Edge Detection Operator

Here, edge detection function i.e., $e[P_{IJ}] = E_{IJ}$ is regarded as edge operator comprise of 4 operators functions i.e., (i) conditioning (c), (ii) feature extracting ($\mathbf{f}$), (iii) blending (b) and (iv) scaling (s) as shown in Equation 5.19.

$$e = c \circ \mathbf{f} \circ b \circ s \quad \text{Equation 5.19}$$

The procedure of the edge operator is summarized as a sequence of operations from Step 1 to Step 4 as follows. The operation sequence flow of an edge operator (e) is depicted in Figure 5.3.

Step 1: Conditioning: $c: IJ \times Q \mapsto IJ \times \mathfrak{R}: c[P_{IJ}] = C_{IJ}$ i.e., enhance input image P_{IJ} .

Step 2: Feature extraction: $\mathbf{f}: IJ \times Q \mapsto IJ \times \mathfrak{R}^P: \mathbf{f}[C_{IJ}] = F_{IJ}$ i.e., extract gradients vectors from C_{IJ} into $\mathfrak{R}^P$.

Step 3: Blending: $b: IJ \times \mathfrak{R}^P \mapsto IJ \times \mathfrak{R}: b[\mathbf{F}_{IJ}] = B_{IJ}$, i.e., blends components of gradients vectors in $\mathbf{F}_{IJ}$.

Step 4: Scaling: $IJ \times \mathfrak{R} \mapsto IJ \times Q: s[B_{IJ}] = E_{IJ}$, i.e., scale edge image B_{IJ} to grayscale image with range $[0,255]$.

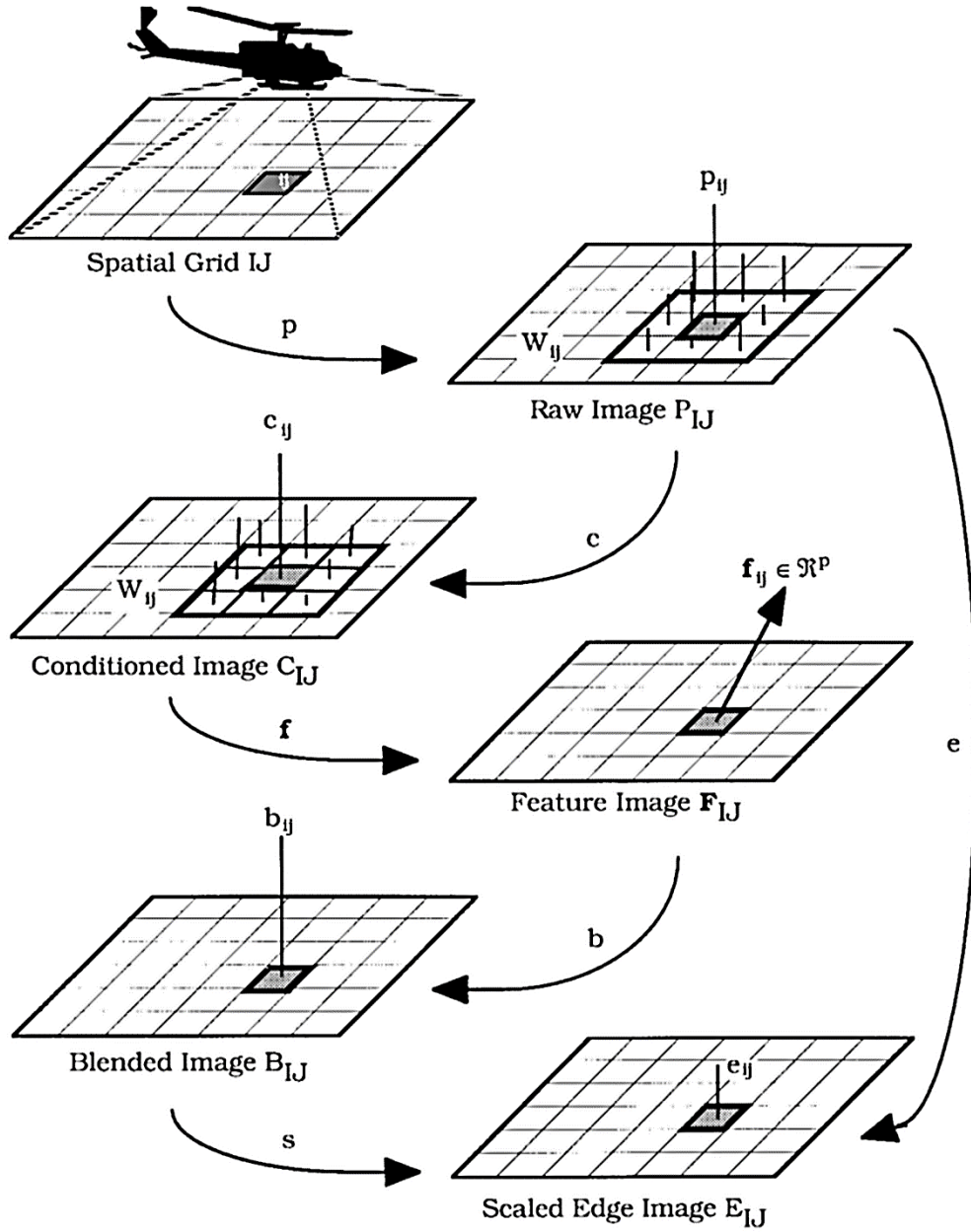


Figure 5.3: Operation sequence flows of an edge operator (e) (Bezdek et al., 1998)

5.4 Waterfall Blending Functions as Monotone Function.

Consider $\mathbf{x}' = x'_i + x_{i \neq i}^t$ where $[i, i] = \{1, 2\}$ and $x'_i \geq x_i^t$. The output of A waterfall function in Equation 5.14 i.e., $b(\mathbf{x})$ is always monotonically increasing w.r.t $x_i \in \mathbf{x}$ if $b(\mathbf{x}') - b(\mathbf{x}) \geq 0$, as shown in Equation 5.20. Noted that $|x'_i| \geq |x_i^t|$ is always true for all $\forall x_{i \in \{1, 2\}} \in [0, 4]$.

$$\begin{aligned}
 & b(\mathbf{x}') - b(\mathbf{x}) \geq 0 \\
 & \frac{1}{1 + e^{-\lambda(\|\mathbf{x}'\|_t - \beta)}} - \frac{1}{1 + e^{-\lambda(\|\mathbf{x}\|_t - \beta)}} \geq 0 \\
 & e^{-\lambda(\|\mathbf{x}'\|_t - \beta)} \geq e^{-\lambda(\|\mathbf{x}\|_t - \beta)} \\
 & \|\mathbf{x}'\|_t \geq \|\mathbf{x}\|_t \\
 & (|x'_i| + |x_{i \neq i}^t|)^{\frac{1}{t}} \geq (|x_i^t| + |x_{i \neq i}^t|)^{\frac{1}{t}} \\
 & |x'_i| \geq |x_i^t|
 \end{aligned}
 \tag{Equation 5.20}$$

A blending function should be at greatest extent to preserve information about intensity differences of each pixel with the same ordered as in feature extraction image. As the argument in Marco-Detchart et al. (2021) research, the nature of edges implies that the bigger an intensity jump is, the larger its influence is in the possible existence of an edge. In this regards, monotone blending function, such as Waterfall blending functions, is an ideal solution to aggregate the information about edges possessed by feature extraction function and preserve information about intensity differences of each pixel at greatest extent.

5.4.1 Image Histogram Analysis of $\mathbf{F}_{IJ}$

The contrast of experimental raw image inputs P_{IJ} is enhanced with a histogram equalization (as in Equation 5.1) at Conditioning function stage and produce C_{IJ} . Then, C_{IJ} are processed with Sobel features extraction to produce a two band $\mathbf{F}_{IJ}$ image.

Demonstration using image histogram to analysis two bands image of $\mathbf{F}_{IJ}$ i.e., F_{IJ}^1 and F_{IJ}^2 , where F_{IJ}^1 and F_{IJ}^2 are mapped into grayscale image, as such, $F_{IJ}^1: IJ \times \mathfrak{R}^1 \mapsto IJ \times Q$ and $F_{IJ}^2: IJ \times \mathfrak{R}^2 \mapsto IJ \times Q$. An image histogram plots the number of pixels distribution for each grayscale intensity value $q \in [0,255]$.

For ease comparison image histogram between $\mathbf{F}_{IJ}$, an image histogram analysis algorithm of $\mathbf{F}_{IJ}$ is introduced in this section. The algorithm determines the highest grayscale intensity of pixels i.e., $q_{\mathcal{T}}$ under a defined range at threshold value $\mathcal{T}$ of first accumulated number of pixels in a cumulative histogram function of image $\mathbf{F}_{IJ}$. Consider cumulative histogram functions i.e., $\mathcal{C}(q) = \mathcal{C}(q - 1) + \mathcal{H}(q)$ and $\mathcal{C}(0) = \mathcal{H}(0)$. $\mathcal{H}(q)$ in an image histogram function representing number of pixels at a grayscale intensity value $q \in Q$ found in image. The algorithms are designed to find out $q_{\mathcal{T}}$ for the first accumulated number of pixels, i.e., $\mathcal{C}(q_{\mathcal{T}} = q)$, hits the threshold value $\mathcal{T}$ i.e., $\mathcal{C}(q)/(m \times n) < \mathcal{T} \in [0, 1]$, where $m \times n$ is size of $\mathbf{F}_{IJ}$. A percentage value i.e., $\mathcal{T}_p = \mathcal{C}(q)/(m \times n) \times 100$ is computed to indicate percentage of number pixels over total pixels (i.e., $m \times n$) in an image are fall bellow $q_{\mathcal{T}}$. The algorithm is summaried in Figure 5.4. In this experiment $\mathcal{T}$ is set to 0.98.

Input: $\mathcal{H}(q)$

Constant: Threshold value $\mathcal{T} = 0.98$

Output: $q_{\mathcal{T}} = q$ after termination of while loop; $\mathcal{T}_p$

1. Set $\mathcal{C}(0) = \mathcal{H}(0)$
 2. **while** $\mathcal{C}(q)/(m \times n) < \mathcal{T}$
 3. $q = q + 1$
 4. $\mathcal{C}(q) = \mathcal{C}(q - 1) + \mathcal{H}(q)$
 5. **end while**
 6. Compute value of $\mathcal{T}_p$
-

Figure 5.4: Algorithms to determine grayscale intensity $q_{\mathcal{T}}$ at set threshold $\mathcal{T}$

In this section, experiment compares two C_{IJ} i.e., “*Lenna*” image and a mammographic Image “*mdb313*” from database of MIAS, as shown in Figure 5.5 and Figure 5.6, respectively. The image histogram of “*Lenna*” and “*mdb313*” image is shown in Figure 5.7 and Figure 5.8. Compare Image histogram in Figure 5.7 and Figure 5.8 shown that C_{IJ} of “*Lenna*” shown wider and richer range of grayscale intensity distribution than C_{IJ} of “*mdb313*”. As shown in Figure 5.7, the image histogram demonstrates that the number of pixels in the “*Lenna*” image is distributed more evenly across the grayscale intensity. Meanwhile, the image histogram in Figure 5.8 shows that a high portion of pixels in the “*mdb313*” image are distributed at grayscale intensity values greater than 100.



Figure 5.5: C_{IJ} of “*Lenna*” Image

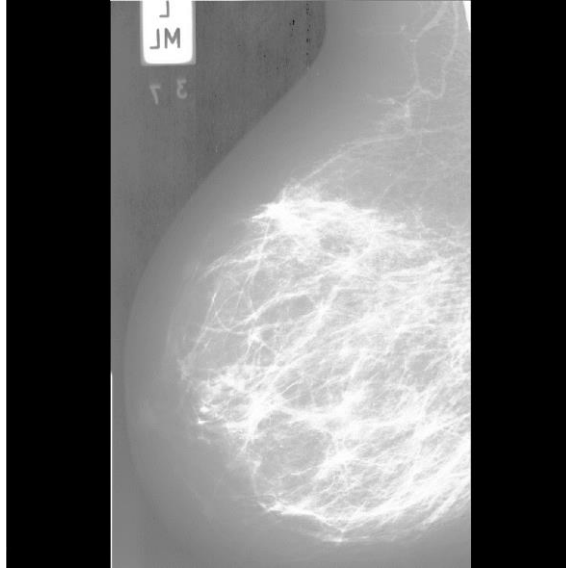


Figure 5.6: C_{IJ} of mammographic Image “mdb313” from database of MIAS

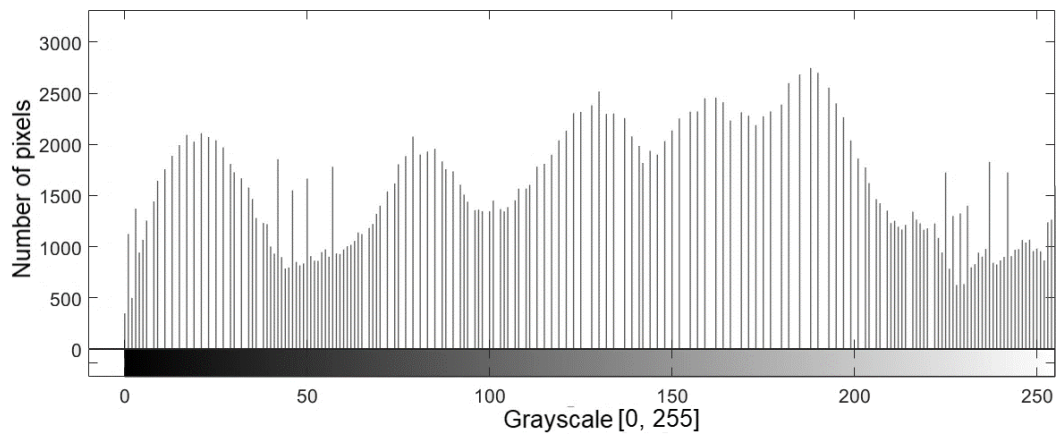


Figure 5.7: Image hitrogram of C_{IJ} “Lenna” Image

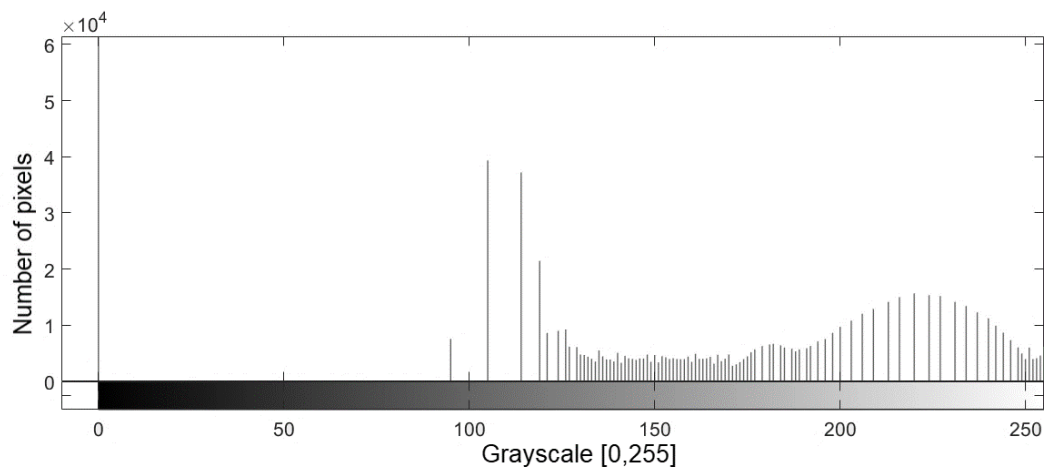


Figure 5.8: Image hitrogram of C_{IJ} “mdb313” image

A Sobel feature extraction function is used in this example. Noted that the extracted feature images for “*Lenna*” image and “*mdb313*”, i.e., $\mathbf{F}_{IJ} = [F_{IJ}^1, F_{IJ}^2]$, are mapped into grayscale $Q \in [0, 255]$, as such, $F_{IJ}^1: IJ \times \mathbb{R}^1 \mapsto IJ \times Q$ and $F_{IJ}^2: IJ \times \mathbb{R}^2 \mapsto IJ \times Q$. The result of F_{IJ}^1 and F_{IJ}^2 for “*Lenna*” image are presented in Figure 5.9 and Figure 5.10, respectively. Both F_{IJ}^1 and F_{IJ}^2 of “*Lenna*” image shown image histogram with right skewed number of pixels distribution, as attached in in Figure 5.9 and Figure 5.10, respectively. Image histogram in Figure 5.9 shown that approximate 98% number of pixels in F_{IJ}^1 have the grayscale intensity fall bellow $q_T = 67$. Image histogram as depicted in Figure 5.10 shown the shown that approximate 98% number of pixels in F_{IJ}^2 have the grayscale intensity fall bellow $q_T = 92$.

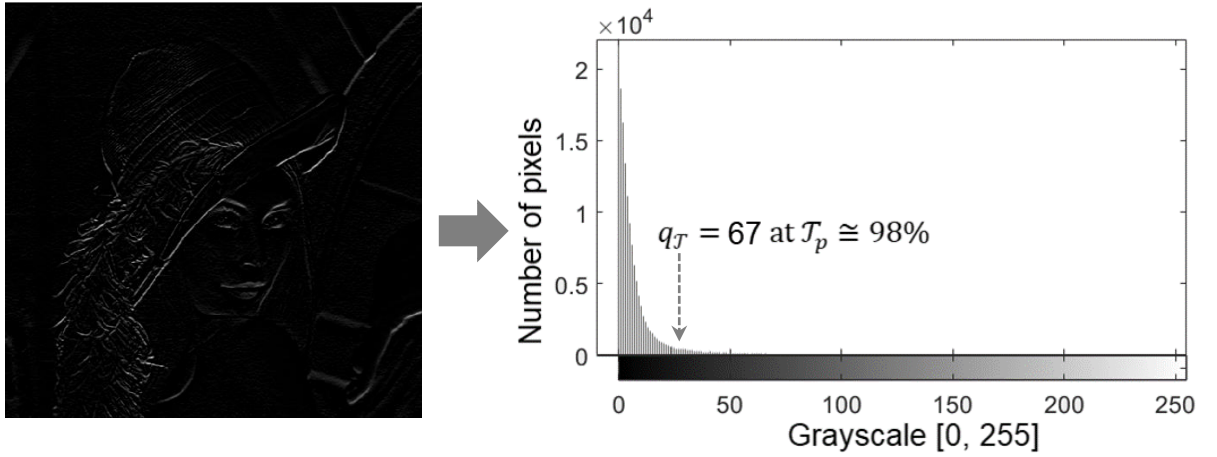


Figure 5.9: F_{IJ}^1 and image histogram of F_{IJ}^1 for “*Lenna*” image

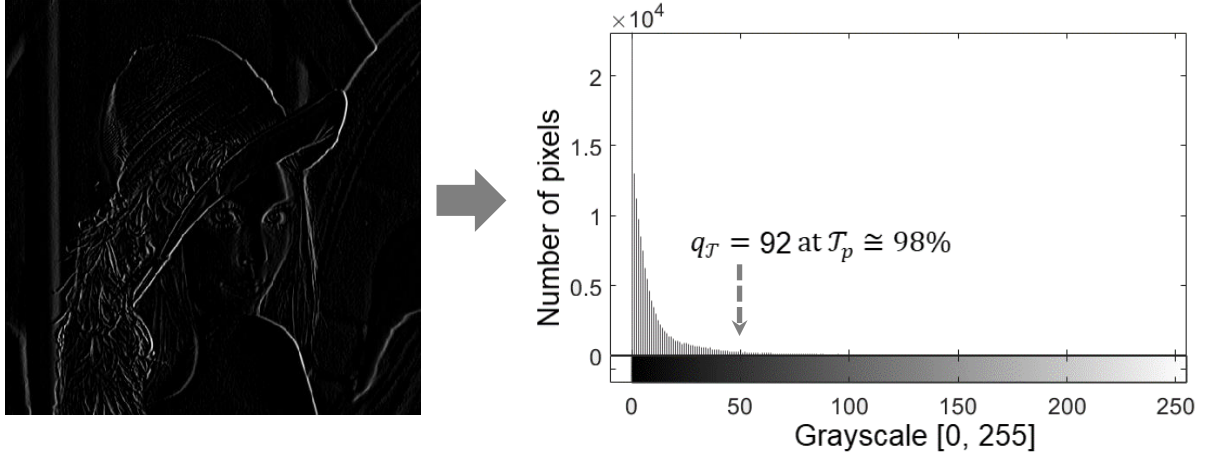


Figure 5.10: F_{IJ}^2 and image histogram of F_{IJ}^2 for “Lenna” image

The results of F_{IJ}^1 and F_{IJ}^2 for “mdb313” image are depicted in Figure 5.11 and Figure 5.12, respectively. Again, both F_{IJ}^1 and F_{IJ}^2 of “mdb313” image display the image histogram with a right skewed number of pixels distribution, as attached in Figure 5.11 and Figure 5.12, respectively. Image histogram in Figure 5.11 shown that approximate 98% number of pixels in F_{IJ}^1 have the grayscale intensity fall bellow $q_T = 17$. Meanwhile, Image histogram as depicted in Figure 5.12 shown the shown that approximate 98% number of pixels in F_{IJ}^2 have the grayscale intensity fall bellow $q_T = 16$.

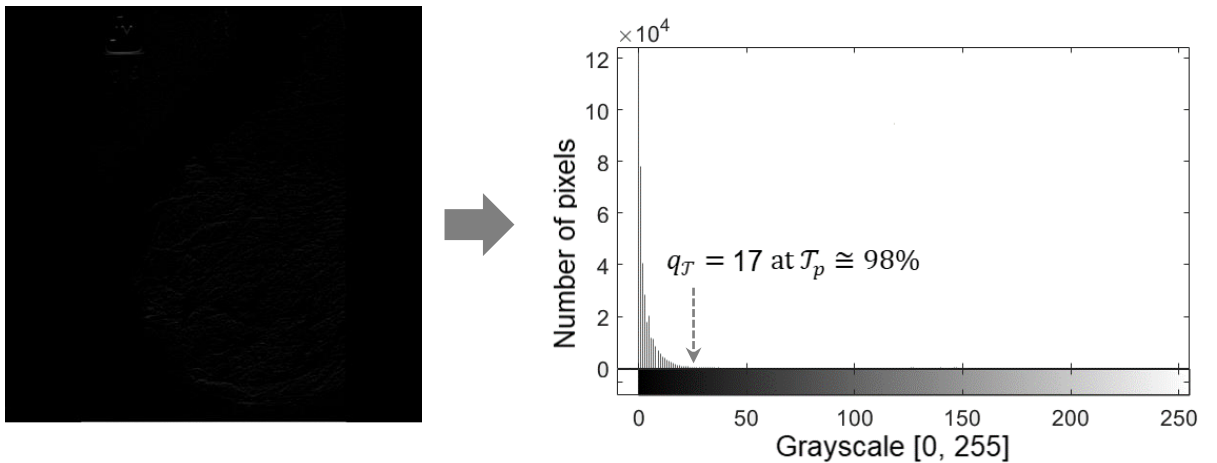


Figure 5.11: F_{IJ}^1 and image histogram of F_{IJ}^1 for “mdb313” image

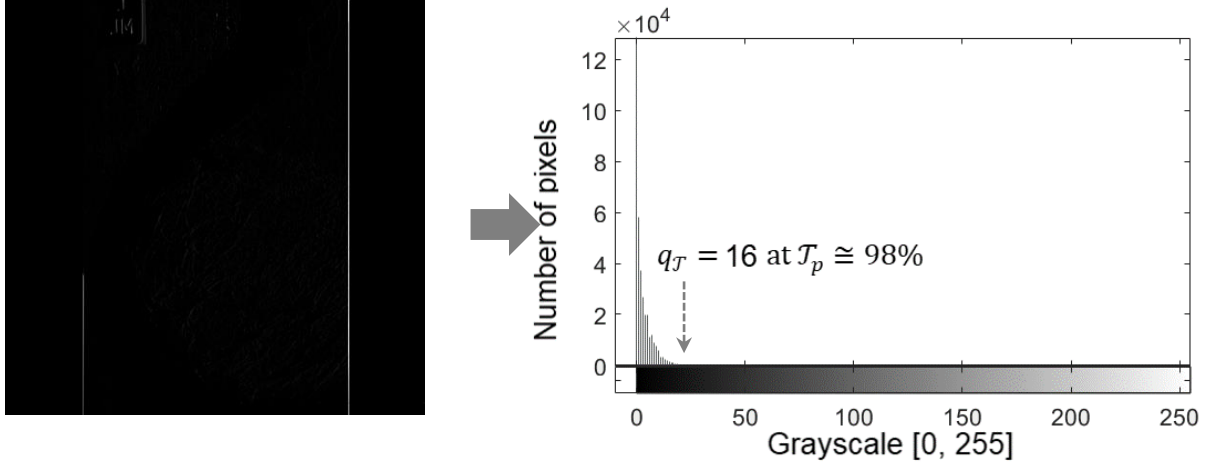


Figure 5.12: F_{IJ}^2 and image histogram of F_{IJ}^2 for “mdb313” image

Since number of pixels of an image histogram is a discrete value at q , it is difficult to get a precise $\mathcal{T}_p = \mathcal{T} \times 100$. Thus, approximate value i.e., $\mathcal{T}_p \cong \mathcal{T} \times 100$ is used instead this case. The result of $q_{\mathcal{T}}$ of $\mathbf{F}_{IJ} = [F_{IJ}^1, F_{IJ}^2]$ at $\mathcal{T} = 0.98$ for image “Lenna” and “mdb313” are summaries in Table 5.1. As shown in Table 5.1, image “Lenna” has a higher $q_{\mathcal{T}}$ for both F_{IJ}^1 and F_{IJ}^2 compared to image “mdb313”.

Table 5.1: $q_{\mathcal{T}}$ of $\mathbf{F}_{IJ}$ at $\mathcal{T} = 0.98$ for image “Lenna” and “mdb313”

Image	Extracted Feature Image	$\mathcal{T}_p$ at $\mathcal{T} = 0.98$	$q_{\mathcal{T}}$ at $\mathcal{T} = 0.98$
“Lenna”	F_{IJ}^1	98.0209%	67
	F_{IJ}^2	98.0137%	92
“mdb313”	F_{IJ}^1	98.2366%	17
	F_{IJ}^2	98.2552%	16

5.4.2 Edge Detection Operator with Waterfall Blending Functions

Experiment in this section explains the parameter design of Waterfall blending function based on image histogram analysis output of $[F_{IJ}^1, F_{IJ}^2] \in \mathbf{F}_{IJ}$. Examples are presented in this section to demonstrate impact of manipulate parameter λ and β at fix constant $t = 2$ in $b\left(\mathbf{x}: \|\mathbf{x}\|_t, \lambda, \beta\right)$ to the concave or convex region near origin in feature

space $\mathfrak{R}^1 \times \mathfrak{R}^2$. Waterfall Blending Functions with 3 different sets of parameters setup are presented i.e., $b(\mathbf{x}: \|\mathbf{x}\|_2, 5, 1)$, and $b(\mathbf{x}: \|\mathbf{x}\|_2, 8, 0.5)$, and $b(\mathbf{x}: \|\mathbf{x}\|_2, 10, 0.1)$ as depicted in Figure 5.13, Figure 5.14 and Figure 5.15 respectively. Surface plot output of Waterfall Blending Functions shown that $b(\mathbf{x}: \|\mathbf{x}\|_2, 10, 0.1)$ in Figure 5.15 has the sharpest change at concave or convex region near origin in feature space $\mathfrak{R}^1 \times \mathfrak{R}^2$, follows with $b(\mathbf{x}: \|\mathbf{x}\|_2, 8, 0.5)$ in Figure 5.14 and $b(\mathbf{x}: \|\mathbf{x}\|_2, 5, 1)$ in Figure 5.13. Noted that, input $\mathbf{x} = [x_1, x_2]$ of the Waterfall blending function are $[0, 4] \times [0, 4]$ space. For $\mathbf{F}_{IJ}$ processed with Sober extraction function i.e., $|f_{sh}(\mathbf{w})|$ and $|f_{sv}(\mathbf{w})|$, divide $|f_{sh}(\mathbf{w})|$ and $|f_{sv}(\mathbf{w})|$ by 255 to remap the $\mathfrak{R}^1 \times \mathfrak{R}^2 \mapsto [0, 4] \times [0, 4]$ where $\mathbf{w} \in c_{ij}$ and c_{ij} is pixels intensity at spatial location ij in C_{IJ} .

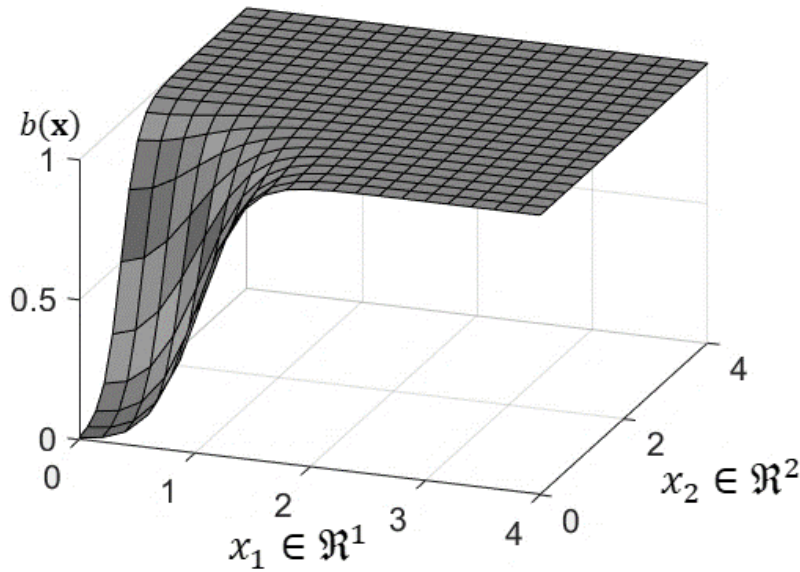


Figure 5.13: $b(\mathbf{x}: \|\mathbf{x}\|_2, 5, 1)$ blending function output at $\mathbf{x} \in [0, 4] \times [0, 4]$

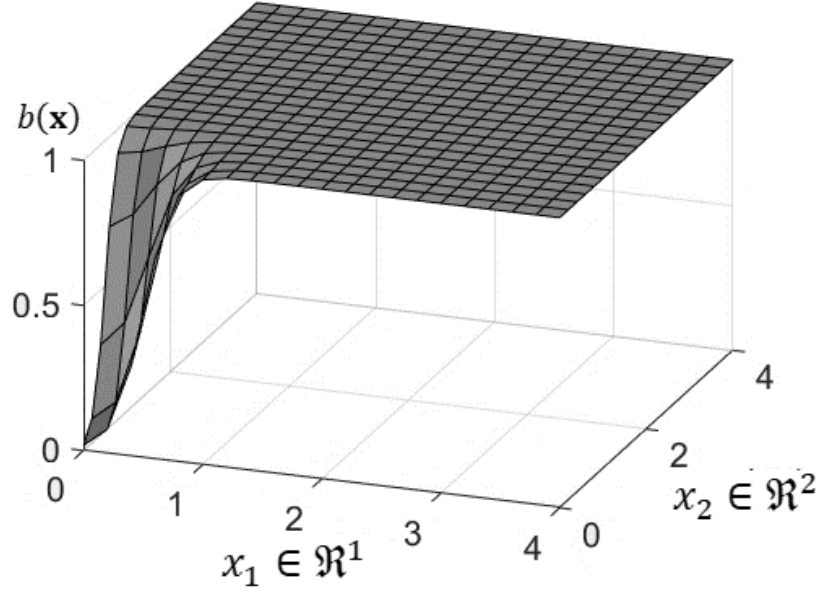


Figure 5.14: $b\left(\mathbf{x}: \|\mathbf{x}\|_2, 8, 0.5\right)$ blending function output at $\mathbf{x} \in [0,4] \times [0,4]$

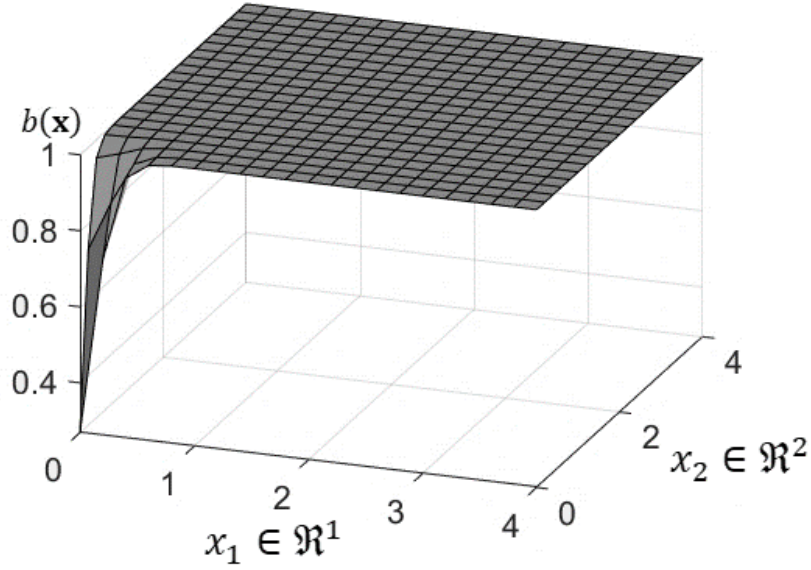


Figure 5.15: $b\left(\mathbf{x}: \|\mathbf{x}\|_2, 10, 0.1\right)$ blending function output at $\mathbf{x} \in [0,4] \times [0,4]$

Experiment use $\mathbf{F}_{IJ}$ of “Lenna” and “mdb3l3” image from Section 5.4.1 as the input of blending function. The results are presented as edge image E_{IJ} after processed by scaling function i.e., $s[B_{IJ}] = E_{IJ}$, where B_{IJ} is blended image. Remind that pixel c_{ij} with high intensity jumps with adjacent pixels, also the indicate high possible existence of an edge

feature in f_{ij} . In this experiment, the Waterfall blending function tend to map edge feature in f_{ij} to $e_{ij} \in E_{IJ}$ with brighter grayscale intensity. Meanwhile, Waterfall blending function tend to map f_{ij} with low intensity are mapped $e_{ij} \in E_{IJ}$ with lower grayscale intensity as the background pixels. This is to create an E_{IJ} with visionally strong contrast between edge feature and non-edge feature (i.e., background pixels)

(A) Edge Image Output E_{IJ} of “*Lenna*” Image

The E_{IJ} with attached image histogram of “*Lenna*” image after processed by $b(\mathbf{x}: ||\mathbf{x}||_2, 5, 1)$, $b(\mathbf{x}: ||\mathbf{x}||_2, 8, 0.5)$ and $b(\mathbf{x}: ||\mathbf{x}||_2, 10, 0.1)$ are depicted in Figure 5.16, Figure 5.17 and Figure 5.18, respectively. Waterfall blending function $b(\mathbf{x}: ||\mathbf{x}||_2, 8, 0.5)$ in Figure 5.17 produce a best edge features details compare to $b(\mathbf{x}: ||\mathbf{x}||_2, 5, 1)$ and $b(\mathbf{x}: ||\mathbf{x}||_2, 10, 0.1)$. As presented in Figure 5.17, $b(\mathbf{x}: ||\mathbf{x}||_2, 5, 1)$ produce visually clear edge feature details in E_{IJ} , meantime prevents the background in E_{IJ} from corrupted by white noise. Image histogram in Figure 5.17 shown a distribution at just right balance between edge feature pixels and background image. Waterfall blending function $b(\mathbf{x}: ||\mathbf{x}||_2, 5, 1)$ in Figure 5.16 produce E_{IJ} edge features details which slightly inferior than E_{IJ} as presented in Figure 5.17. On the other hand, Waterfall blending function $b(\mathbf{x}: ||\mathbf{x}||_2, 10, 0.1)$ in Figure 5.18, produce E_{IJ} background overwhelmed by white noise and covering the edge features details. Image histogram in Figure 5.18 shown that are large portion of e_{ij} in E_{IJ} are mapped to high grayscale intensity.

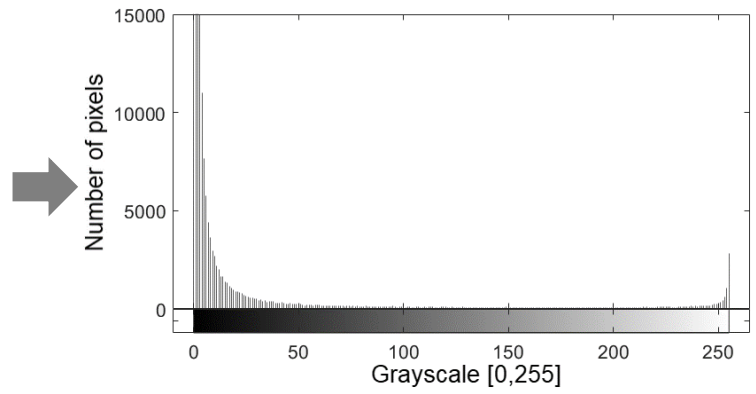


Figure 5.16: E_{IJ} of “Lenna” image and image histogram of E_{IJ} of after processed by $b(\mathbf{x}: \|\mathbf{x}\|_2, 5, 1)$

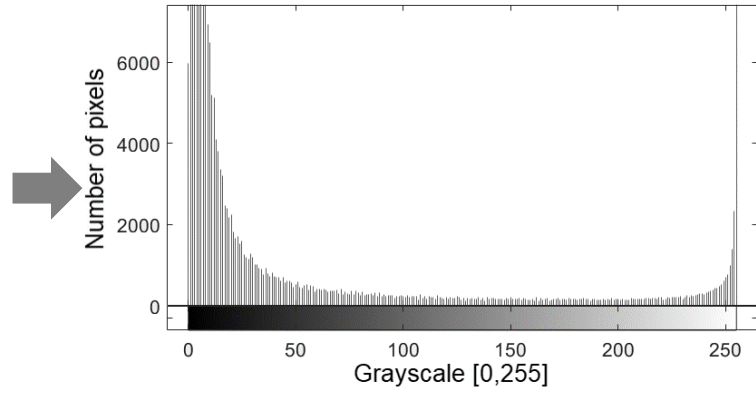


Figure 5.17: E_{IJ} of “Lenna” image and image histogram of E_{IJ} of after processed by $b(\mathbf{x}: \|\mathbf{x}\|_2, 8, 0.5)$

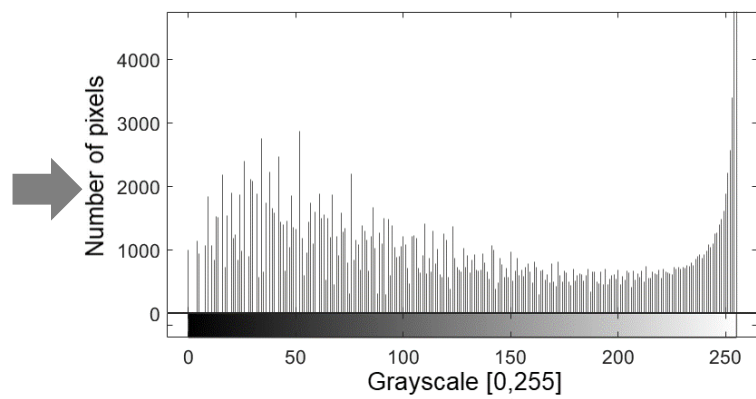
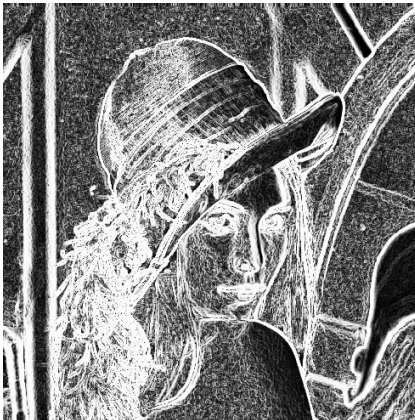


Figure 5.18: E_{IJ} of “Lenna” image and image histogram of E_{IJ} of after processed by $b(\mathbf{x}: \|\mathbf{x}\|_2, 10, 0.1)$

(B) Edge Image Output E_{IJ} of “mdb313” Image

The E_{IJ} with attached image histogram of “mdb313” image after processed by $b(\mathbf{x}: ||\mathbf{x}||_2, 5, 1)$, $b(\mathbf{x}: ||\mathbf{x}||_2, 8, 0.5)$ and $b(\mathbf{x}: ||\mathbf{x}||_2, 10, 0.1)$ are depicted in Figure 5.19, Figure 5.20 and Figure 5.21. Waterfall blending function $b(\mathbf{x}: ||\mathbf{x}||_2, 10, 0.1)$ in Figure 5.21 produce a best edge feature details compared to $b(\mathbf{x}: ||\mathbf{x}||_2, 8, 0.5)$ and $b(\mathbf{x}: ||\mathbf{x}||_2, 10, 0.1)$. As presented in Figure 5.21, $b(\mathbf{x}: ||\mathbf{x}||_2, 5, 1)$ produces a visually clear breast tissue structure with detail edge feature is presented in E_{IJ} . Figure 5.21 shows the appearance of white noise at the background in E_{IJ} . Image histogram in Figure 5.21 shows a shown wider and richer range of grayscale intensity distribution in comparison to image histogram in Figure 5.19, Figure 5.20. On the other hand, the waterfall blending function $b(\mathbf{x}: ||\mathbf{x}||_2, 5, 1)$ in Figure 5.19 produces an E_{IJ} with edge features that are hardly visually observable. In Figure 5.20, the waterfall blending function $b(\mathbf{x}: ||\mathbf{x}||_2, 8, 0.5)$ produces an E_{IJ} with edge features that are barely observable. No visually observable white noise appears in E_{IJ} , as presented in Figure 5.19 and Figure 5.20.

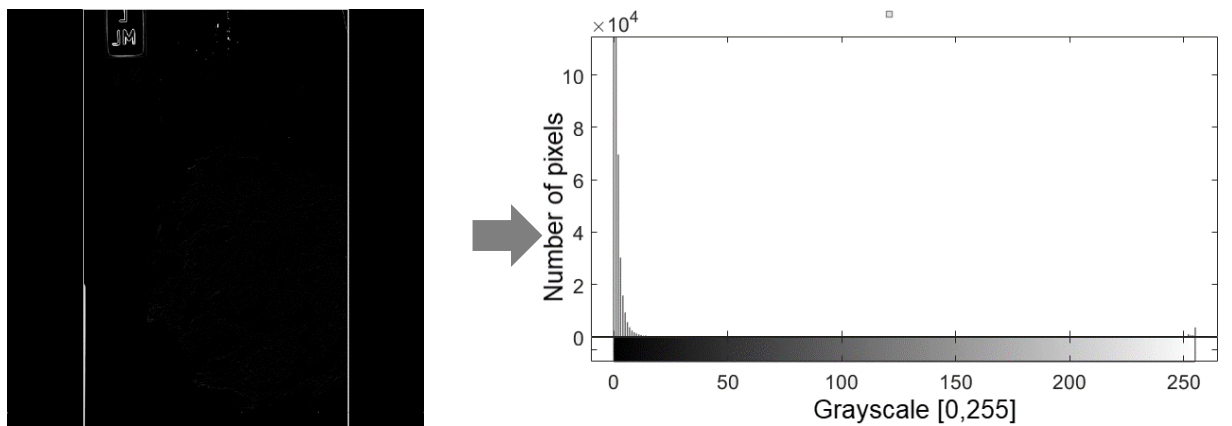


Figure 5.19: E_{IJ} of “mdb313” image and image histogram of E_{IJ} of after processed by $b(\mathbf{x}: ||\mathbf{x}||_2, 5, 1)$

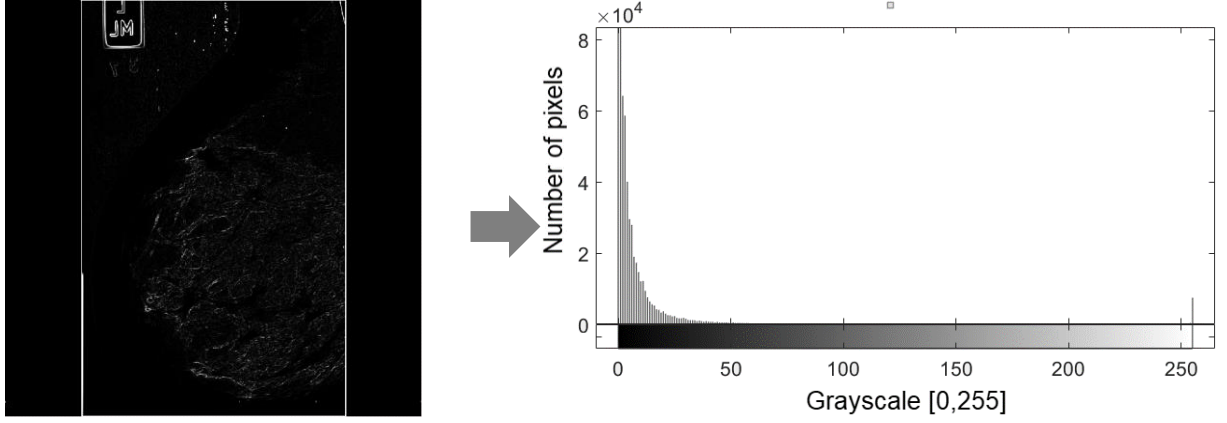


Figure 5.20: E_{IJ} of “mdb313” image and image histogram of E_{IJ} of after processed by $b(\mathbf{x}: \|\mathbf{x}\|_2, 8, 0.5)$

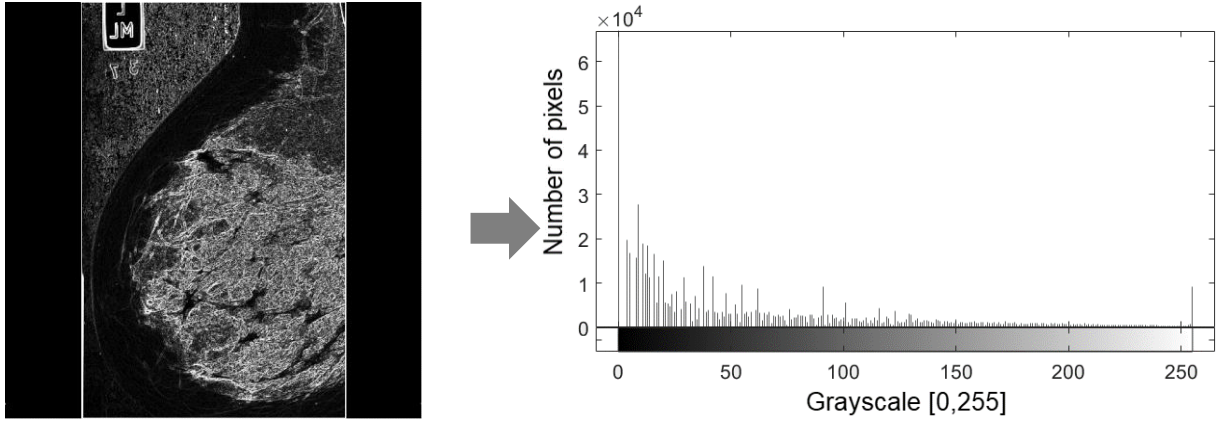


Figure 5.21: E_{IJ} of “mdb313” image and image histogram of E_{IJ} of after processed by $b(\mathbf{x}: \|\mathbf{x}\|_2, 10, 0.1)$

(C) E_{IJ} Comparison at Different Waterfall Bending Function Setting

Results in Figure 5.17 shows “Lenna” image with the waterfall blending function setting $b(\mathbf{x}: \|\mathbf{x}\|_2, 5, 1)$ presenting an E_{IJ} with visually clear edge features and a low white noise background. However, the same setting of $b(\mathbf{x}: \|\mathbf{x}\|_2, 5, 1)$ produces an E_{IJ} with edge features barely observable for the “mdb313” image, as presented in Figure 5.20. On the other hand, the waterfall blending function with the setting $b(\mathbf{x}: \|\mathbf{x}\|_2, 10, 0.1)$ applied to the “mdb313” image produced an E_{IJ} with a visually clear breast tissue structure and detailed

edge features, as presented in Figure 5.18. In contrast, the E_{IJ} of the “*Lenna*” image with the setting $b(\mathbf{x}: \|\mathbf{x}\|_2, 10, 0.1)$, as presented in Figure 5.18, shows an E_{IJ} background overwhelmed by white noise.

Comparing the E_{IJ} outputs of the “*Lenna*” and “*mdb313*” images at different parameter settings of the waterfall blending function $b(\mathbf{x}: \|\mathbf{x}\|_t, \lambda, \beta)$ shows that the λ and β settings are relevant in controlling the balance between edge feature details and background white noise in an E_{IJ} . Recalling the results in Table 5.1, both $F_{IJ}^1: IJ \times \mathfrak{R}^1 \mapsto IJ \times Q$ and $F_{IJ}^2: IJ \times \mathfrak{R}^2 \mapsto IJ \times Q$ for “*mdb313*” image display right-skewed image histograms with narrower distributions compared to the “*Lenna*” image. In this regard, a waterfall blending function with a sharp change at the concave or convex region near the origin in feature space $\mathfrak{R}^1 \times \mathfrak{R}^2$ (e.g., $b(\mathbf{x}: \|\mathbf{x}\|_t, 10, 0.1)$ as presented in Figure 5.15) seems to be more suitable for the “*mdb313*” image to produce more detailed edge features in E_{IJ} .

5.5 Fuzzy Blending Function with Monotone T1 TSK FIS

TSK FIS blending function of Bezdek *et. al* (1998) framework (as presented in Section 5.3.3(B)) is already with MFM design fulfilled the *Condition 1* in Theorem 3.2. However, the parameters of the fuzzy rule’s consequent of TSK FIS blending function are yet to be resolved. In this regard, *Condition 2* and *Condition 3* in Theorem 3.2 are simplified to facilitate the T1 monotone TSK FIS blending function modelling in Bezdek *et. al* (1998) edge detection framework. Recall the fuzzy rule of TSK FIS blending function of Bezdek *et. al* (1998) framework as in Equation 5.16, the Remark 5.1 are derived. Noted that, $\mathcal{B}^{(j_1, j_2)}$

is the consequent function for rule $R^{(j_1, j_2)}$ where $j_i \in \{1, 2\}$ represent the j_i -th FMF in input x_i and $\mathcal{B}^{(j_1, j_2)}$ is the consequent function for rule $R^{(j_1, j_2)}$. Noted that, the consequents $\mathcal{B}^{(1, 2)}(\mathbf{x}) = \chi$, $\mathcal{B}^{(2, 1)}(\mathbf{x}) = \gamma$ and $\mathcal{B}^{(2, 2)}(\mathbf{x}) = \omega$ in TSK FIS blending function are positive constants.

Remark 5.1: Consider that inputs of TSK FIS blending function i.e., $\mathbf{x} = \{x_1, x_2\} \in \{[0, 4] \times [0, 4]\}$, a T1 monotone TSK FIS blending function fulfilled the *Condition 2* in *Condition 3* in **Theorem 3.2** should meet the *Condition 1* and *Condition 2* as shown follows.

Condition 1: The first derivative $\mathcal{B}^{(j_1, j_2)}$ w.r.t to x_i i.e., $d(\mathcal{B}^{(j_1, j_2)}(\mathbf{x}))/dx_i \geq 0$, for $\forall i \in \{1, 2\}$.

Condition 2: $\mathcal{B}^{(j_1, j_2)}$ fulfilled the order, as such, $\omega \geq \gamma \geq \chi \geq \sup(\mathcal{B}^{(1, 1)}(\mathbf{x}))$ where $\sup(\mathcal{B}^{(1, 1)}(\mathbf{x})) = 2 \times 4^t$.

On regards *Condition 1* in Remark 5.1, the first derivative $\mathcal{B}^{(1, 1)} = x_1^t + x_2^t$ w.r.t to x_i i.e., $d(\mathcal{B}^{(1, 1)}(\mathbf{x}))/dx_i \geq 0$ should hold for $\forall i \in \{1, 2\}$ consider inputs $\mathbf{x}$ are the positive value denoted by $\mathbf{x} \in \{[0, 4] \times [0, 4]\}$. Consider that $\mathcal{B}^{(j_1, j_2)} = \{\mathcal{B}^{(1, 2)}(\mathbf{x}), \mathcal{B}^{(2, 1)}(\mathbf{x}), \mathcal{B}^{(2, 2)}(\mathbf{x})\}$ are constant value, the first derivative $\mathcal{B}^{(j_1, j_2)}$ w.r.t to x_i i.e., $d(\mathcal{B}^{(j_1, j_2)}(\mathbf{x}))/dx_i \geq 0$, is true for $\forall i \in \{1, 2\}$. On regards *Condition 2* in Remark 5.1, consider that the TSK FIS blending function only consider inputs $\mathbf{x} \in \{[0, 4] \times [0, 4]\}$, $\sup(\mathcal{B}^{(1, 1)}(\mathbf{x}))$ is simplified to 2×4^t .

At next section, experiments are conducted to evaluate the necessity of T1 TSK FIS blending function in edge detection framework. Experiments also cover the evaluation the

impact of T1 TSK FIS blending function at different parameter settings to edge feature detail for image output E_{IJ} .

5.5.1 Necessity of T1 Monotone TSK FIS Blending Function in Edge Detection.

In this section, demonstrations compare both monotone and non-monotone T1 TSK FIS blending function to study the relevant impact to the edge feature detail for E_{IJ} . A computer-generated image (as in Figure 5.22) is used as input image P_{IJ} to examine TSK FIS blending function. The computer generated image is designed as such, vertical grey strips with predetermined grayscale intensity $q \in \{a_1, \dots, a_i, \dots, a_n\}$ is separated by white stripes i.e., $q = 255$, where $a_1 = 0$, $a_n = 250$ and $a_i = a_{i-1} + 10$. Noted that the size of grayscale strips is 25×255 and the white stripes 5×200 . The image histogram of P_{IJ} is presented in Figure 5.23.

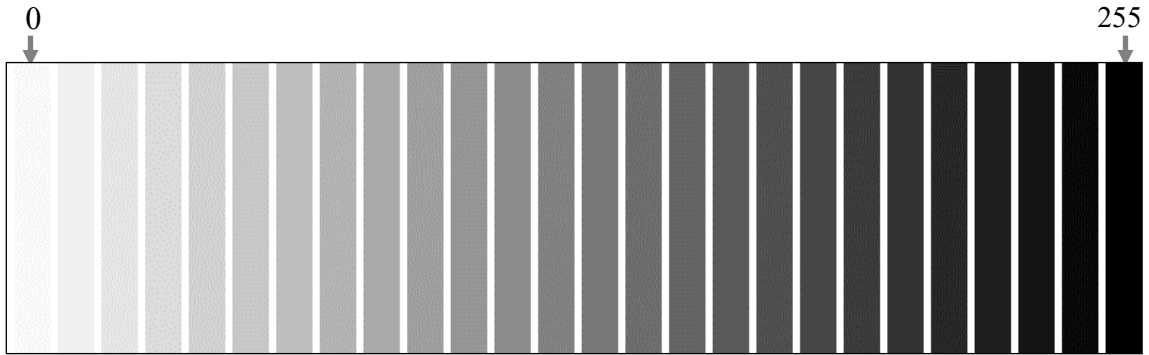


Figure 5.22: Computer-generated vertical grey strips and white stripes image, P_{IJ}

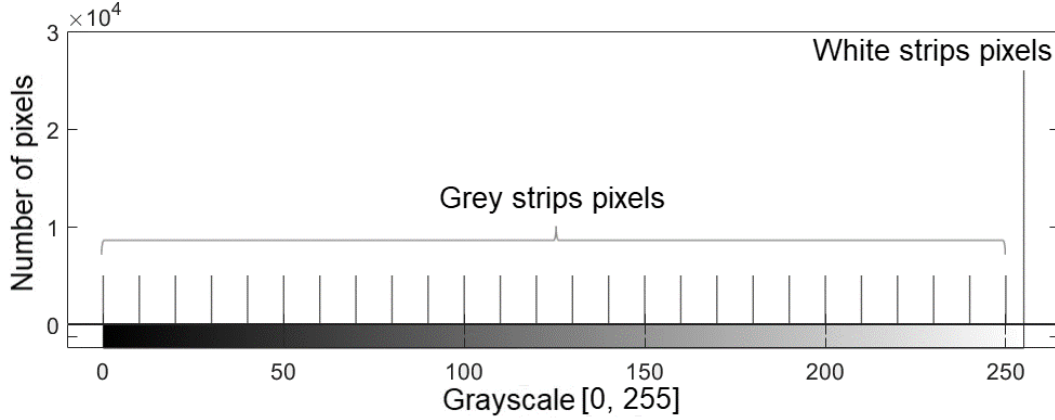


Figure 5.23: Image histogram of vertical grey strips and white strips image, P_{IJ}

The purpose of the computer-generated image P_{IJ} is used to visually inspect the blending function at different intensity jumps between the vertical grey strips and white strips. As in Figure 5.22, the vertical grey strips at right hand side are designed to have higher intensity jump at the boundary between vertical grey strips and white strips, in comparison to left hand side. As the argument in Marco-Detchart et al. (2021) research, the nature of edges implies that the bigger an intensity jump is, the larger its influence is in the possible existence of an edge. Thus, higher grayscale intensity which also indicates higher possible existence of edge feature, is expected to be more obvious at the right-hand side, in comparison to the left-hand side of the edge image output E_{IJ} .

In the demonstration, no conditional function is applied to P_{IJ} . P_{IJ} is direct feed in feature extraction function, as such, $\mathbf{f}[C_{IJ} = P_{IJ}] = \mathbf{F}_{IJ}$. Demonstration apply Sobel feature extractor function (as in Equation 5.6) to produce feature image $\mathbf{F}_{IJ}$. The inputs of $b(\mathbf{x}; t, \chi, \gamma, \omega)$ denoted as $\mathbf{x} = \{x_1, x_2\} \in \{|f_{sh}(\mathbf{w}_{ij})|, |f_{sv}(\mathbf{w}_{ij})|\}$. Consider P_{IJ} is a vertical grayscale strips and white strips image and thus, $f_{sv}(\mathbf{w}_{ij}) = 0$ for $\mathbf{w}_{ij} \in \{\forall c_{ij}\}$. The edge image output i.e., $E_{IJ} = s[B_{IJ}]$ is used to evaluate the $b(\mathbf{x}; t, \chi, \gamma, \omega)$ where s is image scaling function and B_{IJ} is blended image output of $b(\mathbf{x}; t, \chi, \gamma, \omega)$.

(A) TSK FIS Blending Function Design in Bezdek et al. (1998)

Consider that $x_2 \in |f_{sv}(\mathbf{w}_{ij})| = 0$, $b(\mathbf{x}; t, \chi, \gamma, \omega)$ with parameter settings from Bezdek et al. (1998) are adopted in the demonstration in this section i.e., $b(x_1: 0.25, 2, 2, 3)$, $b(x_1: 1, 2, 2, 3)$ and $b(x_1: 4, 2, 2, 3)$. The graph plot result of $b(x_1: 0.25, 2, 2, 3)$ are depicted in Figure 5.24 and edge image output E_{IJ} for $b(x_1: 0.25, 2, 2, 3)$ is depicted in Figure 5.25. The graph plot result of $b(x_1: 1, 2, 2, 3)$ in comparison to edge image output E_{IJ} for $b(x_1: 1, 2, 2, 3)$ are depicted in Figure 5.26 and Figure 5.27, respectively. The graph plot results of $b(x_1: 4, 2, 2, 3)$ are depicted in Figure 5.28 and edge image output E_{IJ} for $b(x_1: 4, 2, 2, 3)$ is depicted in Figure 5.29.

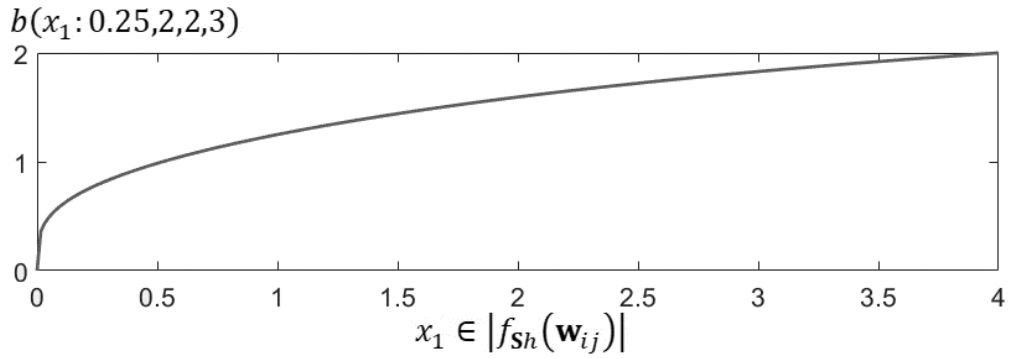


Figure 5.24: Graph plot output of $b(x_1: 0.25, 2, 2, 3)$ at $t = 0.25$ and $|f_{sv}(\mathbf{w}_{ij})| = 0$

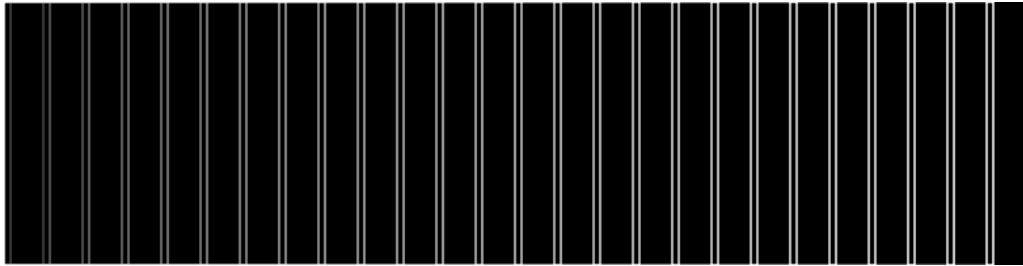


Figure 5.25: Edge image output E_{IJ} for $b(x_1: 0.25, 2, 2, 3)$ at $t = 0.25$ and $|f_{sv}(\mathbf{w}_{ij})| = 0$

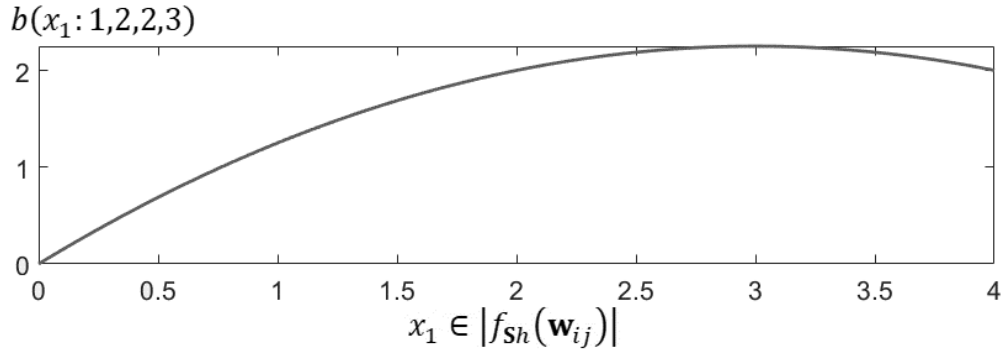


Figure 5.26: Graph plot output of $\mathbf{b}(x_1: 1,2,2,3)$ at $t = 1$ and $|\mathbf{f}_{sv}(\mathbf{w}_{ij})| = 0$

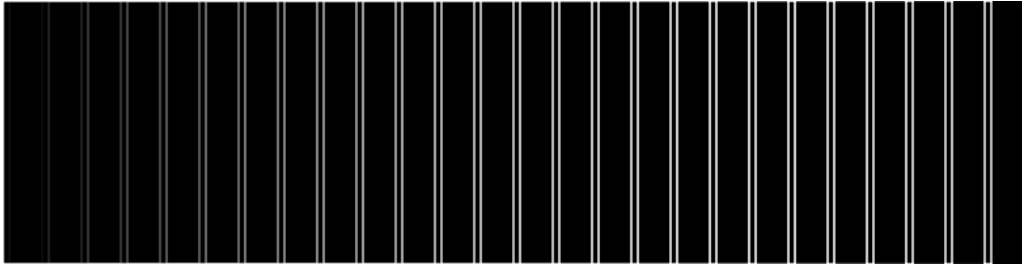


Figure 5.27: Edge image output E_{IJ} for $\mathbf{b}(x_1: 1,2,2,3)$ at $t = 1$ and $|\mathbf{f}_{sv}(\mathbf{w}_{ij})| = 0$

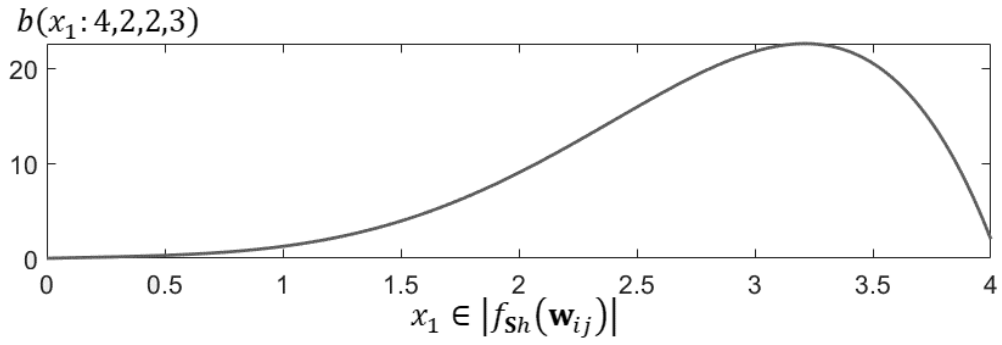


Figure 5.28: Surface plot output of $b(x_1: 4,2,2,3)$ at $t = 4$ and $\mathbf{f}_{sv}(\mathbf{w}_{ij}) = 0$

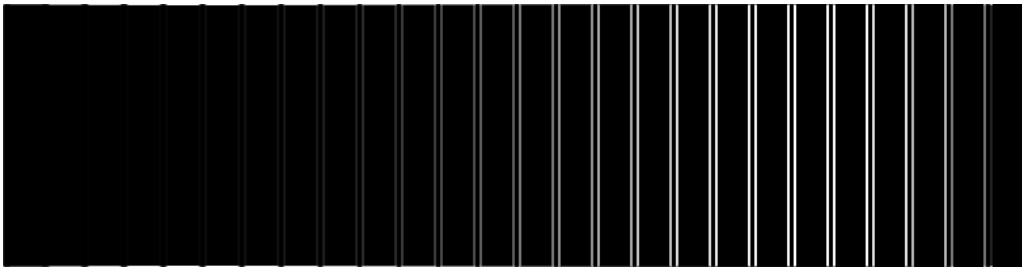


Figure 5.29: E_{IJ} for $b(x_1: 4,2,2,3)$ at $t = 4$ and $|\mathbf{f}_{sv}(\mathbf{w}_{ij})| = 0$

Consider that $b(x_1: 0.25, 2, 2, 3)$ shows a monotone output at $|f_{sv}(\mathbf{w}_{ij})| = 0$, as shows in Figure 5.24. Thus, Figure 5.25 for $b(x_1: 0.25, 2, 2, 3)$ shows that the border of white strips in P_{IJ} , after being processed to E_{IJ} , displays an increase in grayscale intensity from left to right. In contrast, the graph plot results of $b(x_1: 1, 2, 2, 3)$ and $b(x_1: 4, 2, 2, 3)$, as shown in Figures 5.26 and 5.28, respectively, show non-monotone outputs. Observations in Figures 5.27 and 5.29 show that the border of white strips in P_{IJ} , after being processed to E_{IJ} , displays brighter (i.e., higher grayscale intensity) in the middle compared to the left and right sides of E_{IJ} . Obviously, $b(x_1: 1, 2, 2, 3)$ and $b(x_1: 4, 2, 2, 3)$ do not abide by the condition that the larger an intensity jump in $p_{ij} \in P_{IJ}$ with adjacent pixels, the greater its influence in the possible existence of an edge feature in $e_{ij} \in E_{IJ}$.

(B) T1 Monotone TSK FIS Blending Function Design

Here, Remark 5.1 is adopted to facilitate the formation of the T1 monotone TSK blending function for $b(\mathbf{x}: t, \chi, \gamma, \omega)$. The demonstration examines 3 different t parameter settings, i.e., $t = 0.25$, $t = 1$ and $t = 4$. Adopted *Condition 2* from Remark 5.1, $b(\mathbf{x}: t, \chi, \gamma, \omega)$ achieves a monotone output when $\omega \geq \gamma \geq \chi \geq 2 \times 4^t$ is fulfilled. Demonstration considers only minimum requirement that satisfies $\omega = \gamma = \chi \geq 2 \times 4^t$ to allow $b(\mathbf{x}: t, \chi, \gamma, \omega)$ achieves monotone output. Table 5.2 shows the ω, γ and χ parameter settings, comparing the original $b(\mathbf{x}: t, \chi, \gamma, \omega)$ from Bezdek et al. (1998) and the $b(\mathbf{x}: t, \chi, \gamma, \omega)$ governed by Remark 5.1. Detailed calculations determining the ω, γ and χ parameter settings for ω, γ and χ with guidance from Remark 5.1 are included in Table 5.2. Note that a ceiling value of $2 \times 4^{0.25}$ (i.e., approximately 3) is used for ω, γ and χ at $t = 0.25$.

Table 5.2: Comparison of ω, γ and χ parameter settings of $b(\mathbf{x}: t, \chi, \gamma, \omega)$ from Bezdek et al. (1998) and $b(\mathbf{x}: t, \chi, \gamma, \omega)$ governed with Remark 5.1

t	Original $b(\mathbf{x}: t, \chi, \gamma, \omega)$ parameter settings from Bezdek et. al (1998)	Facilitate formation of $b(\mathbf{x}: t, \chi, \gamma, \omega)$ with Remark 5.1	
		Minimum value for ω, γ and χ	$b(\mathbf{x}: t, \chi, \gamma, \omega)$
0.25	$b(\mathbf{x}: 0.25, 2, 2, 3)$	$\omega = \gamma = \chi \geq 2 \times 4^{0.25} \approx 3$	$b(\mathbf{x}: 0.25, 3, 3, 3)$
1	$b(\mathbf{x}: 1, 2, 2, 3)$	$\omega = \gamma = \chi \geq 2 \times 4^1 = 8$	$b(\mathbf{x}: 1, 8, 8, 8)$
4	$b(\mathbf{x}: 4, 2, 2, 3)$	$\omega = \gamma = \chi \geq 2 \times 4^4 = 528$	$b(\mathbf{x}: 4, 528, 528, 528)$

The computer-generated image P_{IJ} as in Figure 5.22 is used to examine the formulated three $b(\mathbf{x}: t, \chi, \gamma, \omega)$ function with governing of Remark 5.1. Consider that $x_2 \in |f_{sv}(\mathbf{w}_{ij})| = 0$, the three $b(\mathbf{x}: t, \chi, \gamma, \omega)$ function are denoted as such, i.e., $b(x_1: 0.25, 3, 3, 3)$, $b(x_1: 1, 8, 8, 8)$ and $b(x_1: 4, 528, 528, 528)$. The graph plot results of $b(x_1: 0.25, 3, 3, 3)$ are depicted in Figure 5.30 and edge image output E_{IJ} for $b(x_1: 0.25, 3, 3, 3)$ is depicted in Figure 5.31. The graph plot results of $b(x_1: 1, 8, 8, 8)$ in comparison to edge image output E_{IJ} for $b(x_1: 1, 8, 8, 8)$ are depicted in Figure 5.32 and Figure 5.33, respectively.

The graph plot results of $b(x_1: 4, 528, 528, 528)$ are depicted in Figure 5.34 and edge image output E_{IJ} for $b(x_1: 4, 528, 528, 528)$ is depicted in Figure 5.35. The graph plot result of $b(x_1: 0.25, 3, 3, 3)$, $b(x_1: 1, 8, 8, 8)$ and $b(x_1: 4, 528, 528, 528)$ shows the monotone output at $|f_{sv}(\mathbf{w}_{ij})| = 0$, as depicted in Figure 5.30, Figure 5.32 and Figure 5.34, respectively. Thus, Figure 5.31, Figure 5.33 and Figure 5.35 all show that boarder of white strips in P_{IJ} after process to E_{IJ} , displaying an increase of the grayscale intensity from left to right.

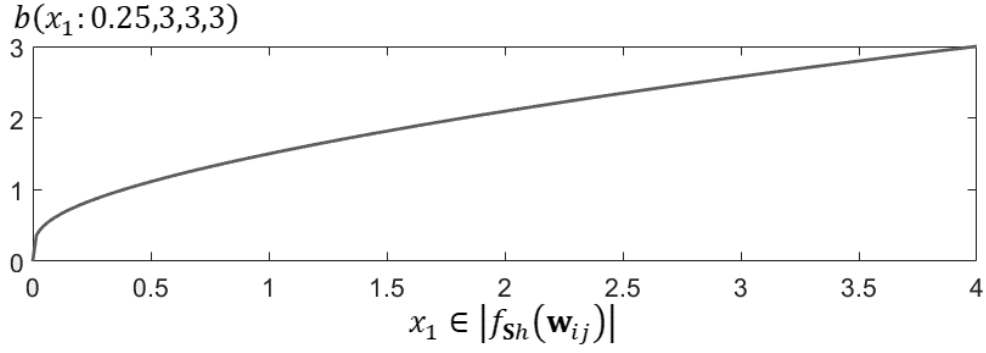


Figure 5.30: Graph plot output of $\mathbf{b}(x_1: 0.25, 3, 3, 3)$ at $t = 0.25$ and $|\mathbf{f}_{sv}(\mathbf{w}_{ij})| = 0$

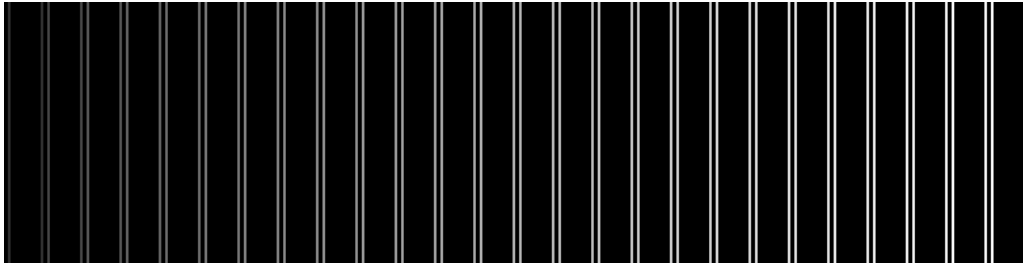


Figure 5.31: E_{IJ} for $\mathbf{b}(x_1: 0.25, 3, 3, 3)$ at $t = 0.25$ and $|\mathbf{f}_{sv}(\mathbf{w}_{ij})| = 0$

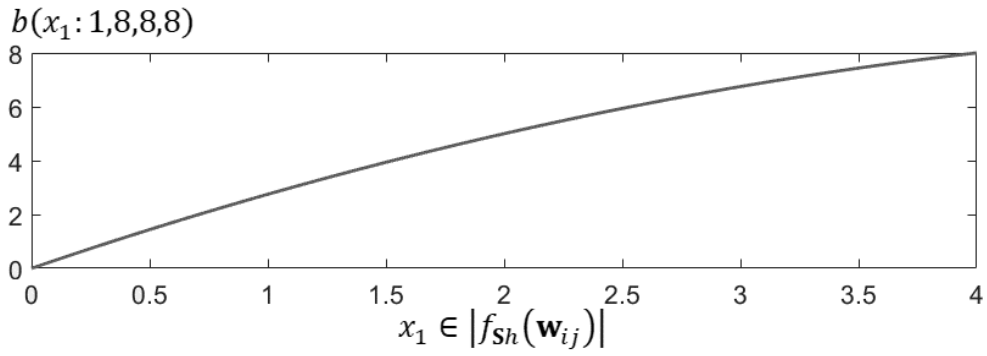


Figure 5.32: Graph plot output of $\mathbf{b}(x_1: 1, 8, 8, 8)$ at $t = 1$ and $|\mathbf{f}_{sv}(\mathbf{w}_{ij})| = 0$

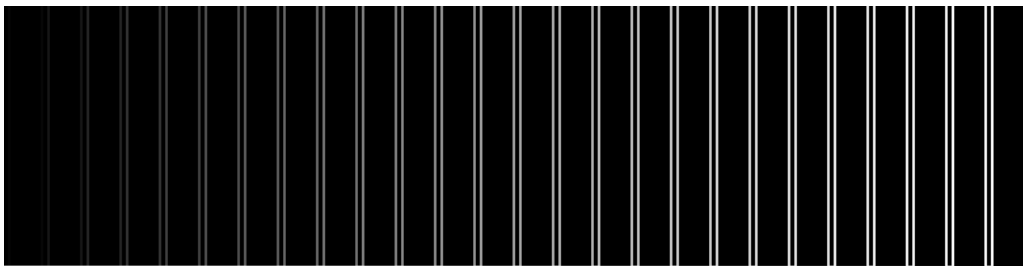


Figure 5.33: E_{IJ} for $\mathbf{b}(x_1: 1, 8, 8, 8)$ at $t = 1$ and $|\mathbf{f}_{sv}(\mathbf{w}_{ij})| = 0$

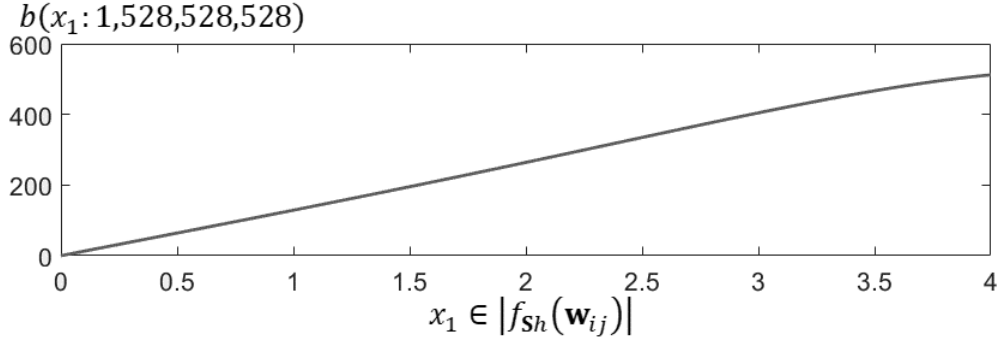


Figure 5.34: Graph plot output of $b(x_1: 1,528,528,528)$ at $t = 1$ and $f_{sv}(w_{ij}) = 0$

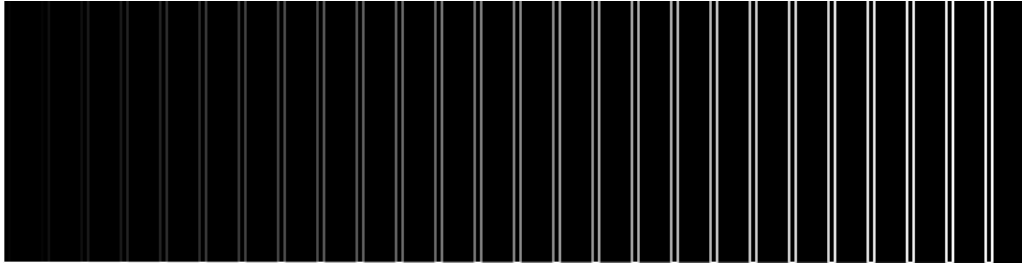


Figure 5.35: E_{IJ} for $b(x_1: 4,528,528,528)$ at $t = 4$ and $|f_{sv}(w_{ij})| = 0$

At next section, Monotonicity test is applied to evaluate the T1 TSK FIS blending function. Experiments are conducted to examine both monotone and non-monotone T1 TSK FIS blending function to the edge image output.

5.5.2 Monotonicity Test on TSK FIS Blending Function.

For easy explanation, the TSK FIS blending function is parameterized as $b(\mathbf{x}: t, \chi, \gamma, \omega)$. Monotonicity test approach from Tay and Lim (2011b), Tay et al. (2012a, 2012b) and Teh et al. (2018) is adopted to evaluate the output of $b(\mathbf{x}: t, \chi, \gamma, \omega)$. The procedure is as follows.

Step 1: Divide input range of $x_{\forall i \in \{1,2\}} = [0, 4]$ into $n = 200$ subintervals. As such, fixed step size as denoted as $\delta_{\forall i \in \{1,2\}} = (4 - 0)/200 = 0.02$ form total spaced points $\{x_{i,0}, \dots x_{i,m_i}, \dots x_{i,200}\} \in [0, 4]$ where $x_{i,m_i} = 0 + 0.02 \times m_i$.

Step 2: Let's denote $j = \{1,2\}$ and $j \neq i$, evaluate any pair $b(x_j, x_i: t, \chi, \gamma, \omega)$ and

$b(x_j, x'_i: t, \chi, \gamma, \omega)$ as in Equation 5.21, where $[x_i, x'_i] \in \{x_{i,0}, \dots, x_{i,m_i}, \dots, x_{i,200}\}$.

For a selected $x_i = x_{i,m_i}$, $x' = x_{i,m_i+1}$ should always true where $m_i < 200$.

$$\begin{aligned} & \text{Monotone} \left(b(x_j, x_i: t, \chi, \gamma, \omega), b(x_j, x'_i: t, \chi, \gamma, \omega) \right) \\ &= \begin{cases} 1 & \text{if } b(x_j, x'_i: t, \chi, \gamma, \omega) \geq b(x_j, x_i: t, \chi, \gamma, \omega) \\ 0 & \text{otherwise} \end{cases} \end{aligned} \quad \text{Equation 5.21}$$

Step 3: Evaluate $b(\mathbf{x}: t, \chi, \gamma, \omega)$ for all combination of grid points for $\forall i = \{1,2\}$ as in as in

Equation 5.21 i.e., total $n(n+1) = 200(200+1) = 40200$ compare-able pairs are available for $n = 200$. Compute the Monotone Index between y and inputs x_i as in Equation 5.22.

$$\begin{aligned} & \text{Monotone_Index}(x_i) \\ &= \frac{\sum_{m_j \neq i=0}^{m_j \neq i=n} \sum_{m_i=0}^{m_i=n-1} \left(\text{monotone} \left(b(x_j, x_i: t, \chi, \gamma, \omega), b(x_j, x'_i: t, \chi, \gamma, \omega) \right) \right)}{n(n+1)} \end{aligned} \quad \text{Equation 5.22}$$

(A) TSK FIS Blending Function Design in Bezdek et al. (1998)

Demonstration adopted examples of $b(\mathbf{x}: t, \chi, \gamma, \omega)$ functions from Bezdek *et. al* (1998) with 3 different t parameter settings i.e., $t = 0.25$, $t = 1$ and $t = 4$, with parameters $\chi = \gamma = 2$ and $\omega = 3$ are set to be same for all 3 examples. Here, the 3 examples are represented by the function as such, $b(\mathbf{x}: 0.25, 2, 2, 3)$, $b(\mathbf{x}: 1, 2, 2, 3)$ and $b(\mathbf{x}: 4, 2, 2, 3)$, with the monotonicity test result are presented in Table 5.3, Table 5.4 and Table 5.5, respectively. Result in Table 5.3, Table 5.4 and Table 5.5 shown all three TSK FIS blending function i.e., $b(\mathbf{x}: 0.25, 2, 2, 3)$, $b(\mathbf{x}: 1, 2, 2, 3)$ and $b(\mathbf{x}: 4, 2, 2, 3)$ fail to achieve the result $\text{Monotone_index}(x_{i \in \{1,2\}}) = 1$. The surface plot of $b(\mathbf{x}: 0.25, 2, 2, 3)$, $b(\mathbf{x}: 1, 2, 2, 3)$ and $b(\mathbf{x}: 4, 2, 2, 3)$ are presented in Figure 5.36, Figure 5.37 and Figure 5.38, respectively.

Table 5.3: Monotonicity test for $b(\mathbf{x}: 0.25, 2, 2, 3)$ at $t=0.25$

Input x_i	Number of monotone compare-able pairs	Monotone_Index (x_i)
x_1	38985	$\frac{38985}{40200} = 0.97$
x_2	38985	$\frac{38985}{40200} = 0.97$

Table 5.4: Monotonicity test for $b(\mathbf{x}: 1, 2, 2, 3)$ at $t = 1$

Input x_i	Number of monotone compare-able pairs	Monotone_Index (x_i)
x_1	29680	$\frac{29680}{40200} = 0.74$
x_2	29680	$\frac{29680}{40200} = 0.74$

Table 5.5: Monotonicity test for $b(\mathbf{x}: 4, 2, 2, 3)$ at $t = 4$

Input x_i	Number of monotone compare-able pairs	Monotone_Index (x_i)
x_1	17701	$\frac{17701}{40200} = 0.44$
x_2	17701	$\frac{17701}{40200} = 0.44$

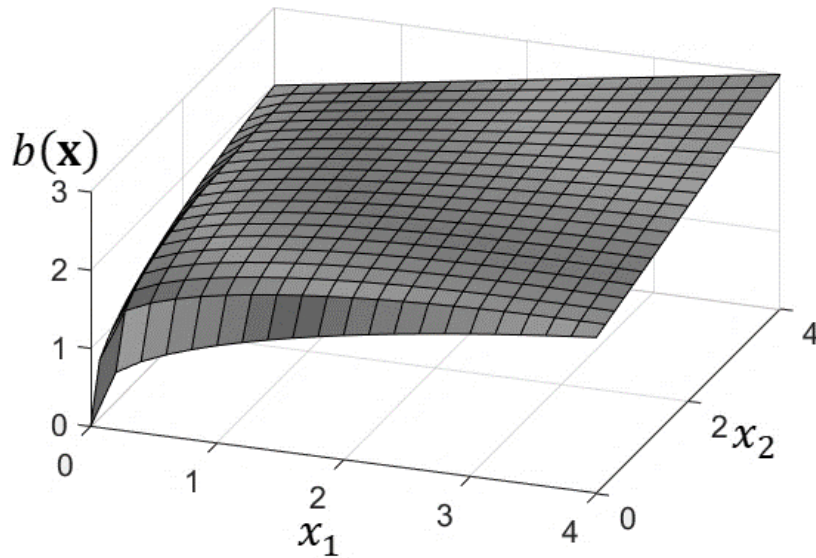


Figure 5.36: Surface plot output of $b(\mathbf{x}: 0.25, 2, 2, 3)$ at $t = 0.25$

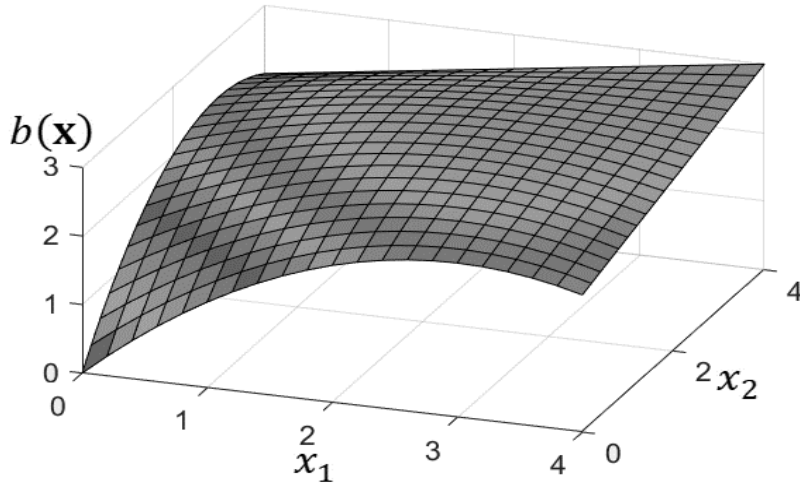


Figure 5.37: Surface plot output of $b(\mathbf{x}: 1,2,2,3)$ at $t = 1$

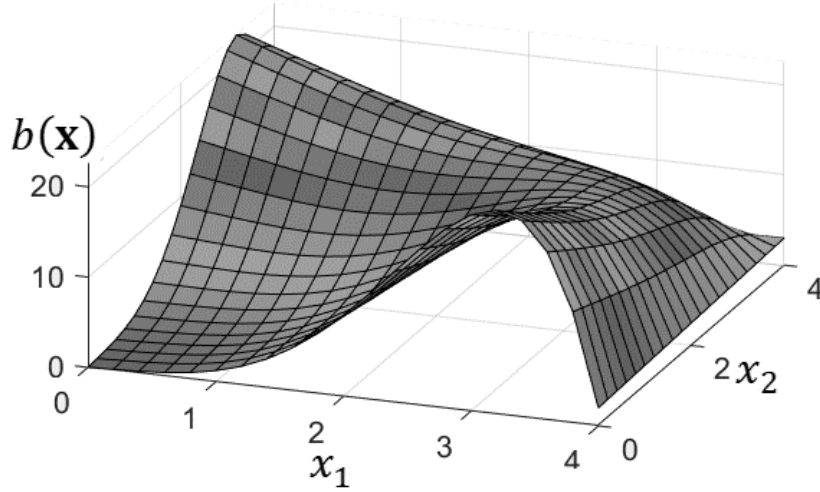


Figure 5.38: Surface plot output of $b(\mathbf{x}: 4,2,2,3)$ at $t = 4$

(B) T1 Monotone TSK FIS Blending Function Design.

The T1 monotone TSK blending function to $b(\mathbf{x}: t, \chi, \gamma, \omega)$ from Table 5. are adopted to the demonstration in this section i.e., $b(\mathbf{x}: 0.25, 3, 3, 3)$, $b(\mathbf{x}: 1, 8, 8, 8)$ and $b(\mathbf{x}: 4, 528, 528, 528)$. Results in Table 5.6, Table 5.7 and Table 5.8 show all three TSK FIS blending function i.e., $b(\mathbf{x}: 0.25, 3, 3, 3)$, $b(\mathbf{x}: 1, 8, 8, 8)$ and $b(\mathbf{x}: 4, 528, 528, 528)$ achieve the result $\text{Monotone_index}(x_{i \in \{1,2\}}) = 1$. The surface plot of $b(\mathbf{x}: 0.25, 3, 3, 3)$, $b(\mathbf{x}: 1, 8, 8, 8)$

and $b(\mathbf{x}: 4,528,528,528)$ as presented in Figure 5.39, Figure 5.40 and Figure 5.41, respectively.

Table 5.6: Monotonicity test for $b(\mathbf{x}: 0.25,3,3,3)$ at $t = 0.25$

Input x_i	Number of monotone compare-able pairs	Monotone_Index (x_i)
x_1	40200	$\frac{40200}{40200} = 1$
x_2	40200	$\frac{40200}{40200} = 1$

Table 5.7: Monotonicity test for $b(\mathbf{x}: 1,8,8,8)$ at $t = 1$

Input x_i	Number of monotone compare-able pairs	Monotone_Index (x_i)
x_1	40200	$\frac{40200}{40200} = 1$
x_2	40200	$\frac{40200}{40200} = 1$

Table 5.8: Monotonicity test for $b(\mathbf{x}: 4,528,528,528)$ at $t = 4$

Input x_i	Number of monotone compare-able pairs	Monotone_Index (x_i)
x_1	40200	$\frac{40200}{40200} = 1$
x_2	40200	$\frac{40200}{40200} = 1$

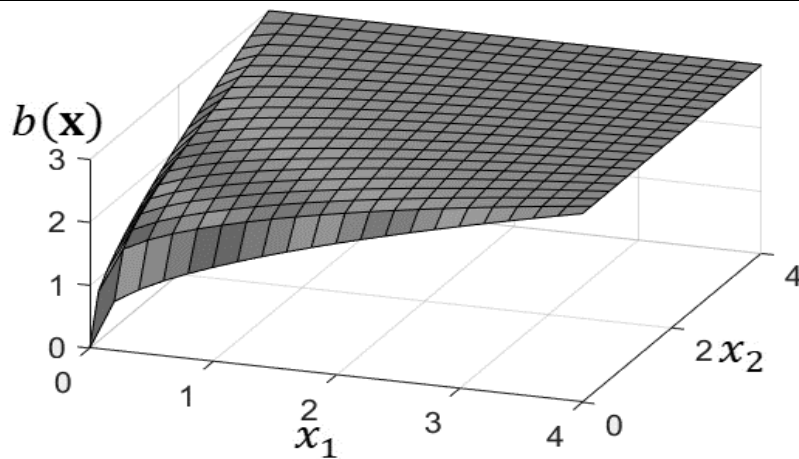


Figure 5.39: Surface plot output of $b(\mathbf{x}: 0.25,3,3,3)$ at $t = 0.25$

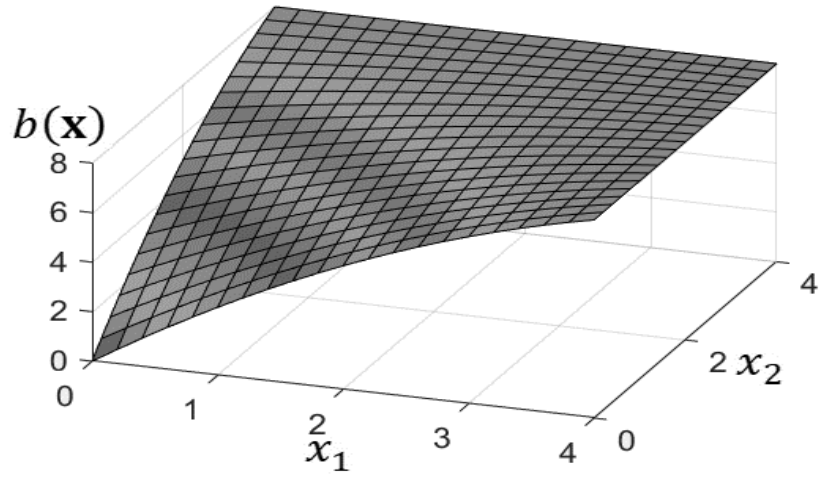


Figure 5.40: Surface plot output of $b(\mathbf{x}: 1,8,8,8)$ at $t = 1$

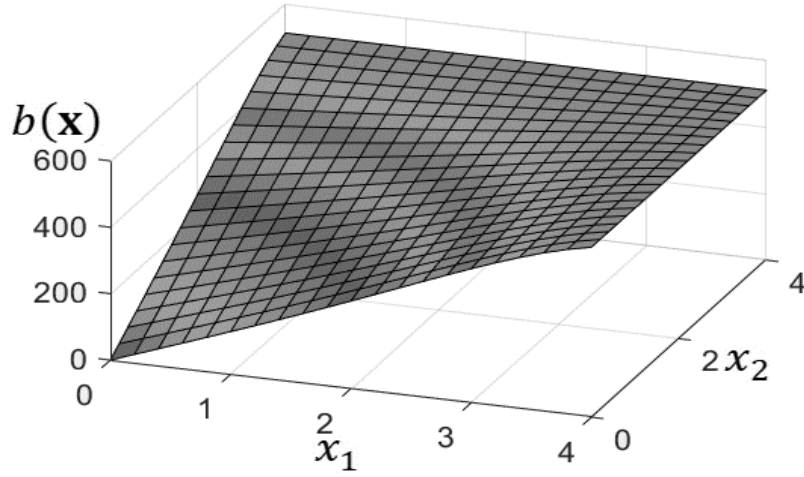


Figure 5.41: Surface plot output of $b(\mathbf{x}: 4,528,528,528)$ at $t = 4$

As in Figure 5.39, Figure 5.40 and Figure 5.41, $b(\mathbf{x}: t, \chi, \gamma, \omega)$ governed with Remark 5.1 displays profile view in analogy to a waterfall blending function (as in Section 5.3.3(A)). Consider $t = 1$ as reference point and $b(\mathbf{x}: t, \chi, \gamma, \omega)$ is governed with Remark 5.1, $b(\mathbf{x}: t, \chi, \gamma, \omega)$ with $t > 1$ displays a more flat plane in surface plot, e.g., $t = 4$ as in Figure 5.41. In contrast, surface plot of $b(\mathbf{x}: t, \chi, \gamma, \omega)$ with $t < 1$ displays a sharper change at concave or convex region near origin in feature space $\mathbf{x} = \{x_1, x_2\} \in \mathfrak{R}^1 \times \mathfrak{R}^2$, e.g., $t = 0.25$ as in Figure 5.39. In short, parameter t setting changes the output behaviour at concave or convex region near origin in feature space $\mathfrak{R}^1 \times \mathfrak{R}^2$. Demonstrations are presented in the

next section to explain the usefulness of the parameter t setting in the $b(\mathbf{x}; t, \chi, \gamma, \omega)$ with governing Remark 5.1 to improve the edge feature output in E_{IJ} .

5.6 Application edge detection framework on digitized mammogram

This section is devoted to examples of edge images E_{IJ} produced using various parameter t setting in the $b(\mathbf{x}; t, \chi, \gamma, \omega)$ with governing Remark 5.1 as discussed in section 5.5.2. The contrast of experimental raw image inputs P_{IJ} are enhanced with histogram equalization (as in Equation 5.1) at Conditioning function stage and produce C_{IJ} . 4 digitized mammograms of a normal patient from the MIAS database are selected in the demonstration i.e., image “*mdb107*”, “*mdb108*”, “*mdb279*” and “*mdb280*”, with the corresponding conditioned output image C_{IJ} are presented in Figure 5.42, Figure 5.43, Figure 5.44 and Figure 5.45, respectively.

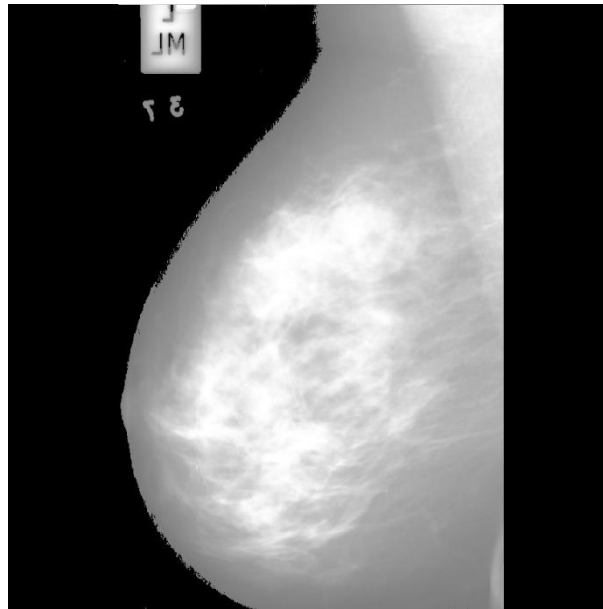


Figure 5.42: C_{IJ} of mammographic Image “*mdb107*” from database of MIAS

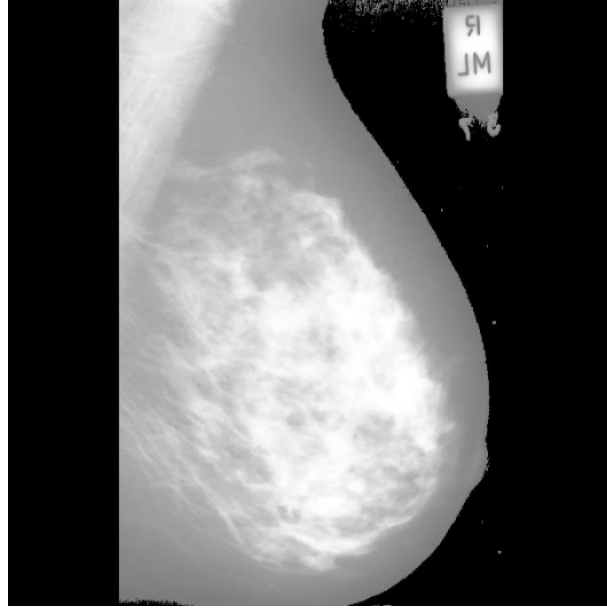


Figure 5.43: C_{IJ} of mammographic Image “mdb108” from database of MIAS

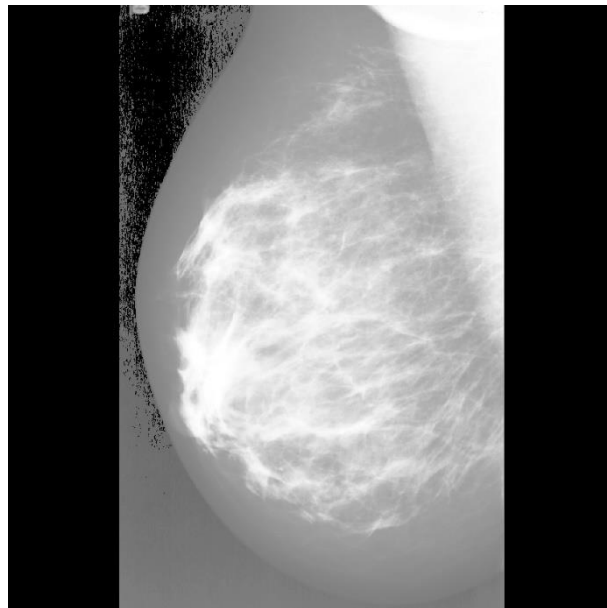


Figure 5.44: C_{IJ} of mammographic Image “mdb279” from database of MIAS

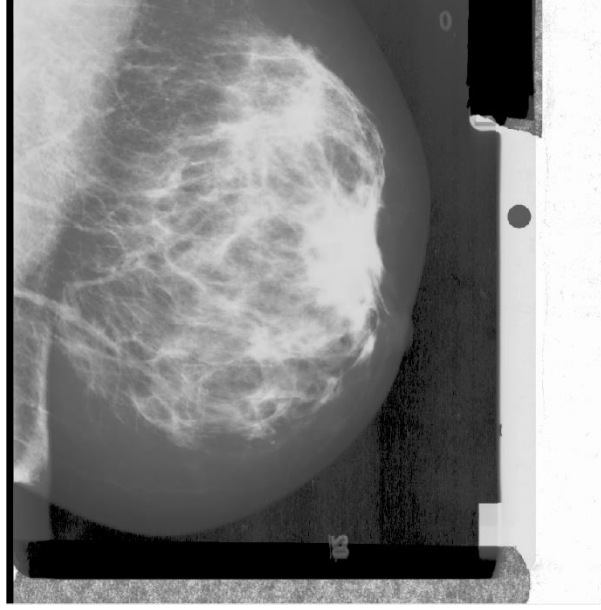


Figure 5.45: C_{IJ} of mammographic Image “mdb280” from database of MIAS

5.6.1 Image Histogram Analysis for Feature Extraction Image

Demonstration examines both Sobel operator and Prewitt operator to produce extracted feature image F_{IJ} . Image histogram analysis algorithm from Section 5.4.1 are adopted to interpreting F_{IJ} . Noted that, image histogram analysis algorithm required $F_{IJ} = [F_{IJ}^1, F_{IJ}^2]$ are mapped into grayscale $Q \in [0, 255]$. For case Sobel operator is implemented in feature extraction process, image histogram of $F_{IJ} = [F_{IJ}^1, F_{IJ}^2]$ for image “mdb107”, “mdb108”, “mdb279” and “mdb280” are presented in Figure 5.46, Figure 5.47, Figure 5.48 and Figure 5.49, respectively. Overall, observations on Figure 5.46, Figure 5.47, Figure 5.48 and Figure 5.49 show that all the image histogram of F_{IJ} for image “mdb107”, “mdb108”, “mdb109” and “mdb110” display the right skewed number of pixels distribution.

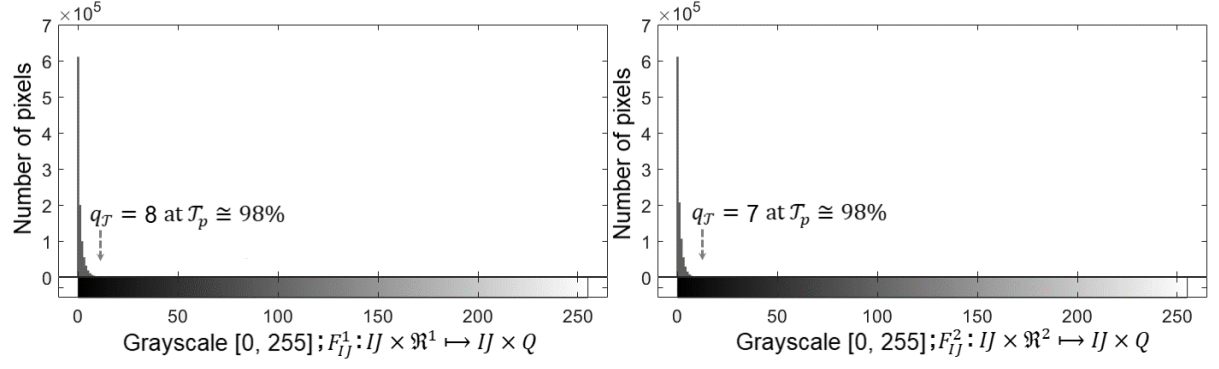


Figure 5.46: Image histogram of $\mathbf{F}_{IJ}$ processed Sobel operator for “mdb107” image

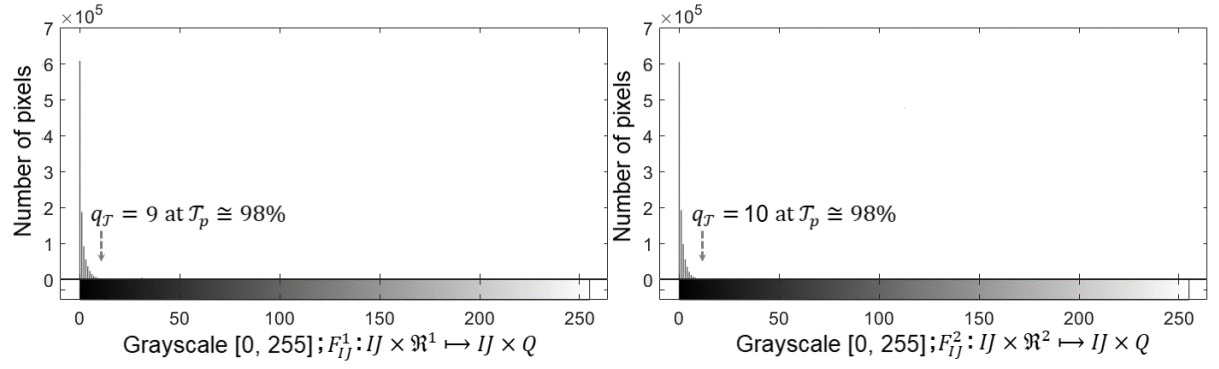


Figure 5.47: Image histogram of $\mathbf{F}_{IJ}$ processed Sobel operator for “mdb108” image

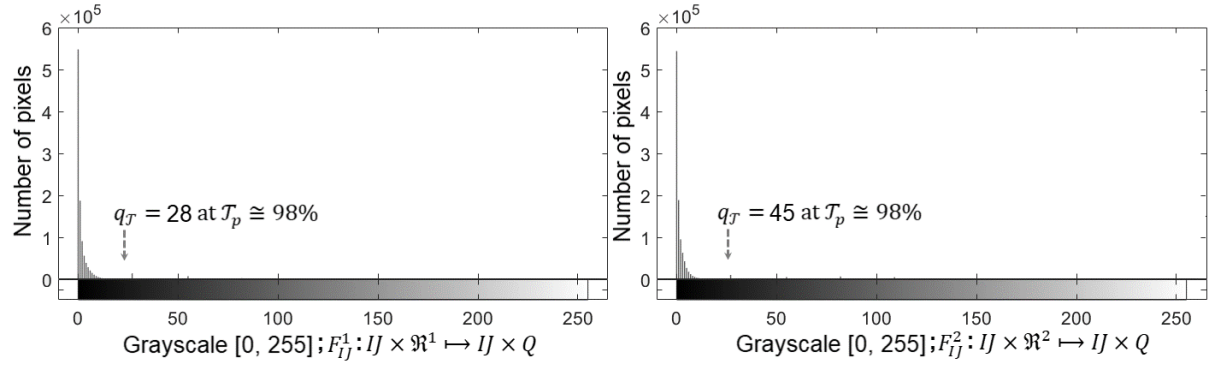


Figure 5.48: Image histogram of $\mathbf{F}_{IJ}$ processed Sobel operator for “mdb279” image

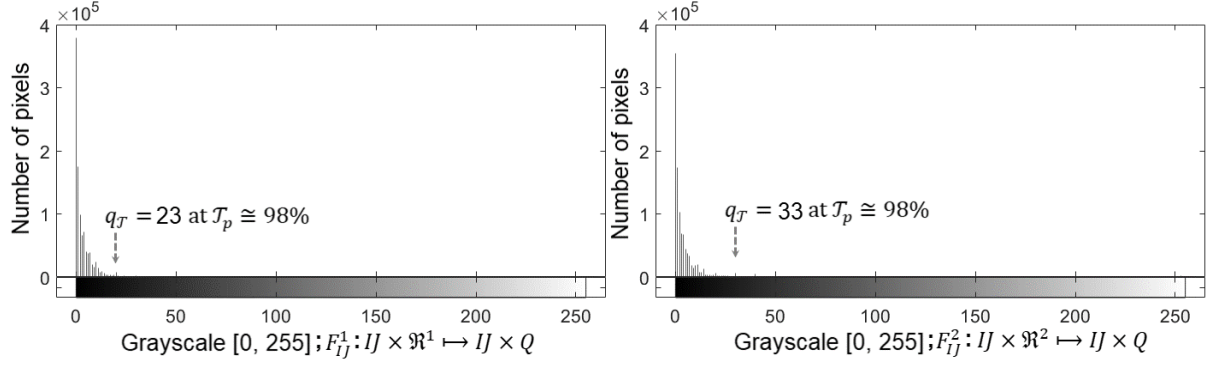


Figure 5.49: Image histogram of $\mathbf{F}_{IJ}$ processed Sobel operator for “mdb280” image

Remind that $q_{\mathcal{T}}$ is the highest grayscale intensity for the first accumulated number of pixels at set threshold approximate 98%. The result of $q_{\mathcal{T}}$ of $\mathbf{F}_{IJ}$ for image “mdb107”, “mdb108”, mdb279” and “mdb280” are summarized in Table 5.9. The example image “mdb107” in Table 5.9 and corresponding image histogram of $\mathbf{F}_{IJ}$ in Figure 5.46 are referred, approximately 98% pixels of feature image i.e., $f_{ij}^1 \in F_{IJ}^1$ fall below grayscale intensity $q_{\mathcal{T}} = 8$. Meanwhile, approximately 98% pixels of feature image for image “mdb107” i.e., $f_{ij}^2 \in F_{IJ}^2$ are below grayscale intensity $q_{\mathcal{T}} = 7$.

Table 5.9: $q_{\mathcal{T}}$ of $\mathbf{F}_{IJ}$ processed Sobel operator at $\mathcal{T} = 0.98$ for image “mdb107”, “mdb108”, “mdb279” and “mdb280”

Image	Extracted Feature Image	$\mathcal{T}_p$ at $\mathcal{T} = 0.98$	$q_{\mathcal{T}}$ at $\mathcal{T} = 0.98$
“mdb107”	F_{IJ}^1	98.3383%	8
	F_{IJ}^2	98.1891%	7
“mdb108”	F_{IJ}^1	98.1039%	9
	F_{IJ}^2	98.1942%	9
“mdb279”	F_{IJ}^1	98.6927%	28
	F_{IJ}^2	98.0006%	46
“mdb280”	F_{IJ}^1	98.0694%	23
	F_{IJ}^2	98.0217%	33

For case Prewitt operator is implemented in feature extraction process, image histogram of $\mathbf{F}_{IJ} = [F_{IJ}^1, F_{IJ}^2]$ for image “mdb107”, “mdb108”, “mdb109” and “mdb110”, are presented in Figure 5.50, Figure 5.51, Figure 5.52 and Figure 5.53 respectively. Again, all the image histogram of $\mathbf{F}_{IJ}$ for image “mdb107”, “mdb108”, “mdb279” and “mdb280” as depicted in Figure 5.50, Figure 5.51, Figure 5.52 and Figure 5.53, respectively, display the right skewed number of pixels distribution.

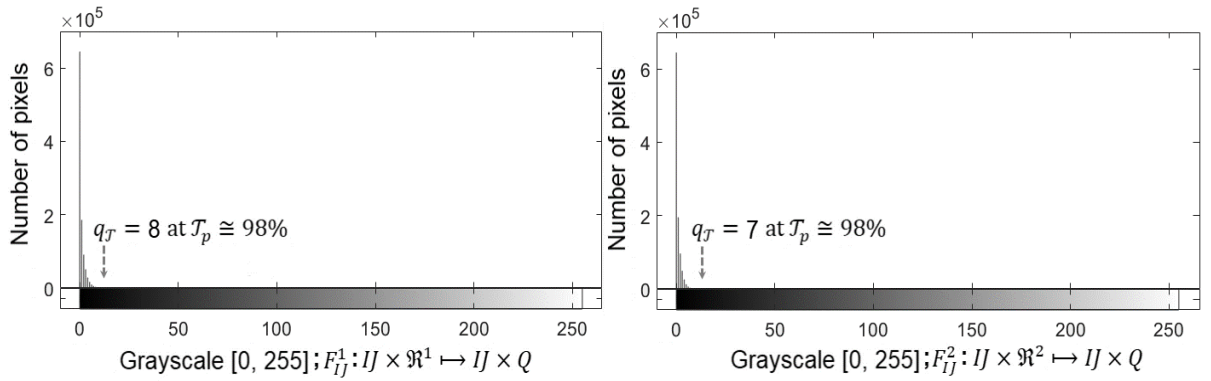


Figure 5.50: Image histogram of $\mathbf{F}_{IJ}$ processed Prewitt operator for “mdb107” image

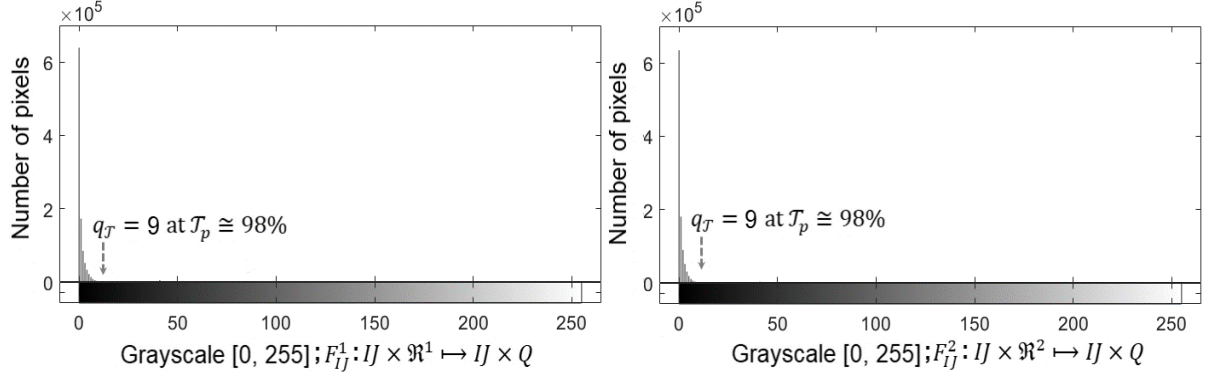


Figure 5.51: Image histogram of $\mathbf{F}_{IJ}$ processed Prewitt operator for “mdb108” image

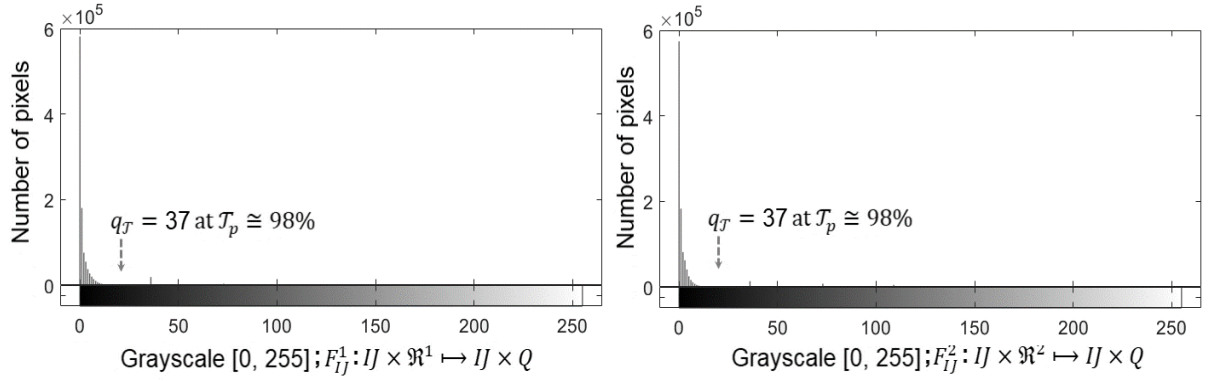


Figure 5.52: Image histogram of $\mathbf{F}_{IJ}$ processed Prewitt operator for “mdb279” image

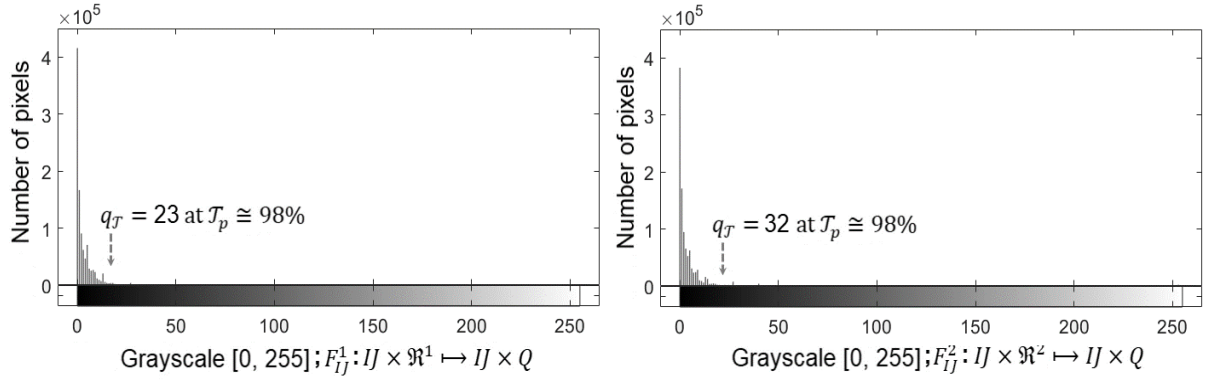


Figure 5.53: Image histogram of $\mathbf{F}_{IJ}$ processed Prewitt operator for “mdb280” image

The result of q_T of $\mathbf{F}_{IJ}$ for image “mdb107”, “mdb108”, “mdb279” and “mdb280” are summarized in Table 5.10. Overall, there are no vast variations between result in Table 5.9 and Table 5.10. For some case, e.g., “mdb107” and “mdb280”, same result of q_T are obtained for both F_{IJ}^1 and F_{IJ}^2 in Table 5.9 and Table 5.10. However, T_p value from the F_{IJ}^1 of

image “mdb279” shown the obvious divergence from the expected value 98%, i.e., 99.3309%.

Table 5.10: q_T of $\mathbf{F}_{IJ}$ processed Prewitt operator at $T = 0.98$ for image “Lenna” and “mdb313”

Image	Extracted Feature Image	$\mathcal{T}_p$ at $T = 0.98$	q_T at $T = 0.98$
“mdb107”	F_{IJ}^1	98.4456%	8
	F_{IJ}^2	98.2981%	7
“mdb108”	F_{IJ}^1	98.2201%	9
	F_{IJ}^2	98.0324%	9
“mdb279”	F_{IJ}^1	99.3309%	37
	F_{IJ}^2	98.2750%	37
“mdb280”	F_{IJ}^1	98.1120%	23
	F_{IJ}^2	98.0524%	33

5.6.2 Parameter t value and T1 Monotone TSK FIS blending function design.

At the previous section, feature image i.e., $\mathbf{F}_{IJ}$ displays an image histogram with the right skewed number of pixels distribution for all image “mdb107”, “mdb108”, “mdb109” and “mdb110”. Thus, $b(\mathbf{x}; t, \chi, \gamma, \omega)$ with $t \leq 1$ is more suitable as the blending function in the edge detection framework in this case. In this section, 4 parameter settings for $b(\mathbf{x}; t, \chi, \gamma, \omega)$ are examined i.e., $t = 0.4$, $t = 0.3$, $t = 0.2$ and $t = 0.1$, with the governing of Remark 5.1.

Table 5.11 show the ω, γ and χ parameter settings for $b(\mathbf{x}; t, \chi, \gamma, \omega)$ governed with Remark 5.1. Detail calculation on determining ω, γ and χ parameter settings for $b(\mathbf{x}; t, \chi, \gamma, \omega)$ with guidance from Remark 5.1 are included in Table 5.11. Noted that, a ceil result with 2 decimal point is considered for 2×4^t to ensure the $\omega = \gamma = \chi \geq 2 \times 4^t$ is fulfilled. As example for $t = 0.01$ in Table 5.11, the ceil result with 2 decimal point for $2 \times 4^{0.1} \approx 2.297$ is approximated to 2.30.

Table 5.11. ω, γ and χ parameter settings of $b(\mathbf{x}: t, \chi, \gamma, \omega)$ from Bezdek *et. al* (1998) and $b(\mathbf{x}: t, \chi, \gamma, \omega)$ governed with Remark 5.1

t	Facilitate formation of $b(\mathbf{x}: t, \chi, \gamma, \omega)$ with Remark 5.1	
	Minimum value for ω, γ and χ	$b(\mathbf{x}: t, \chi, \gamma, \omega)$
0.4	$\omega = \gamma = \chi \geq 2 \times 4^{0.4} = 3.49$	$b(\mathbf{x}: 0.4, 3.49, 3.49, 3.49)$
0.3	$\omega = \gamma = \chi \geq 2 \times 4^{0.3} = 3.04$	$b(\mathbf{x}: 0.3, 3.04, 3.04, 3.04)$
0.2	$\omega = \gamma = \chi \geq 2 \times 4^{0.2} = 2.64$	$b(\mathbf{x}: 0.2, 2.64, 2.64, 2.64)$
0.1	$\omega = \gamma = \chi \geq 2 \times 4^{0.1} = 2.30$	$b(\mathbf{x}: 0.1, 2.3, 2.3, 2.3)$

The monotonicity test is adopted in this section and monotone index for $b(\mathbf{x}: 0.4, 3.49, 3.49, 3.49)$, $b(\mathbf{x}: 0.3, 3.04, 3.04, 3.04)$, $b(\mathbf{x}: 0.2, 2.64, 2.64, 2.64)$ and $b(\mathbf{x}: 0.1, 2.3, 2.3, 2.3)$ are show in Table 5.12, Table 5.13, Table 5.14 and Table 5.15 respectively. Despite different t value, results in Table 5.12, Table 5.13, Table 5.14 and Table 5.15 shown all TSK FIS blending function achieve the result $\text{Monotone_index}(x_{i \in \{1,2\}}) = 1$.

Table 5.12: Monotonicity test for $b(\mathbf{x}: 0.4, 3.49, 3.49, 3.49)$ at $t = 0.4$

Input x_i	Number of monotone compare-able pairs	Monotone_Index (x_i)
x_1	40200	$\frac{40200}{40200} = 1$
x_2	40200	$\frac{40200}{40200} = 1$

Table 5.13: Monotonicity test for $b(\mathbf{x}: 0.3, 3.04, 3.04, 3.04)$ at $t = 0.3$

Input x_i	Number of monotone compare-able pairs	Monotone_Index (x_i)
x_1	40200	$\frac{40200}{40200} = 1$
x_2	40200	$\frac{40200}{40200} = 1$

Table 5.14: Monotonicity test for $b(\mathbf{x}: 0.2, 2.64, 2.64, 2.64)$ at $t = 0.2$

Input x_i	Number of monotone compare-able pairs	Monotone_Index (x_i)
x_1	40200	$\frac{40200}{40200} = 1$
x_2	40200	$\frac{40200}{40200} = 1$

Table 5.15: Monotonicity test for $b(\mathbf{x}: 0.1, 2.3, 2.3, 2.3)$ at $t = 0.1$

Input x_i	Number of monotone compare-able pairs	Monotone_Index (x_i)
x_1	40200	$\frac{40200}{40200} = 1$
x_2	40200	$\frac{40200}{40200} = 1$

The surface plot of $(\mathbf{x}: 0.4, 3.49, 3.49, 3.49)$, $b(\mathbf{x}: 0.3, 3.04, 3.04, 3.04)$, $b(\mathbf{x}: 0.2, 2.64, 2.64, 2.64)$ and $b(\mathbf{x}: 0.1, 2.3, 2.3, 2.3)$ are presented in Figure 5.54, Figure 5.55, Figure 5.56 and Figure 5.57 respectively. Noted that $b(\mathbf{x}: 0.1, 2.3, 2.3, 2.3)$ as in Figure 5.57 show the sharpest change at concave or convex region near origin in feature space $\{x_1, x_2\} \in \mathbb{R}^1 \times \mathbb{R}^2$, followed by $b(\mathbf{x}: 0.2, 2.64, 2.64, 2.64)$ in Figure 5.56, $b(\mathbf{x}: 0.3, 3.04, 3.04, 3.04)$ in Figure 5.55 and $(\mathbf{x}: 0.4, 3.49, 3.49, 3.49)$ in Figure 5.54.

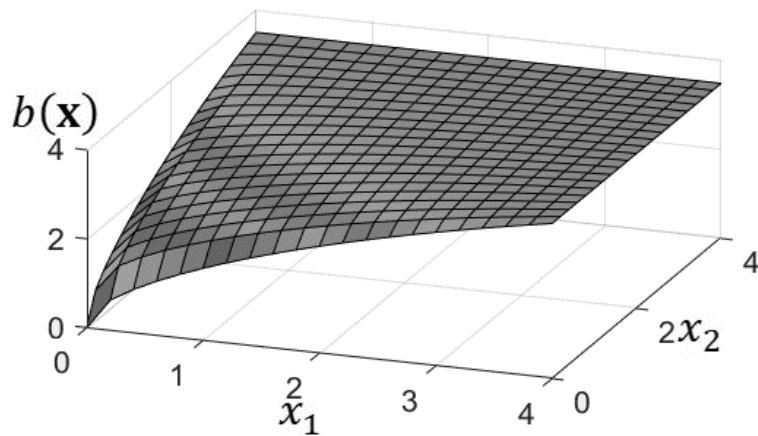


Figure 5.54: Surface plot output of $b(\mathbf{x}: 0.4, 3.49, 3.49, 3.49)$ at $t = 0.4$

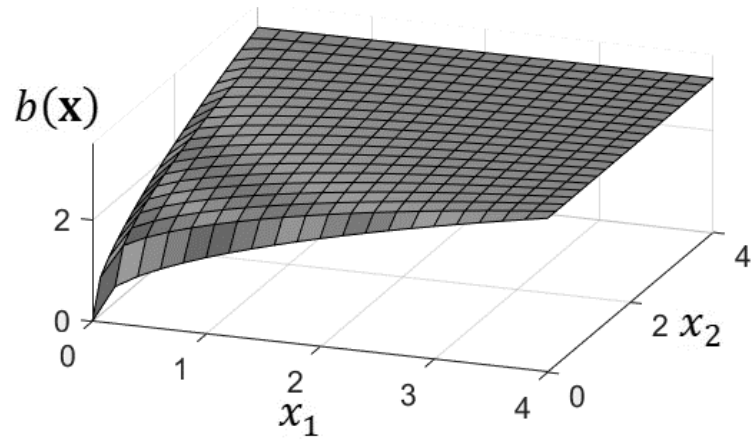


Figure 5.55: Surface plot output of $\mathbf{b}(\mathbf{x}: 0.3, 0.3, 3.04, 3.04, 3.04)$ at $t = 0.1$

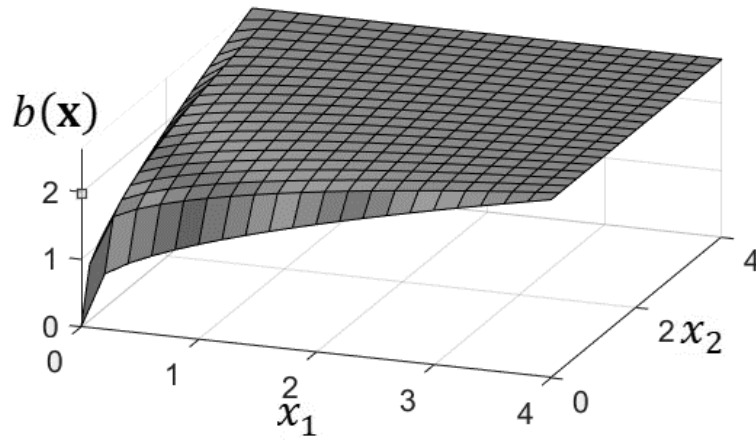


Figure 5.56: Surface plot output of $\mathbf{b}(\mathbf{x}: 0.2, 2.64, 2.64, 2.64)$ at $t = 0.2$

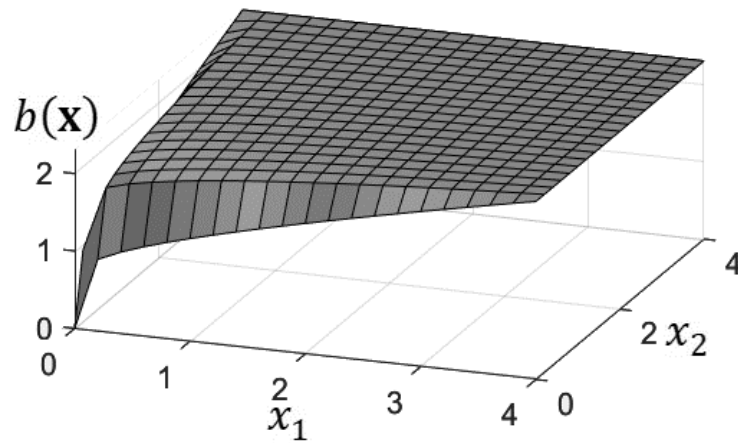


Figure 5.57: Surface plot output of $\mathbf{b}(\mathbf{x}: 0.1, 2.3, 2.3, 2.3)$ at $t = 0.1$

5.6.3 Application T1 Monotone TSK FIS blending on F_{IJ} processed with Sober operator.

In this section, two conditioned images C_{IJ} of digitized mammogram i.e., image “*mdb107*”, “*mdb108*”, “*mdb279*” and “*mdb280*” are selected in this demonstration. In the demonstration, C_{IJ} of image “*mdb107*”, “*mdb108*”, “*mdb279*” and “*mdb280*” are processed with the Sober operators to produce featured image $F_{IJ} = [F_{IJ}^1, F_{IJ}^2]$. Noted that inputs of $b(\mathbf{x}; t, \chi, \gamma, \omega)$ is denoted by $\mathbf{x} = \{x_1, x_2\} \in \{[0,4] \times [0,4]\}$. As in Equation 5.12, the F_{IJ} contains image $[F_{IJ}^1, F_{IJ}^2]$ in space with a range denoted by $\mathfrak{R}^1 \times \mathfrak{R}^2 = [0,1020] \times [0,1020]$. Thus, mapping $\mathfrak{R}^1 \times \mathfrak{R}^2 \mapsto [0,4] \times [0,4]$ requires dividing each pixel in F_{IJ} i.e., $f_{ij}^1 \in F_{IJ}^1$ and $f_{ij}^2 \in F_{IJ}^2$ by 255.

4 parameter settings for $b(\mathbf{x}; t, \chi, \gamma, \omega)$ as in Table 5.11 are examined, i.e., $(\mathbf{x}: 0.4, 3.49, 3.49, 3.49)$, $b(\mathbf{x}: 0.3, 3.04, 3.04, 3.04)$, $b(\mathbf{x}: 0.2, 2.64, 2.64, 2.64)$ and $b(\mathbf{x}: 0.1, 2.3, 2.3, 2.3)$. The edge image E_{IJ} processed by $b(\mathbf{x}; t, \chi, \gamma, \omega)$ at parameter settings $t = 0.4$, $t = 0.3$, $t = 0.2$ and $t = 0.1$ for image “*mdb107*”, “*mdb108*”, “*mdb279*” and “*mdb280*” are show in Figure 5.58, Figure 5.59, Figure 5.60 and Figure 5.61, respectively. As shown in Figure 5.58, Figure 5.59, Figure 5.60 and Figure 5.61, reducing the t value setting from 0.4 to 0.1 show the improve the edge feature details in E_{IJ} . Meantime, it also increases the risk of creating overexposure-like effect in the E_{IJ} . As the example in Figure 5.58, significant overexposure-like effect is observed in the E_{IJ} processed by $b(\mathbf{x}; t, \chi, \gamma, \omega)$ at $t = 0.1$ compared other settings at $t = 0.4$, $t = 0.3$ and $t = 0.2$. Similar result are observed in case image “*mdb108*”, “*mdb279*” and “*mdb280*” as shown in Figure 5.59, Figure 5.60 and Figure 5.61, respectively.

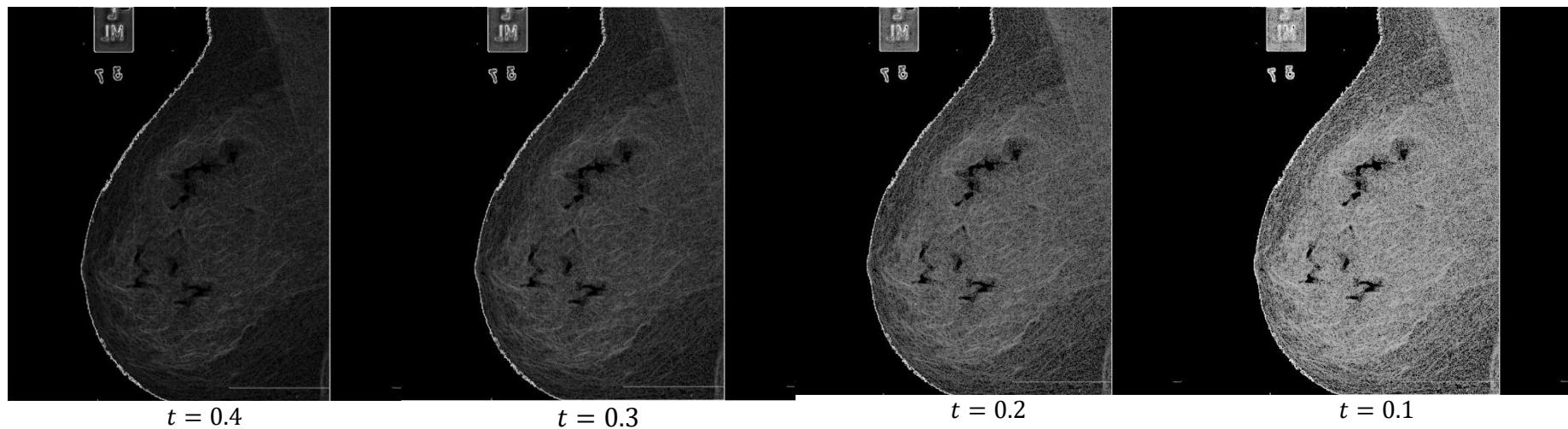


Figure 5.58: E_{IJ} for image “mdb107” processed by $\mathbf{b}(\mathbf{x}; t, \chi, \gamma, \omega)$ at parameter setting $t = 0.4, t = 0.3, t = 0.2$ and $t = 0.1$

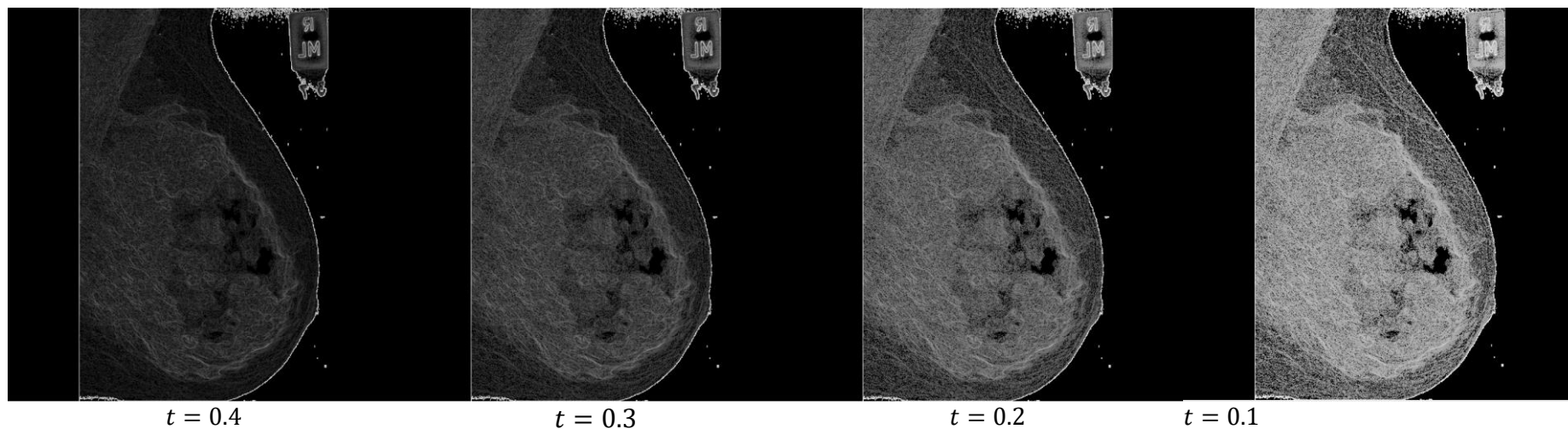


Figure 5.59: E_{IJ} image “mdb108” processed by $\mathbf{b}(\mathbf{x}; t, \chi, \gamma, \omega)$ at parameter setting $t = 0.4, t = 0.3, t = 0.2$ and $t = 0.1$

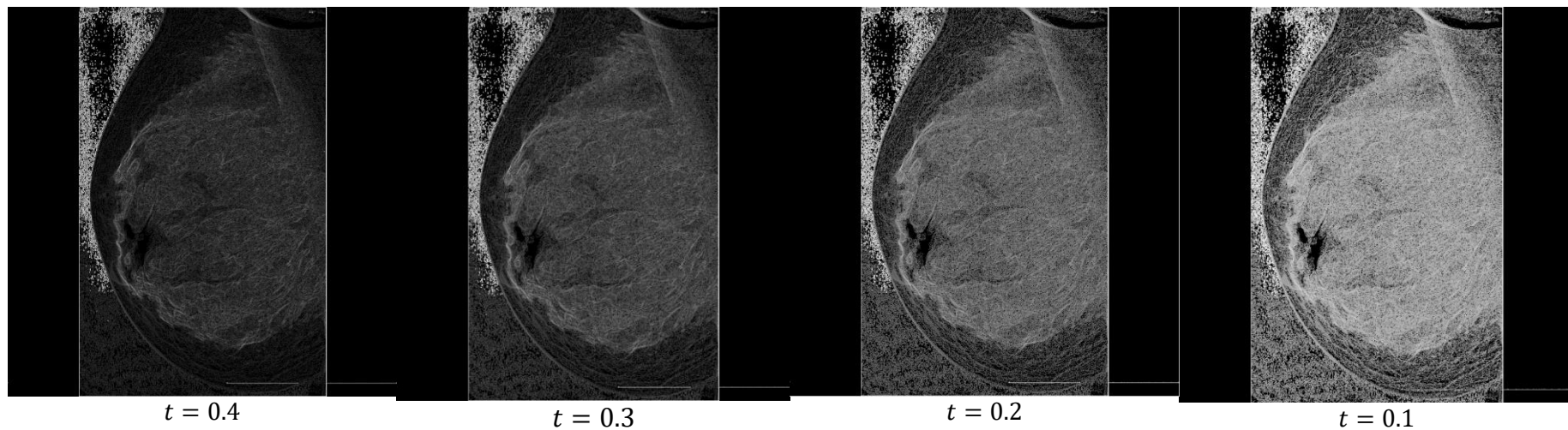


Figure 5.60: E_{IJ} image “mdb279” processed by $\mathbf{b}(\mathbf{x}; t, \chi, \gamma, \omega)$ at parameter setting $t = 0.4, t = 0.3, t = 0.2$ and $t = 0.1$

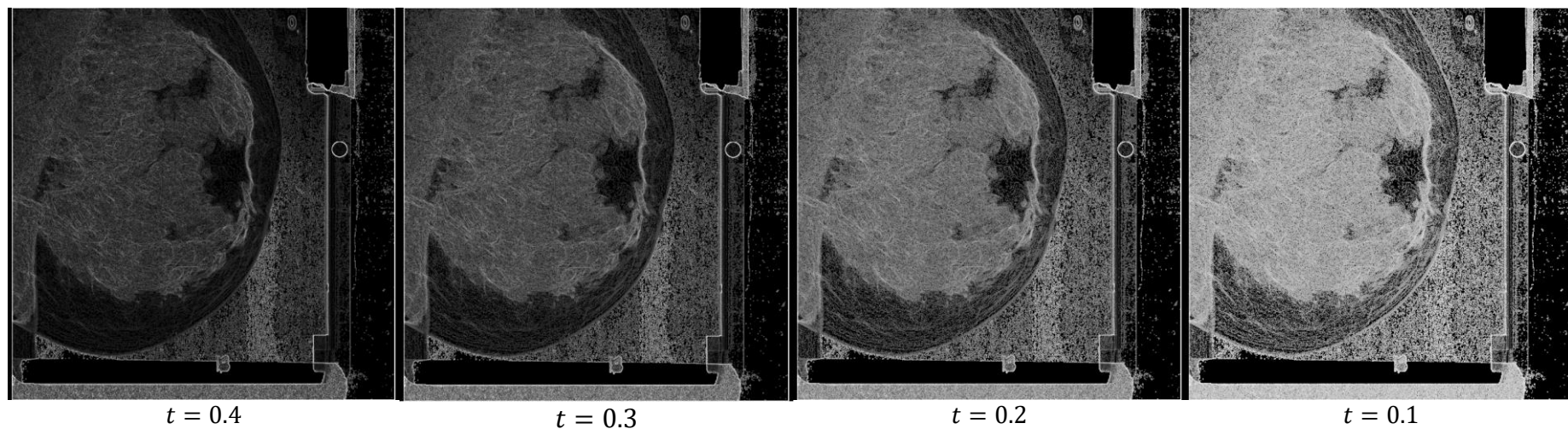


Figure 5.61: E_{IJ} image “mdb280” processed by $\mathbf{b}(\mathbf{x}; t, \chi, \gamma, \omega)$ at parameter setting $t = 0.4, t = 0.3, t = 0.2$ and $t = 0.1$

5.6.4 Application T1 Monotone TSK FIS blending on F_{IJ} processed with Prewitt operator.

In this section, demonstration continued with the Prewitt operators to process the conditioned image C_{IJ} of digitized mammogram i.e., image “*mdb107*”, “*mdb108*”, “*mdb109*” and “*mdb110*” to featured image $\mathbf{F}_{IJ} = [F_{IJ}^1, F_{IJ}^2]$. Consider the $\mathbf{F}_{IJ}$ containing image $[F_{IJ}^1, F_{IJ}^2]$ in a space with a range denoted by $\mathfrak{R}^1 \times \mathfrak{R}^2 = [0,765] \times [0,765]$ as in Equation 5.13. Thus, mapping $\mathfrak{R}^1 \times \mathfrak{R}^2 \mapsto [0,4] \times [0,4]$ requires multiple each pixel in $\mathbf{F}_{IJ}$ i.e., $f_{ij}^1 \in F_{IJ}^1$ and $f_{ij}^2 \in F_{IJ}^2$ by 4/765.

There are 4 parameter settings for $b(\mathbf{x}: t, \chi, \gamma, \omega)$ as in Table 5.11 are examined i.e., $b(\mathbf{x}: 0.40, 3.49, 3.49, 3.49)$, $b(\mathbf{x}: 0.30, 3.04, 3.04, 3.04)$, $b(\mathbf{x}: 0.20, 2.64, 2.64, 2.64)$ and $b(\mathbf{x}: 0.1, 2.3, 2.3, 2.3)$. The edge image E_{IJ} processed by $b(\mathbf{x}: t, \chi, \gamma, \omega)$ at parameter settings $t = 0.4$, $t = 0.3$, $t = 0.2$ and $t = 0.1$ for images “*mdb107*” “*mdb108*” “*mdb279*” and “*mdb280*” are shown in Figure 5.62, Figure 5.63, Figure 5.64, and Figure 5.65, respectively. Again, reducing the t value setting from 0.4 to 0.1 improves the edge feature details in E_{IJ} , as observed in Figure 5.58, Figure 5.59, Figure 5.60, and Figure 5.61. However, it also increases the risk of creating an overexposure-like effect in E_{IJ} as the t value setting in $b(\mathbf{x}: t, \chi, \gamma, \omega)$ is reduced from 0.4 to 0.1. Results in Figure 5.62 show a significant overexposure-like effect on the E_{IJ} processed by $b(\mathbf{x}: t, \chi, \gamma, \omega)$ at $t = 0.1$ compared to other settings at $t = 0.4$, $t = 0.3$ and $t = 0.2$. Similar results are observed in images “*mdb108*”, “*mdb279*”, and “*mdb280*”, as shown in Figure 5.63, Figure 5.64, and Figure 5.65, respectively. Comparing the results in sections 5.6.3 and 5.6.4, there are no significant differences between the Sobel operator and the Prewitt operator in the application of the edge detection framework in digitized mammograms.

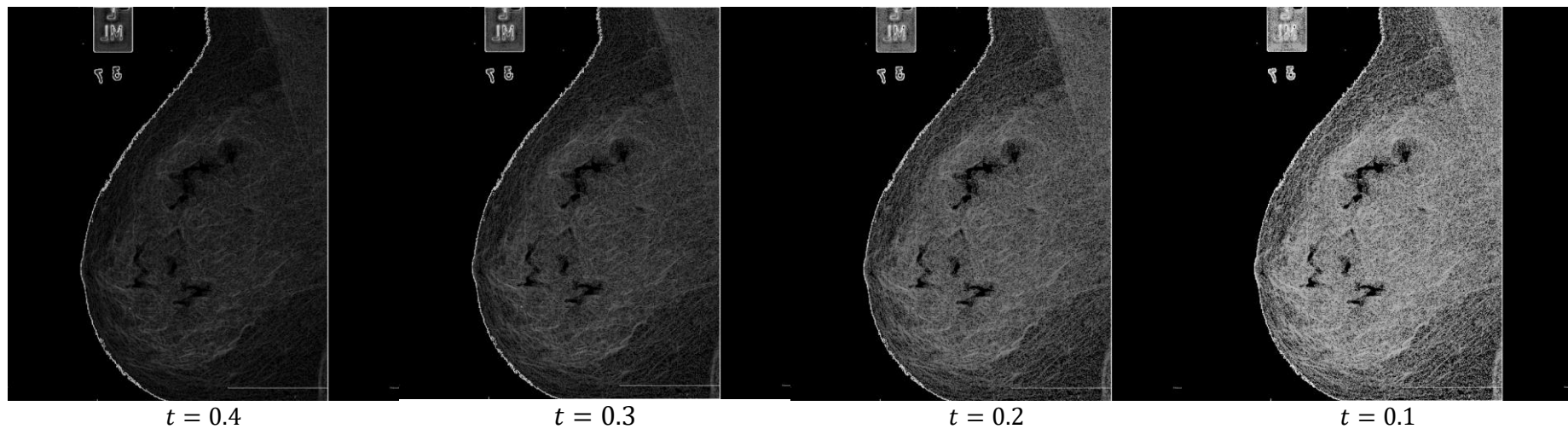


Figure 5.62: E_{IJ} for image “mdb107” processed by $b(\mathbf{x}; t, \chi, \gamma, \omega)$ at parameter setting $t = 0.4, t = 0.3, t = 0.2$ and $t = 0.1$

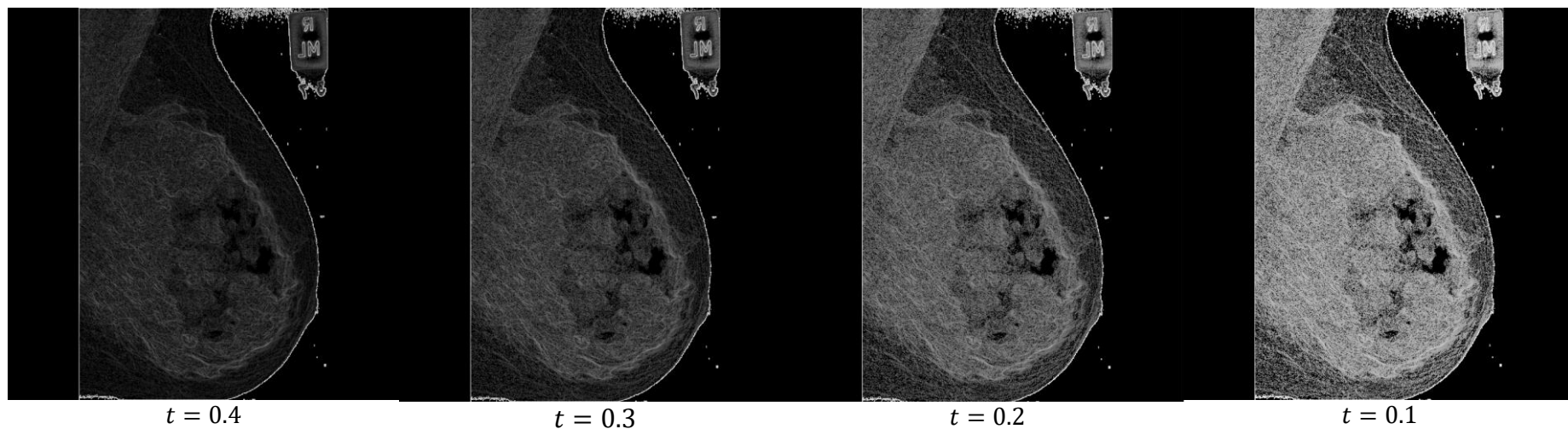


Figure 5.63: E_{IJ} image “mdb108” processed by $b(\mathbf{x}; t, \chi, \gamma, \omega)$ at parameter setting $t = 0.4, t = 0.3, t = 0.2$ and $t = 0.1$

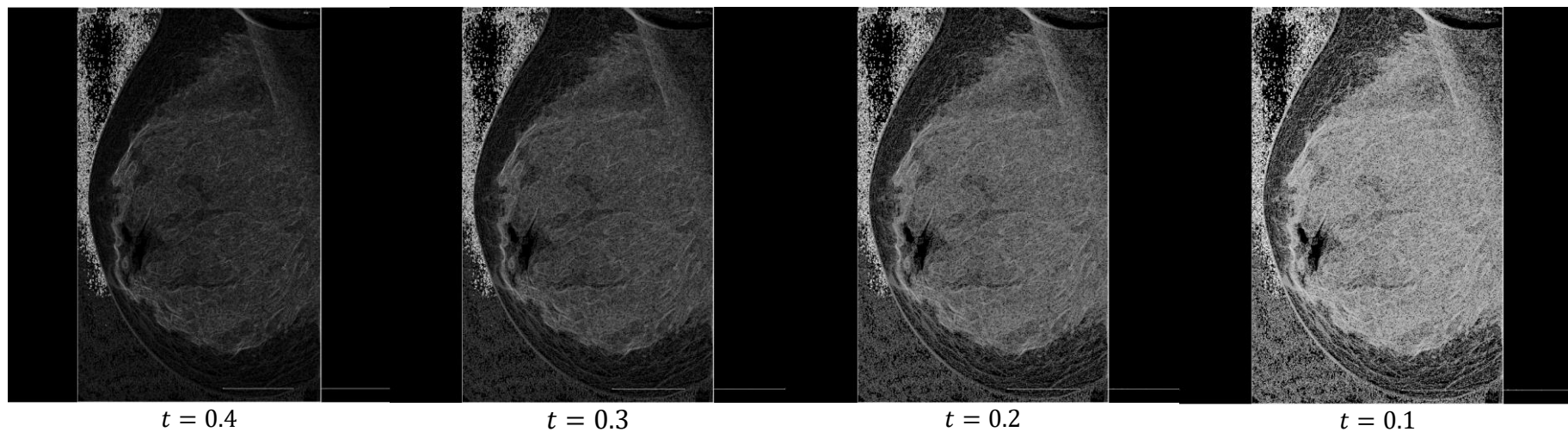


Figure 5.64: E_{IJ} image “mdb279” processed by $b(\mathbf{x}; t, \chi, \gamma, \omega)$ at parameter setting $t = 0.4, t = 0.3, t = 0.2$ and $t = 0.1$

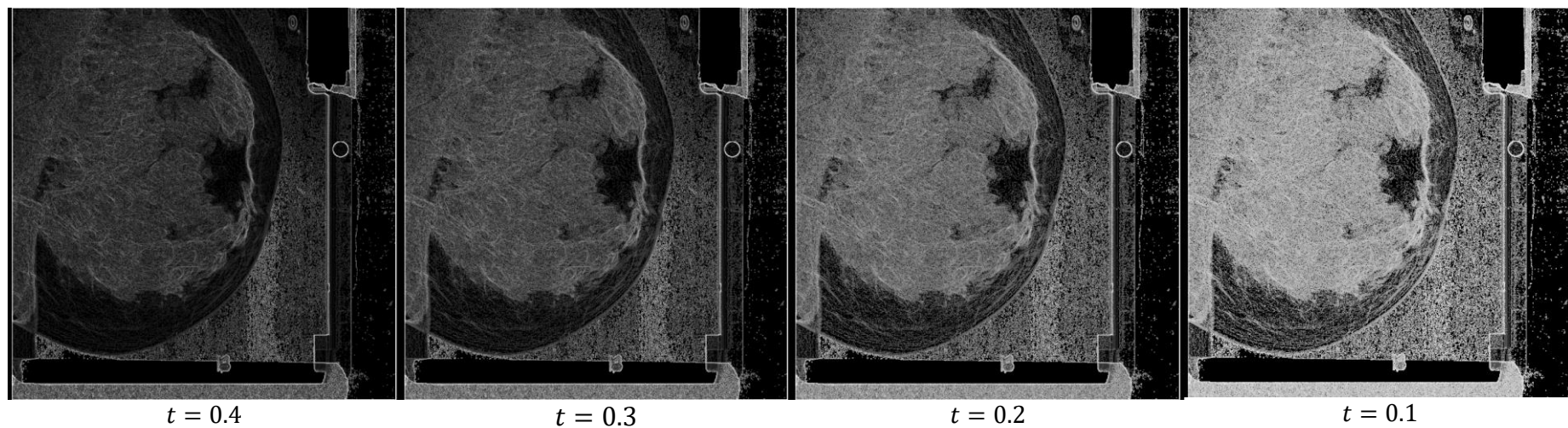


Figure 5.65: E_{IJ} image “mdb280” processed by $b(\mathbf{x}; t, \chi, \gamma, \omega)$ at parameter setting $t = 0.4, t = 0.3, t = 0.2$ and $t = 0.1$

5.7 Summary

In this chapter, the edge detection framework developed by Bezdek et al. (1998) was explored in depth, with a focus on applying monotone T1 TSK FIS to image processing tasks. The chapter provided detailed coverage of implementing feature extraction functions, including the Sobel and Prewitt Operators. An image histogram analysis algorithm was introduced to interpret right-skewed pixel distributions, which are crucial for designing the blending function. The findings in Sections 5.4.1 and 5.6 indicated that the image histogram $\mathbf{F}_{IJ}$ consistently displays a right-skewed pixel distribution, supporting the use of the waterfall blending function for processing $\mathbf{F}_{IJ}$ to produce blended image B_{IJ} and E_{IJ} .

The chapter also examined the use of the TSK FIS blending function as an approximation to the waterfall blending function. Section 5.5 addressed the formulation of the T1 monotone TSK FIS blending function, guided by simplified principles from previous research by Won et al. (2002), Tay and Lim (2011a), and Won and Karray (2014). Section 5.6 demonstrated the effectiveness of the T1 monotone TSK FIS blending function through various experiments.

This chapter successfully achieved the RO3 applying the parametric conditions of monotone T1 TSK FIS to solve problems in image processing, specifically focusing on edge detection. The detailed study, formulation, validation, and application of the monotone T1 TSK FIS blending function to edge detection demonstrate the fulfillment of this RO. In the next chapter, relevant theorems that facilitate the formulation of the monotone Type 2-FIS-based model is presented, along with simulated experiments to showcase the theorem's utility.

CHAPTER 6

MONOTONICITY PROPERTY ON IT2 FIS MODELLING

6.1 Overview

Chapter 3 examines the monotonicity properties of T1 FIS models. This chapter delves into monotonicity properties in the context of IT2 FIS models, discussing sufficient conditions for achieving monotone output. This chapter is organized as follows.

Section 6.2 introduces the IT2 FIS model. Section 6.3 details the Monotone IT2 FIS and its sufficient fuzzy parameter conditions. Section 6.4 discusses parametric conditions for trapezoidal and Gaussian IT2 FMFs design in monotone IT2 TSK FIS. Section 6.5 presents simulated experiments on SISO Monotone IT2 FIS with trapezoidal and Gaussian FMFs. Section 6.6 utilizes case studies from Chapter 4 to exemplify Gaussian and trapezoidal IT2 FMF design in MISO Monotone IT2 FIS models.

6.2 Background

T2 FIS-based models gained popularity for their capability to model high levels of uncertainties and produce more complex input–output mappings, while still retaining the advantages of conventional FIS models (i.e., FIS-based models) (Li et al., 2014). Meanwhile, IT2 FIS-based models (Liang and Mendel, 2000; Mendel et al., 2006; Wu et al., 2012; Pagola et al., 2013; Li et al., 2014) gained widespread attention for their reduced computational complexity and ease of theoretical analysis compared to other T2 FIS-based models. The additional degree of freedom provided by the Footprint of Uncertainty (FOU) in IT2 Fuzzy Sets (FSs) allowed IT2 FIS-based models to achieve better performance than T1 FIS-based

models (Mendel and Wu, 2010; Castillo and Melin, 2012; Kumbasar and Hagra, 2014; Yesil, 2014). An IT2 FIS-based model employed IT2 FSs in the fuzzy rule base, resulting in IT2 FSs output in the fuzzy inference process. A type reduction procedure was required to convert IT2 FSs output to T1 FS before the defuzzification procedure. The KM type-reducer (also known as iterative Karnik-Mendel algorithms) (Karnik et al., 1999; Liu and Mendel, 2011) was commonly used for type reduction in IT2 FIS-based models.

This chapter was dedicated to examining the monotonicity properties of IT2 TSK FIS-based models with the KM type-reducer, referred to simply as IT2 FIS-based models for easy explanation. Inspired by mathematical analyses in Li et al. (2014) and Li et al. (2011), these mathematical conditions were adopted to facilitate the construction of a monotone IT2 FIS-based model. The effectiveness of these conditions was examined through numerical experiments in this section. Monotone IT2 TSK FIS with Trapezoidal and Gaussian IT2 FMFs design were demonstrated in this chapter. Gaussian IT2 FMFs offered advantages such as simpler representation and ease of optimization, while trapezoidal IT2 FMFs provided greater flexibility in construction, simpler analytical structures, and had been preliminarily studied for monotonicity properties (Wu, 2012).

6.3 Monotone IT2 FIS model

This chapter focuses on review monotonicity properties of SISO monotone IT2 FIS-based model and MISO monotone IT2 FIS-based model. Let consider $y = f(\mathbf{x})$ as function mapping input space X , to an output space Y , where variables $\mathbf{x} \in X$ and $y \in Y$. An SISO monotone IT2 FIS-based model obeyed the monotonicity property between the input and output variables as follows:

Definition 6.1 (SISO monotone IT2 FIS-based model): SISO monotone IT2 FIS-based model i.e., $y = f(x)$ with x as a sole input in the range $(\underline{x} \leq x \leq \bar{x}) \in X$ and y is the output in the range $(\underline{y} \leq y \leq \bar{y}) \in Y$. Then, $y = f(x)$ considerable to fulfil the monotonicity property between X and Y , if $\forall (x > x') \in X$ fulfil either solely $f(x) \geq f(x')$ or $f(x) \leq f(x')$.

Extending Definition 6.1 for MISO monotone TSK FIS has Definition 6.2 as shown follows:

Definition 6.2 (MISO monotone IT2 FIS-based model): SISO monotone IT2 FIS-based model i.e., $y = f(\mathbf{x})$ is considered, where $\mathbf{x} = (x_1, x_2, \dots, x_n)$ is the input in space $X = X_1 \times X_2 \times \dots \times X_n$ and $y \in Y$ is the output. Note that input x_i is in the range $(\underline{x}_i \leq x_i \leq \bar{x}_i) \in X_i$ and output y is in the range $(\underline{y} \leq y \leq \bar{y}) \in Y$. Let denoted $\mathbf{x}' = (x_1, \dots, x_i', \dots, x_n)$ where $\forall (x_i > x_i') \in X_i$. $y = f(\mathbf{x})$ is considerable to fulfil the monotonicity property between X_i and Y , if $\forall (\mathbf{x} > \mathbf{x}') \in X$ fulfil either solely $f(\mathbf{x}) \geq f(\mathbf{x}')$ or $f(\mathbf{x}) \leq f(\mathbf{x}')$ for all for all possible combinations $i \in \{1, \dots, n\}$.

The monotone IT2 FIS-based model could be either monotonically increasing or monotonically decreasing. For easy explanation for following section, only the monotonically increasing case is considered.

6.3.1 Sufficient conditions for SISO monotone IT2 FIS model

An SISO monotone IT2 FIS-based model as in Definition 6.1 is discussed in this section i.e., $y = f(x)$ with x is the sole input. The fuzzy rule R^j of the SISO monotone IT2 FIS-based model, applicable for $j \in \{1, \dots, m\}$, is defined in Equation 6.1. The fuzzy rule

R^j is explained as mapping $R^j: \tilde{A}^j \rightarrow y = [\underline{b}^j, \bar{b}^j]$, where $\tilde{A}^j$ as IT2 fuzzy set from input space $x \in X$ to a consequent $\tilde{b}^j = [\underline{b}^j, \bar{b}^j]$ at output space Y .

$$R^j: \text{If } x \text{ is } \tilde{A}^j \text{ then } y = [\underline{b}^j, \bar{b}^j] \quad \text{Equation 6.1}$$

The antecedent of this fuzzy rule in the input space X is represented as $\tilde{A}^j$. $\tilde{A}^j$ is an IT2 fuzzy set with the upper and lower MF denoted as $\underline{\mu}^j(x) \in [\underline{x}, \bar{x}]$ and $\bar{\mu}^j(x) \in [\bar{x}, \underline{x}]$, respectively. The interpretation of this fuzzy rule varies depending on the model. In the IT2 TSK FIS model with the KM method, $[\underline{b}^j, \bar{b}^j]$ are viewed as constants, subject to the conditions $\underline{b}^j \leq \bar{b}^j$ is fulfilled. On other hand in the IT2 Mamdani FIS model, $[\underline{b}^j, \bar{b}^j]$ in the rule base is considerable as singleton IT2 fuzzy sets in the consequent of the rule base.

The KM type-reducer with center-of-sets (COS) type-reduction method is utilized for type reduction in the SISO monotone IT2 FIS-based model. The membership function $\mu^j(x) \in \{\underline{\mu}^j(x), \bar{\mu}^j(x)\}$ is used to determine sequence of $y_l^k(x)$ and $y_r^k(x)$ as in Equation 6.2 and Equation 6.3, respectively. The number of possible combinations of sequence of $y_l^k(x)$ and $y_r^k(x)$ are denoted by $k = 1, \dots, M$ where $M = 2^m$ and m is number of IT2 fuzzy set in input space $x \in X$.

$$y_l^k(x) = \frac{\sum_{j=1}^m (\mu^j(x) \times \underline{b}^j)}{\sum_{j=1}^m \mu^j(x)} \quad \text{Equation 6.2}$$

$$y_r^k(x) = \frac{\sum_{j=1}^m (\mu^j(x) \times \bar{b}^j)}{\sum_{j=1}^m \mu^j(x)} \quad \text{Equation 6.3}$$

The center-of-sets (COS) type-reduction method is used to ascertain the type-reducer i.e., to determine the left and right points of the interval outputs of the model, denoted as by $y_l(x)$ and $y_r(x)$, respectively. Both $y_l^k(x)$ and $y_r^k(x)$ are then used in Equation 6.4 and Equation 6.5 to compute type-reducer of SISO monotone IT2 FIS-based model i.e., $y_l(x)$

and $y_r(x)$. The output of the model i.e., $y = f(x)$ is calculated using Equation 6.6. In situations where the $\underline{\mu}^j(x) = \bar{\mu}^j(x)$ and $\bar{b}^j = \underline{b}^j$, the SISO monotone IT2 FIS-based model IT2 FLS can be interpreted as an SISO monotone T1 TSK FIS.

$$y_l(x) = \min_{k=1}^M y_l^k(x) \quad \text{Equation 6.4}$$

$$y_r(x) = \max_{k=1}^M y_r^k(x) \quad \text{Equation 6.5}$$

$$y = f(x) = \frac{1}{2} [y_l(x) + y_r(x)] \quad \text{Equation 6.6}$$

Remark 6.1: Theorem from Li et al., (2014) and Li et al., (2011) is simplified, SISO monotone IT2 FIS-based model (as outlined in Equation 6.2 to Equation 6.6) is monotonically increasing w.r.t x as in Definition 6.1 if the following conditions are satisfied:

Condition 1: $\mu^p(x) \cdot \mu^q(x') \geq \mu^p(x') \cdot \mu^q(x)$ should meet for all $1 \leq p \leq q \leq m$ and

$\forall (x > x') \in X = [\underline{x}, \bar{x}]$, where $\mu^{j \in \{p, q\}}(x) \in \{\underline{\mu}^j(x), \bar{\mu}^j(x)\}$

Condition 2: $\underline{b}^q \geq \underline{b}^p$ and $\bar{b}^q \geq \bar{b}^p$ should meet for all $1 \leq p \leq q \leq m$.

6.3.2 Sufficient conditions for MISO monotone IT2 FIS model

An MISO monotone IT2 FIS model as outlined in Definition 6.2 is considered in this section. i.e., $y = f(\mathbf{x})$ with input vector $\mathbf{x} = (x_1, x_2, \dots, x_n)$. The fuzzy rule associated with the MISO Monotone FIS is given as $R^j = (j_1, j_2, \dots, j_n)$, as detailed in Equation 6.7. R^j is explained as mapping $\tilde{A}^j$ from input space input space $\mathbf{x} \in X_1 \times X_2 \dots \times X_n$ to a consequent b^j at output space $y \in Y$.

$$R^j = (j_1, j_2, \dots, j_n): \text{If } (x_1 \text{ is } \tilde{A}_1^{j_1}) \text{ and } (x_2 \text{ is } \tilde{A}_2^{j_2}) \dots (x_n \text{ is } \tilde{A}_n^{j_n}) \text{ then } y = \tilde{b}^j \quad \text{Equation 6.7}$$

An antecedent of fuzzy rule at input space X is represented as $\tilde{A}^j = \tilde{A}_1^{j_1} \times \tilde{A}_2^{j_2} \dots \tilde{A}_n^{j_n}$, where $\mathbf{j} = (j_1, j_2, \dots, j_n)$ is a integer vector and each j_i belongs to the set $\{1, \dots, m_i\}$. Here,

$\tilde{A}_i^{j_i}$ refers to an IT2 fuzzy set, with the upper and lower MF symbolized as $\underline{\mu}_i^{j_i}(x_i) \in [\underline{x}_i, \underline{x}_i]$ and $\bar{\mu}_i^{j_i}(x_i) \in [\bar{x}_i, \bar{x}_i]$, respectively. The fuzzy rule can be interpreted in two ways. In the IT2 TSK FIS model, $[\underline{b}^j, \bar{b}^j]$ are viewed as constants, subject to the conditions $\underline{b}^j \leq \bar{b}^j$ is fulfilled. Conversely, in the IT2 Mamdani FIS model, $\tilde{b}^j = [\underline{b}^j, \bar{b}^j]$ in the rule base are viewed as singleton IT2 fuzzy sets in the consequent of the rule base.

Again, the KM type-reducer is utilized for type reduction in the MISO monotone IT2 FIS model. Let's denoted membership function $\mu_i^{j_i}(x_i) \in \{\underline{\mu}_i^{j_i}(x_i), \bar{\mu}_i^{j_i}(x_i)\}$ for $\tilde{A}_i^{j_i}$. The interval firing strength of R^j i.e., $[f^j(x), \bar{f}^j(x)]$ can be calculated by the product operation as in Equation 6.8 and Equation 6.9.

$$\underline{f}^j(x) = \prod_{i=1}^n \underline{\mu}_i^{j_i}(x_i) \quad \text{Equation 6.8}$$

$$\bar{f}^j(x) = \prod_{i=1}^n \bar{\mu}_i^{j_i}(x_i) \quad \text{Equation 6.9}$$

Let denote $f^j(x) \in \{\underline{f}^j(x), \bar{f}^j(x)\}$, the sequence of $y_l^k(x)$ and $y_r^k(x)$ can be obtained in Equation 6.10 and Equation 6.11 respectively. The number of possible combinations of sequence of $y_l^k(x)$ and $y_r^k(x)$ are denoted by $k = 1, \dots, M$ where $M = 2^{\prod_{i=1}^n m_i}$ and m_i is number of IT2 fuzzy set in input space $x_i \in X_i$.

$$y_l^k(x) = \frac{\sum_{j_1=1}^{m_1} \dots \sum_{j_n=1}^{m_n} (f^j(x) \times \underline{b}^j)}{\sum_{j_1=1}^{m_1} \dots \sum_{j_n=1}^{m_n} f^j(x)} \quad \text{Equation 6.10}$$

$$y_r^k(x) = \frac{\sum_{j_1=1}^{m_1} \dots \sum_{j_n=1}^{m_n} (f^j(x) \times \bar{b}^j)}{\sum_{j_1=1}^{m_1} \dots \sum_{j_n=1}^{m_n} f^j(x)} \quad \text{Equation 6.11}$$

Both $y_l^k(x)$ and $y_r^k(x)$ are then used in Equation 6.12 and Equation 6.13 to compute type-reducer of MISO monotone IT2 FIS-based model i.e., $y_l(x)$ and $y_r(x)$. The output of

the model i.e., $y = f(\mathbf{x})$ is calculated using Equation 6.14. In situations where the $\underline{\mu}_i^{j_i}(x_i) = \bar{\mu}_i^{j_i}(x_i)$ and $\bar{b}^j = \underline{b}^j$, the MISO monotone IT2 FIS-based model IT2 FLS can be interpreted as an MISO monotone T1 TSK FIS.

$$y_l(\mathbf{x}) = \min_{k=1}^M y_l^k(\mathbf{x}) \quad \text{Equation 6.12}$$

$$y_r(\mathbf{x}) = \max_{k=1}^M y_r^k(\mathbf{x}) \quad \text{Equation 6.13}$$

$$y = f(\mathbf{x}) = \frac{1}{2} [y_l(\mathbf{x}) + y_r(\mathbf{x})] \quad \text{Equation 6.14}$$

Remark 6.2: Theorem from Li et al., (2014) and Li et al., (2011) is simplified, MISO monotone IT2 FIS-based model (as outlined in Equation 6.10 to Equation 6.14) is monotonically increasing w.r.t x_i as in Extending Definition 6.1 for MISO monotone TSK FIS has Definition 6.2 as shown follows:

Definition 6.2 if the following conditions are satisfied:

Condition 1: $\mu^p(x_i) \cdot \mu^q(x'_i) \geq \mu^p(x'_i) \cdot \mu^q(x_i)$ should meet for all $1 \leq p \leq q \leq m_i$ and $\forall(x_i > x'_i) \in X_i = [\underline{x}_i, \bar{x}_i]$, where $\mu_i^{j_i \in \{p, q\}}(x_i) \in \{\underline{\mu}_i^{j_i}(x_i), \bar{\mu}_i^{j_i}(x_i)\}$ and $i = 1, \dots, n$.

Condition 2: $\underline{b}^{j+e_i} \geq \underline{b}^j$ and $\bar{b}^{j+e_i} \geq \bar{b}^j$ is true for all $\mathbf{j}$, such that $j_i < m_i$. Note that, $\mathbf{j} + \mathbf{e}_i$ denotes vector $(j_1, \dots, j_i + 1, \dots, j_n)$ and $\mathbf{e}_i$ denotes an n -dimensional unit vector in direction i .

6.4 Fuzzy Membership Functions (FMF)

In this section, *Condition 1* at Remark 6.1 and Remark 6.2 are employed in the design of FMF for monotone IT2 TSK FIS. The section elucidates the parametric conditions for deploying IT2 FMFs with trapezoidal and Gaussian configurations in a monotone IT2 TSK

FIS. Here, T1 FMF and IT2 FMF are distinguished as A and $\tilde{A}$, respectively. $\tilde{A}$ comprises lower and upper lower and upper FMF.

6.4.1 IT2 Trapezoidal FMF

An IT2 Trapezoidal FMF in the X domain in Figure 6.1 is considered, i.e., $\tilde{A} = \mu_{\tilde{A}}(x: \underline{a}, \bar{a}, \underline{b}, \bar{b}, \underline{c}, \bar{c}, \underline{d}, \bar{d}, \underline{h}, 1)$. $\tilde{A}$ comprises lower and upper lower and upper membership functions as detailed in Equation 6.15 and Equation 6.16 respectively. It should be observed that the conditions $\bar{a} \leq \underline{a}, \bar{b} \leq \underline{b}, \bar{c} \leq \underline{c}, \bar{d} \leq \underline{d}$ and $\underline{h} \leq 1$ should be fulfilled. Let's consider $\mu_A(x: a, b, c, d, h) \in \{\underline{\mu}_{\tilde{A}}(x), \bar{\mu}_{\tilde{A}}(x)\}$, IT2 Trapezoidal FMF is articulated as in Equation 6.17. At the condition where $\underline{b} = \underline{c}$ and $\bar{b} = \bar{c}$, the Trapezoidal IT2 FMF is transitioned into a Triangular IT2 FMF. Additionally, IT2 Trapezoidal FMF is treated as an T1 Trapezoidal FMF when $\underline{\mu}_{\tilde{A}}(x) = \bar{\mu}_{\tilde{A}}(x)$.

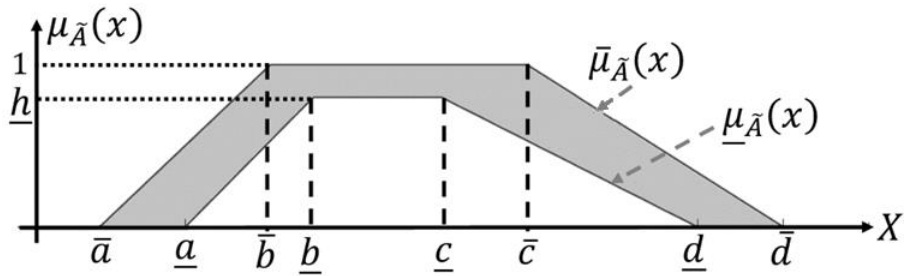


Figure 6.1: IT2 Trapezoidal FMF

$$\underline{\mu}_{\tilde{A}}(x) = \mu_{\tilde{A}}(x: \underline{a}, \underline{b}, \underline{c}, \underline{d}, \underline{h}) \quad \text{Equation 6.15}$$

$$\bar{\mu}_{\tilde{A}}(x) = \mu_{\tilde{A}}(x: \bar{a}, \bar{b}, \bar{c}, \bar{d}, 1) \quad \text{Equation 6.16}$$

$$\mu_A(x: a, b, c, d, h) = \begin{cases} 0, & x \leq \underline{a}; x \geq \bar{d} \\ h \cdot (x - \underline{a}) / (\underline{b} - \underline{a}), & \underline{a} < x \leq \underline{b} \\ h, & \underline{b} < x \leq \underline{c} \\ h \cdot (x - \underline{d}) / (\underline{c} - \underline{d}), & \underline{c} < x < \underline{d} \end{cases} \quad \text{Equation 6.17}$$

Consider an SISO monotone IT2 FIS-based model as outlined in Definition 6.1 which incorporates m IT2 trapezoidal FMFs in the input space $x \in X$. The subsequent Remark 6.3 is derived.

Remark 6.3: (Li et al., 2014) For an IT2 trapezoidal FMF, represented as $\tilde{A} = \mu^j(x: \underline{a}^j, \bar{a}^j, \underline{b}^j, \bar{b}^j, \underline{c}^j, \bar{c}^j, \underline{d}^j, \bar{d}^j, \underline{h}^j, 1)$ in input space X for $1 \leq j \leq m$, *Condition 1* in Remark 6.1 is fulfilled if $\underline{a}^p \leq \bar{a}^q$, $\underline{b}^p \leq \bar{b}^q$, $\underline{c}^p \leq \bar{c}^q$ and $\underline{d}^p \leq \bar{d}^q$ for all $1 \leq p \leq q \leq m$.

Extending Remark 6.3 for MISO monotone IT2 FIS model, Remark 6.4 is deduced:

Remark 6.4: (Li et al., 2014) For an IT2 trapezoidal FMF, represented as $\tilde{A} = \mu_i^{j_i}(x_i: \underline{a}_i^{j_i}, \bar{a}_i^{j_i}, \underline{b}_i^{j_i}, \bar{b}_i^{j_i}, \underline{c}_i^{j_i}, \bar{c}_i^{j_i}, \underline{d}_i^{j_i}, \bar{d}_i^{j_i}, \underline{h}_i^{j_i}, 1)$ in input space X_i for $1 \leq j_i \leq m_i$, *Condition 1* in Remark 6.2 should satisfy if $\underline{a}_i^p \leq \bar{a}_i^q$, $\underline{b}_i^p \leq \bar{b}_i^q$, $\underline{c}_i^p \leq \bar{c}_i^q$ and $\underline{d}_i^p \leq \bar{d}_i^q$ for all $1 \leq p \leq q \leq m_i$.

Simulated experiment on SISO monotone IT2 FIS model demonstrated in section 6.5.1, wherein Remark 6.3 is employed as the governing condition for IT2 trapezoidal FMF design. Subsequently, Section 6.5.3 employs case studies from Chapter 4 to elucidate the utility of Remark 6.4 for IT2 trapezoidal FMF design in MISO monotone IT2 FIS model.

6.4.2 IT2 Gaussian FMF

A IT2 Gaussian FMF in the X domain in Figure 6.2 is considered, i.e., $\tilde{A} = \mu_A(x: \underline{a}, \bar{a}, \underline{b}, \bar{b}, \underline{\sigma}, \bar{\sigma}, \underline{h}, 1)$. $\tilde{A}$ is comprises lower and upper lower and upper membership functions as detailed in Equation 6.18 and Equation 6.19, respectively. It is important to note that the conditions $\bar{a} \leq \underline{a}$, $\bar{b} \leq \underline{b}$, $\underline{\sigma} \leq \bar{\sigma}$, and $\underline{h} \leq 1$ must be satisfied. Let's consider

$\mu_A(x: a, b, \sigma, h) \in \{\underline{\mu}_{\tilde{A}}(x), \bar{\mu}_{\tilde{A}}(x)\}$, IT2 Gaussian FMF is articulated as in Equation 6.20.

Additionally, IT2 FS is treated as a T1 FS when $\underline{\mu}_{\tilde{A}}(x) = \bar{\mu}_{\tilde{A}}(x)$.

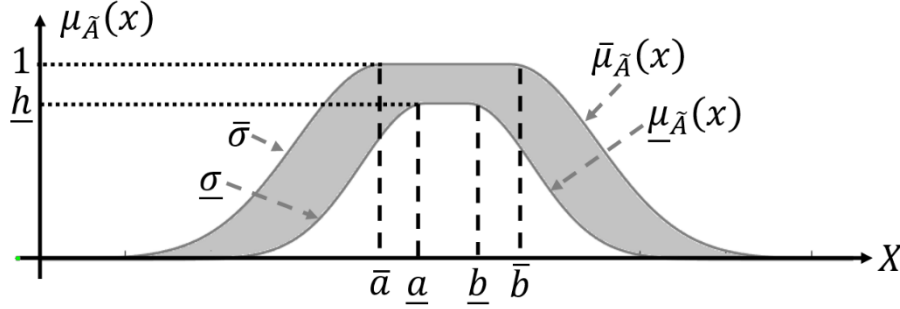


Figure 6.2: Gaussian fuzzy membership function.

$$\underline{\mu}_{\tilde{A}}(x) = \mu_{\tilde{A}}(x: \underline{a}, \underline{b}, \underline{\sigma}, \underline{h}) \quad \text{Equation 6.18}$$

$$\bar{\mu}_{\tilde{A}}(x) = \mu_{\tilde{A}}(x: \bar{a}, \bar{b}, \bar{\sigma}, 1) \quad \text{Equation 6.19}$$

$$\mu_A(x: a, b, \sigma, h) = \begin{cases} h \cdot e^{-\frac{(x-a)^2}{2\sigma^2}}, & x \leq a \\ h, & a < x \leq b \\ h \cdot e^{-\frac{(x-b)^2}{2\sigma^2}}, & x \geq b \end{cases} \quad \text{Equation 6.20}$$

Remark 6.5: (Li et al., 2014) For an IT2 Gaussian FMF, represented as $\tilde{A} = \mu^j(x: \underline{a}^j, \bar{a}^j, \underline{b}^j, \bar{b}^j, \underline{\sigma}^j, \bar{\sigma}^j, \underline{h}^j, 1)$ in input space X for $1 \leq j \leq m$, *Condition 1* in Remark 6.1 is fulfilled if $\underline{a}^p \leq \bar{a}^q$, $\underline{b}^p \leq \bar{b}^q$, $\bar{\sigma}^q = \bar{\sigma}^q = \bar{\sigma}$, $\underline{\sigma}^q = \underline{\sigma}^q = \underline{\sigma}$ and $\underline{h}^p \leq \underline{h}^q = \underline{\sigma}^2 / \bar{\sigma}^2$ for all $1 \leq p \leq q \leq m$.

Extending Remark 6.5 for MISO monotone IT2 FIS model, Remark 6.6 is deduced:

Remark 6.6: (Li et al., 2014) For an IT2 Gaussian FMF, represented as $\tilde{A} = \mu_i^{j_i}(x_i: \underline{a}_i^{j_i}, \bar{a}_i^{j_i}, \underline{b}_i^{j_i}, \bar{b}_i^{j_i}, \underline{\sigma}_i^{j_i}, \bar{\sigma}_i^{j_i}, \underline{h}_i^{j_i})$ in input space X_i for $1 \leq j_i \leq m_i$, *Condition 1* in Remark 6.2 should satisfy if $\underline{a}_i^p \leq \bar{a}_i^q$, $\underline{b}_i^p \leq \bar{b}_i^q$, $\bar{\sigma}_i^p = \bar{\sigma}_i^q = \bar{\sigma}$, $\underline{\sigma}_i^p = \underline{\sigma}_i^q = \underline{\sigma}$ and $\underline{h}_i^p = \underline{h}_i^q = \underline{\sigma}^2 / \bar{\sigma}^2$ for all $1 \leq p \leq q \leq m_i$.

Section 6.5.2 presents simulated experiments on an SISO monotone IT2 FIS model, employing Remark 6.5 as governing condition for IT2 gaussian FMF design. In Section 6.5.4, case studies drawn from Chapter 4 are deployed to illuminate the usefulness of Remark 6.6 in the design of IT2 Gaussian FMFs in MISO monotone IT2 FIS model.

6.5 Simulated experiments in SISO Monotone IT2 FIS

This section presents simulated experiments concentrating on trapezoidal and Gaussian FMF design in SISO Monotone IT2 FIS. Simulated experiments in Section 6.5.1 employ Remark 6.3 as the guiding criterion in IT2 trapezoidal FMF design. In Section 6.5.2, Remark 6.5 serves as the governing condition for IT2 Gaussian FMF design.

6.5.1 SISO monotone IT2 FIS with IT2 Trapezoidal FMF

In this experiment, an SISO monotone IT2 FIS model is examined, characterized by a single input $x \in X = [0,20]$ and integrates six trapezoidal FMFs as illustrated in Figure 6.3. The 6 Trapezoidal FMFs are parameterized as such, $\tilde{A}^1 = \tilde{\mu}^1(x: 0, 0, 0, 0, 0, 0, 2, 3, 0.8, 1)$, $\tilde{A}^2 = \tilde{\mu}^2(x: 0, 0, 3, 3, 3, 3, 6, 6, 0.7, 1)$, $\tilde{A}^3 = \tilde{\mu}^3(x: 4, 4, 7, 7, 7, 7, 10, 10, 1, 1)$, $\tilde{A}^4 = \tilde{\mu}^4(x: 8, 8, 10, 10, 12, 12, 14, 14, 1, 1)$, $\tilde{A}^5 = \tilde{\mu}^5(x: 9, 8, 12, 12, 13, 14, 16, 18, 0.7, 1)$ and $\tilde{A}^6 = \tilde{\mu}^6(x: 14, 14, 18, 18, 20, 20, 20, 20, 0.9, 1)$. The design of $\tilde{A}^j$ for $j \in (1,2,3,4,5,6)$ adheres to Remark 6.3. It is imperative to highlight that $\tilde{A}^1$ and $\tilde{A}^2$ are IT2 triangular FMFs and are encompassed as special cases of IT2 trapezoidal FMFs. Conversely, $\tilde{A}^5$ and $\tilde{A}^6$ are characterized as IT2 trapezoidal FMFs. Furthermore, $\tilde{A}^3$ and $\tilde{A}^4$ are T1 trapezoidal FMFs, which can be contemplated as specialized cases of IT2 trapezoidal FMFs under the condition $\underline{\mu}^j = \bar{\mu}^j$ for $j \in (3,4)$.

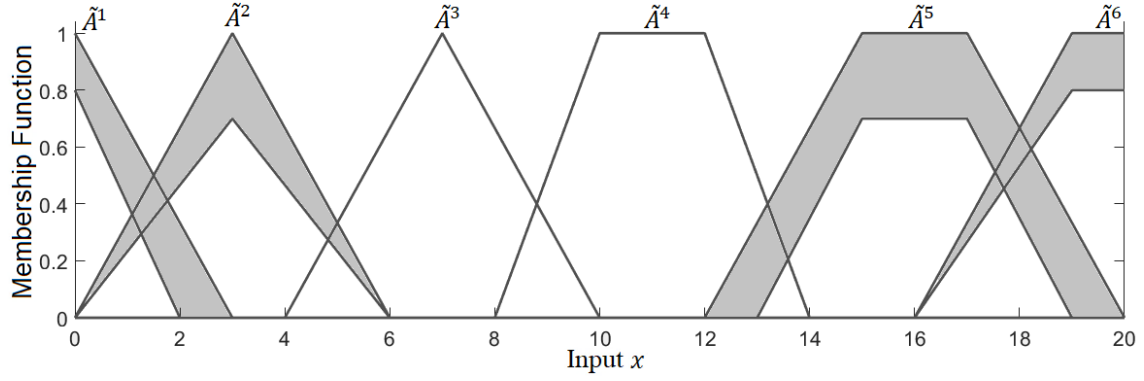


Figure 6.3: IT2 Trapezoidal FMF design of an SISO monotone IT2 FIS

Consequently, the SISO monotone IT2 FIS model constituted six fuzzy rules as specified in Equation 6.21. Let's denote the consequent of fuzzy rules R^j as $[\underline{b}^j, \bar{b}^j]$. The experiment is designed in such that $\underline{b}^q \geq \underline{b}^p$ and $\bar{b}^q \geq \bar{b}^p$ must be satisfied for all $1 \leq p \leq q \leq m$, in accordance with *Condition 2* in Remark 6.1. Utilizing Equation 6.2 and Equation 6.3, the sequences $y_l^k(x)$ and $y_r^k(x)$ are obtained as delineated in Equation 6.22 and Equation 6.23, where $\mu^j(x) \in \{\underline{\mu}^j(x), \bar{\mu}^j(x)\}$. Both sequences $y_l^k(x)$ and $y_r^k(x)$ are subsequently utilized in Equation 6.4 and Equation 6.5 to calculate the type-reducer of the SISO IT2 FIS-based model, denoted as $y_l(x)$ and $y_r(x)$. Finally, the output of the model, denoted as $y = f(x)$, is calculated through Equation 6.6 and this output y is illustrated in Figure 6.4,. As can be observed in Figure 6.4, the output y exhibits a monotonically increasing trend w.r.t x .

$$\begin{aligned}
 R^1: & \text{ If } x \text{ is } \tilde{A}^1 \text{ then } y = [0.5, 1.0] \\
 R^2: & \text{ If } x \text{ is } \tilde{A}^2 \text{ then } y = [1.5, 2.5] \\
 R^3: & \text{ If } x \text{ is } \tilde{A}^3 \text{ then } y = [3.0, 3.5] \\
 R^4: & \text{ If } x \text{ is } \tilde{A}^4 \text{ then } y = [4.0, 6.5] \\
 R^5: & \text{ If } x \text{ is } \tilde{A}^5 \text{ then } y = [7.0, 8.0] \\
 R^6: & \text{ If } x \text{ is } \tilde{A}^6 \text{ then } y = [9.0, 10.0]
 \end{aligned}
 \tag{Equation 6.21}$$

$$y_l^k(x) = \frac{\mu^1(x) \times 0.5 + \mu^2(x) \times 1.5 + \mu^3(x) \times 3 + \dots}{\mu^1(x) + \mu^2(x) + \mu^3(x) + \mu^4(x) + \mu^5(x) + \mu^6(x)} \quad \text{Equation 6.22}$$

$$y_r^k(x) = \frac{\mu^1(x) \times 1.0 + \mu^2(x) \times 2.5 + \mu^3(x) \times 3.5 + \dots}{\mu^1(x) + \mu^2(x) + \mu^3(x) + \mu^4(x) + \mu^5(x) + \mu^6(x)} \quad \text{Equation 6.23}$$

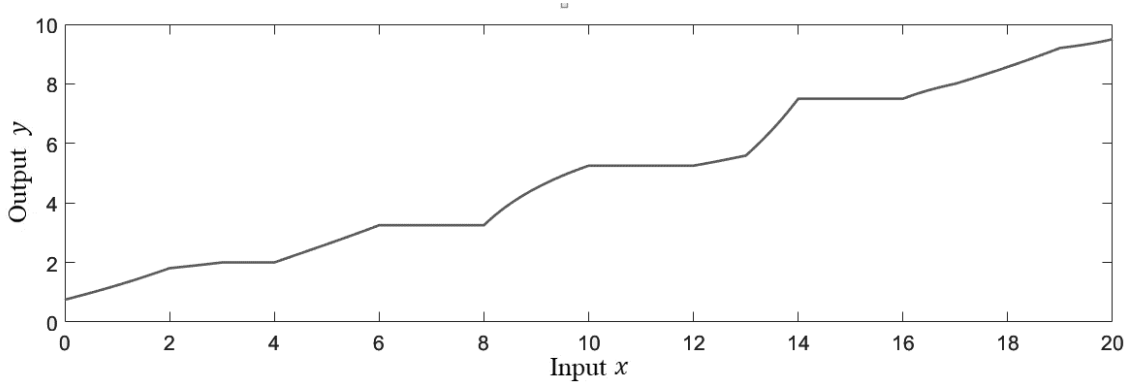


Figure 6.4: Output of SISO monotone IT2 FIS model with IT2Trapezoidal FMF

6.5.2 SISO monotone IT2 FIS with IT2 Gaussian FMF

In this experiment, the SISO monotone IT2 FIS model is characterized by a single input $x \in X = [0,15]$ and integrates 5 Gaussian FMFs as illustrated in Figure 6.5. The 5 Gaussian FMFs are parameterized as such, $\tilde{A}^1 = \tilde{\mu}^1(x: 0,0,0.5,1,0.9,1,0.81,1)$, $\tilde{A}^2 = \tilde{\mu}^2(x: 3,3,4,5,1,1,0.81,1)$, $\tilde{A}^3 = \tilde{\mu}^3(x: 8,8,8,8,0.9,1,0.81,1)$, $\tilde{A}^4 = \tilde{\mu}^4(x: 10,10,10,10,0.9,1,0.81,1)$ and $\tilde{A}^5 = \tilde{\mu}^5(x: 14,13,15,15,0.9,1,0.81,1)$. The design of $\tilde{A}^j = \mu^j(x: \underline{a}^j, \bar{a}^j, \underline{b}^j, \bar{b}^j, \underline{\sigma}^j, \bar{\sigma}^j, \underline{h}^j, 1)$ for $j \in (1,2,3,4,5)$ adheres to Remark 6.5. In this instance, $\underline{\sigma}^j$, $\bar{\sigma}^j$ and $\underline{h}^j$ are set to fix value, as such, $\bar{\sigma}^q = \bar{\sigma}^q = 1$, $\underline{\sigma}^q = \underline{\sigma}^q = 0.9$ and $\underline{h}^p = \underline{h}^q = 0.9^2/1^2 = 0.81$ for all $1 \leq p \leq q \leq m$.

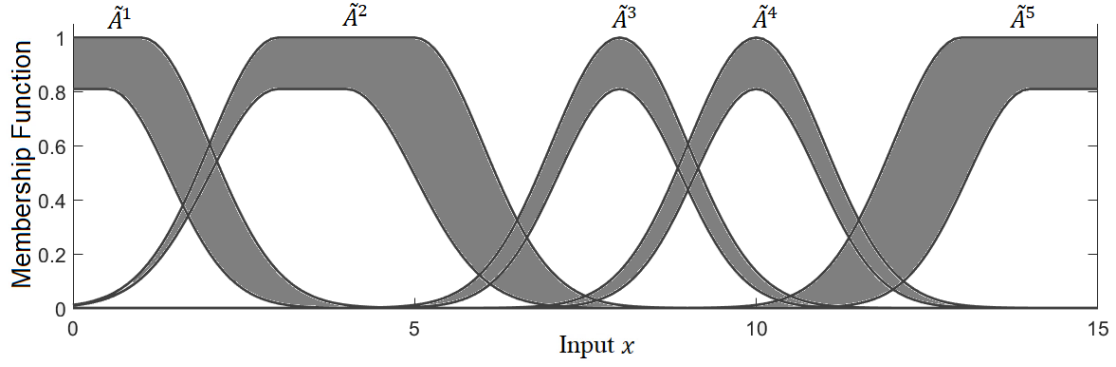


Figure 6.5: IT2 Gaussian FMF design of an SISO monotone IT2 FIS

Subsequently, the SISO monotone IT2 FIS model constituted 5 fuzzy rules as delineated in Equation 6.24. Let's denote the consequent of fuzzy rules R^j as $[\underline{b}^j, \bar{b}^j]$. The experiment is designed in such that $\underline{b}^q \geq \underline{b}^p$ and $\bar{b}^q \geq \bar{b}^p$ must be satisfied for all $1 \leq p \leq q \leq m$, in accordance with *Condition 2* in Remark 6.1.

$$\begin{aligned}
 R^1: & \text{ If } x \text{ is } \tilde{A}^1 \text{ then } y = [0.5, 1.0] \\
 R^2: & \text{ If } x \text{ is } \tilde{A}^2 \text{ then } y = [1.5, 2.5] \\
 R^3: & \text{ If } x \text{ is } \tilde{A}^3 \text{ then } y = [3.0, 3.5] \\
 R^4: & \text{ If } x \text{ is } \tilde{A}^4 \text{ then } y = [7.0, 8.0] \\
 R^5: & \text{ If } x \text{ is } \tilde{A}^5 \text{ then } y = [9.0, 10.0]
 \end{aligned}
 \tag{Equation 6.24}$$

Equation 6.2 and Equation 6.3 are employed in this example, the sequences $y_l^k(x)$ and $y_r^k(x)$ are derived as delineated in Equation 6.25 and Equation 6.26, where $\mu^j(x) \in \{\underline{\mu}^j(x), \bar{\mu}^j(x)\}$. Both sequences $y_l^k(x)$ and $y_r^k(x)$ are subsequently utilized in Equation 6.4 and Equation 6.5 to calculate the type-reducer of the SISO IT2 FIS-based model, denoted as $y_l(x)$ and $y_r(x)$. Finally, the output of the model, denoted as $y = f(x)$, is calculated through Equation 6.6 and this output y is illustrated in Figure 6.6. As can be observed in Figure 6.6, the output y exhibits a monotonically increasing trend w.r.t x .

$$y_l^k(x) = \frac{\mu^1(x) \times 0.5 + \mu^2(x) \times 1.5 + \mu^3(x) \times 3 + \mu^4(x) \times 7 + \mu^5(x) \times 9}{\mu^1(x) + \mu^2(x) + \mu^3(x) + \mu^4(x) + \mu^5(x) + \mu^6(x)}
 \tag{Equation 6.25}$$

$$y_r^k(x) = \frac{\mu^1(x) \times 1.0 + \mu^2(x) \times 2.5 + \mu^3(x) \times 3.5 + \mu^4(x) \times 8 + \mu^5(x) \times 10}{\mu^1(x) + \mu^2(x) + \mu^3(x) + \mu^4(x) + \mu^5(x)} \quad \text{Equation 6.26}$$

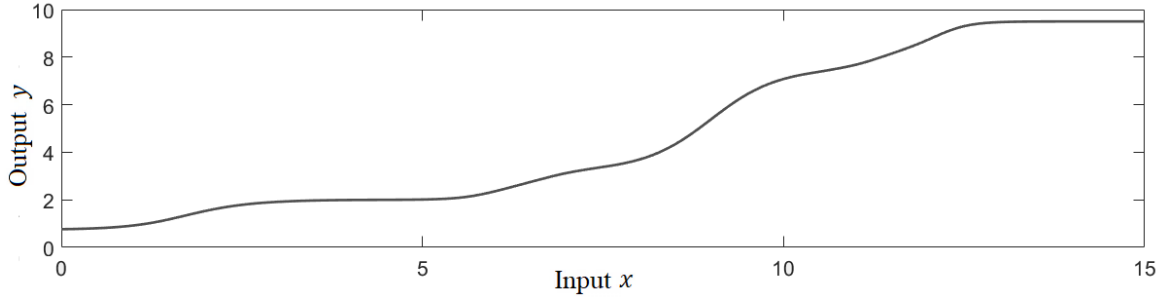


Figure 6.6: Output of SISO monotone IT2 FIS model with IT2Trapezoidal FMF

6.6 MISO monotone IT2 FIS

In this section, case studies drawn from Chapter 4 are leveraged to explicate the applicability of Remark 6.4 in the formulation of IT2 trapezoidal FMF and Remark 6.6 in the synthesis of IT2 Gaussian FMF within the context of MISO monotone IT2 FIS models.

6.6.1 A Monotone IT2 FIS RPN model

In Chapter 4, a monotone T1 FIS-Based RPN model, incorporating 3 inputs i.e., Severity (Sev), Occurrence (Occ), and Detect (Det) is adopted as a quality and transmission risk assessment tool in the proposed Transmission Cause and Effect Analysis (TCEA) procedure. The output of the monotone T1 FIS-based RPN model is a FRPN score denoted by $\text{FRPN} = f(\mathbf{x})$.

In this section, an MISO monotone IT2 FIS-based RPN model is presented for the computation of the FRPN score. IT2 FMF are employed to characterize the linguistic terms for the inputs Sev, Occ, and Det, as detailed in Tables 4.1, 4.2, and 4.3 from Chapter 4. The IT2 FMFs for these inputs are presented in two distinct configurations - IT2 Trapezoidal FMFs and IT2 Gaussian FMFs. Each input is assigned with m_i fuzzy sets, i.e., $\tilde{A}^1, \tilde{A}^{j_i}, \dots, \tilde{A}^{m_i}$

with $i \in \{\text{Sev}, \text{Occ}, \text{Det}\}$ and the membership function of $\tilde{A}^{j_i}$ is denoted by $\tilde{\mu}^{j_i}(x_i)$. Specifically, there are 4 IT2 FMF for input Sev (i.e., $m_{\text{Sev}} = 4$), 4 for input Occ (i.e., $m_{\text{Occ}} = 4$) and 5 for input Det (i.e., $m_{\text{Det}} = 5$), culminating in a total of 80 fuzzy rules, given by $\prod_i^n m_i = 4 \times 4 \times 5$, where $n \in \{\text{Sev}, \text{Occ}, \text{Det}\}$.

The fuzzy rule R^j is explained as a mapping from $\tilde{A}^j$ to $\tilde{b}^j$ i.e., $R^j: \tilde{A}^j \rightarrow \tilde{b}^j$ where $j = (j_{\text{Sev}}, j_{\text{Occ}}, j_{\text{Det}})$. The interval firing strength of R^j , delineated as $[f^j(x), \bar{f}^j(x)]$, is computed via the product operation as expounded in Equation 6.27 and Equation 6.28. It is imperative to note that $\tilde{b}^j$ is singleton IT2 fuzzy sets denoted by $\tilde{b}^j = [\underline{b}^j, \bar{b}^j]$. In this example, the FRPN score (i.e., $\text{FRPN} \in [1, 1000]$) for the MISO monotone IT2 FIS-based RPN model consists of 7 linguistic terms, namely, *Extremely Low*, *Very Low*, *Low*, *Medium*, *High*, *Very High* and *Extremely High*. Each linguistic terms is represented by singleton IT2 fuzzy sets i.e., $[1, 26]$, $[142.5, 192.5]$, $[309, 359]$, $[309, 359]$, $[475.5, 525.5]$, $[642, 692]$, $[808.5, 858.5]$ and $[975, 1000]$, respectively. The $[\underline{b}^j, \bar{b}^j]$ is designed in such a way that *Condition 2* of Remark 6.2 (refer to in Section 6.3.2) is satisfied. The interrelation between the FRPN score and Sev, Occ, and Det is articulated through If-Then fuzzy rules, as depicted in Figure 6.7. The fuzzy rules employed in this demonstration are congruent to fuzzy rules as in Chapter 4

$$\underline{f}^j(x) = \underline{\mu}^{j_{\text{Sev}}} \times \underline{\mu}^{j_{\text{Occ}}} \times \underline{\mu}^{j_{\text{Det}}} \quad \text{Equation 6.27}$$

$$\bar{f}^j(x) = \bar{\mu}^{j_{\text{Sev}}} \times \bar{\mu}^{j_{\text{Occ}}} \times \bar{\mu}^{j_{\text{Det}}} \quad \text{Equation 6.28}$$

R^j : If (Sev is $\tilde{A}^{j_{\text{Sev}}}$) and (Occ is $\tilde{A}^{j_{\text{Occ}}}$) and (Det is $\tilde{A}^{j_{\text{Det}}}$) then FRPN is $\tilde{b}^j = [\underline{b}^j, \bar{b}^j]$

Figure 6.7: If-Then fuzzy rules of an MISO monotone IT2 FIS-based RPN model

Let denote $f^j(\mathbf{x}) \in \{\underline{f}^j(\mathbf{x}), \bar{f}^j(\mathbf{x})\}$, the sequence of $FRPN_l^k(\mathbf{x})$ and $FRPN_r^k(\mathbf{x})$ are derived as per Equation 6.29 and Equation 6.30, respectively. The number of possible combinations of sequence of $FRPN_l^k(\mathbf{x})$ and $FRPN_r^k(\mathbf{x})$ are denoted by $k = 1, \dots, M$ where $M = 2^{\prod_{i=1}^n m_i}$. Here, m_i denotes the count of IT2 fuzzy sets in input space $x_i \in X_i$ and $\prod_{i=1}^n m_i$ amounts to $4 \times 4 \times 5 = 80$

$$FRPN_l^k(\mathbf{x}) = \frac{\sum_{j_{Sev}=1}^{m_{Sev}} \sum_{j_{Occ}=1}^{m_{Occ}} \sum_{j_{Det}=1}^{m_{Det}} (f^j(\mathbf{x}) \times \underline{b}^j)}{\sum_{j_{Sev}=1}^{m_{Sev}} \sum_{j_{Occ}=1}^{m_{Occ}} \sum_{j_{Det}=1}^{m_{Det}} f^j(\mathbf{x})} \quad \text{Equation 6.29}$$

$$FRPN_r^k(\mathbf{x}) = \frac{\sum_{j_{Sev}=1}^{m_{Sev}} \sum_{j_{Occ}=1}^{m_{Occ}} \sum_{j_{Det}=1}^{m_{Det}} (f^j(\mathbf{x}) \times \bar{b}^j)}{\sum_{j_{Sev}=1}^{m_{Sev}} \sum_{j_{Occ}=1}^{m_{Occ}} \sum_{j_{Det}=1}^{m_{Det}} f^j(\mathbf{x})} \quad \text{Equation 6.30}$$

Subsequently, both $FRPN_l^k(\mathbf{x})$ and $FRPN_r^k(\mathbf{x})$ are employed in Equation 6.31 and Equation 6.32 to ascertain the type-reducer of the MISO monotone IT2 FIS-based model, denoted as $y_l(\mathbf{x})$ and $y_r(\mathbf{x})$. The output of the model i.e., $y = f(\mathbf{x})$ is calculated using Equation 6.33.

$$FRPN_l(\mathbf{x}) = \min_{k=1}^M FRPN_l^k(\mathbf{x}) \quad \text{Equation 6.31}$$

$$FRPN_r(\mathbf{x}) = \max_{k=1}^M FRPN_r^k(\mathbf{x}) \quad \text{Equation 6.32}$$

$$FRPN = f(\mathbf{x}) = \frac{1}{2} [FRPN_l(\mathbf{x}) + FRPN_r(\mathbf{x})] \quad \text{Equation 6.33}$$

6.6.2 MISO monotone IT2 FIS with IT2 Trapezoidal FMF

The scale tables for Sev, Occ, and Det in Table 4.1, Table 4.2, and Table 4.3 respectively, of Chapter 4, are invoked in this section. The IT2 Trapezoidal FMF design for Sev, Occ and Det, which scrupulously conforms to Remark 6.4, is elucidated in Figure 6.8, Figure 6.9 and Figure 6.10, respectively. The parameter configurations of the Trapezoidal FMFs for the inputs Sev, Occ, and Det are succinctly encapsulated in Table 6.1.

Table 6.1: Trapezoidal FMFs parameters configuration for input Sev, Occ and Det

Inputs x	Parameters configuration of the Trapezoidal FMFs $\mu^{ji}(x_i) = \mu_i^{ji}(x: \underline{a}_i^{ji}, \bar{a}_i^{ji}, \underline{b}_i^{ji}, \bar{b}_i^{ji}, \underline{c}_i^{ji}, \bar{c}_i^{ji}, \underline{d}_i^{ji}, \bar{d}_i^{ji}, \underline{h}_i^{ji}, 1)$
Sev	• $\tilde{A}_{Sev}^1 = \mu_{Sev}^1(x_{Sev}: 1,1,1,1,3,3,3.75,4,0.8,1)$ • $\tilde{A}_{Sev}^2 = \mu_{Sev}^2(x_{Sev}: 3.25,3,4,4,7,7,7.75,8,0.7,1)$ • $\tilde{A}_{Sev}^3 = \mu_{Sev}^3(x_{Sev}: 7.25,7,8,8,9,9,9.75,10,0.7,1)$ • $\tilde{A}_{Sev}^4 = \mu_{Sev}^4(x_{Sev}: 9.25,9,10,10,10,10,10,10,0.9,1)$
Occ	• $\tilde{A}_{Occ}^1 = \mu_{Occ}^1(x_{Occ}: 1,1,1,1,3,3,4,4,0.7,1)$ • $\tilde{A}_{Occ}^2 = \mu_{Occ}^2(x_{Occ}: 3,3,4,4,6,6,7,7,0.8,1)$ • $\tilde{A}_{Occ}^3 = \mu_{Occ}^3(x_{Occ}: 6,6,7,7,8,8,9,9,0.8,1)$ • $\tilde{A}_{Occ}^4 = \mu_{Occ}^4(x_{Occ}: 8,8,9,9,10,10,10,10,0.7,1)$
Det	• $\tilde{A}_{Det}^1 = \mu_{Det}^1(x_{Det}: 1,1,1,1,1.25,1,1.75,2,0.5,1)$ • $\tilde{A}_{Det}^2 = \mu_{Det}^2(x_{Det}: 1.25,1,2,2,3,3,4,4,0.8,1)$ • $\tilde{A}_{Det}^3 = \mu_{Det}^3(x_{Det}: 3.25,3,4,4,7,7,8,8,0.8,1)$ • $\tilde{A}_{Det}^4 = \mu_{Det}^4(x_{Det}: 7.25,7,8,8,9,9,9.75,10,0.7,1)$ • $\tilde{A}_{Det}^5 = \mu_{Det}^5(x_{Det}: 9.25,9,9.75,10,10,10,10,10,0.5,1)$

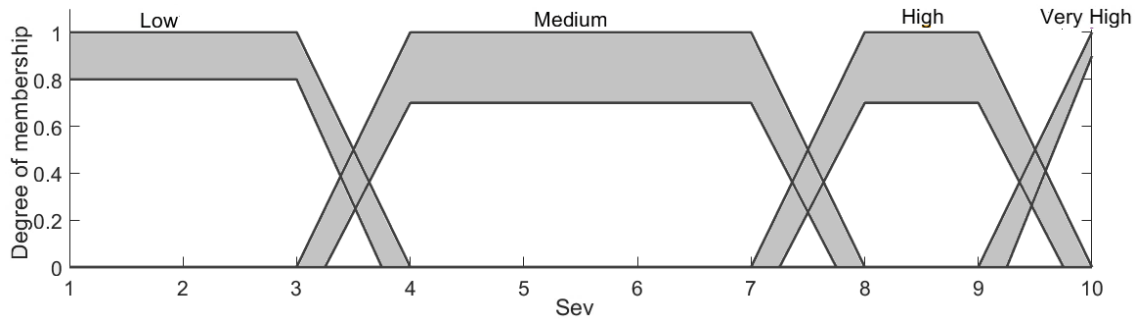


Figure 6.8: IT2 trapezoidal FMF design of Sev

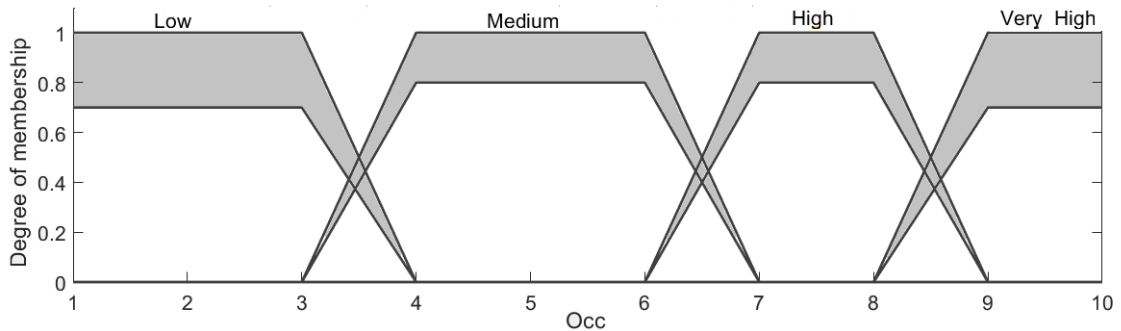


Figure 6.9: IT2 trapezoidal FMF design of Occ

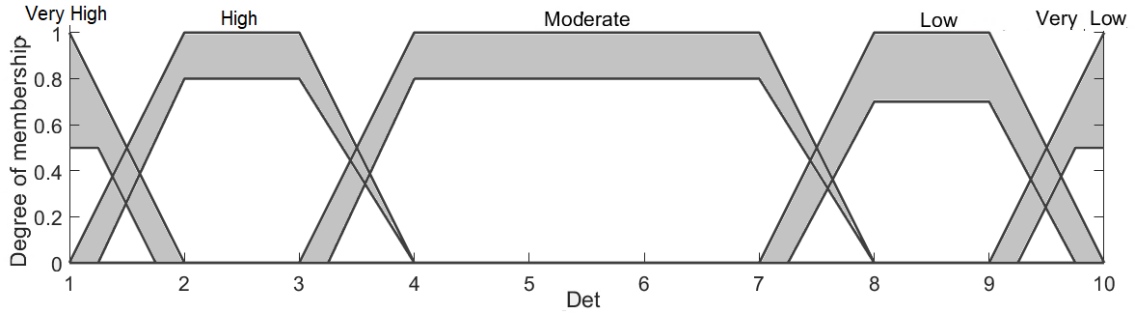


Figure 6.10: IT2 trapezoidal FMF design of Det

(A) Surface plots

Figure 6.11 exhibits the surface plot for the FRPN score versus Occ and Det at Sev is 10. Subsequently, Figure 6.12 elucidates the surface plot delineating the FRPN score against Sev and Det, with Occ held a constant at 10. Moreover, Figure 6.13 portrays the surface plot contrasting the FRPN scores against Sev and Occ, while Det remains at 10. As can be seen in Figure 6.11, Figure 6.12 and Figure 6.13, reveals that the surface plots, representing the MISO IT2 FIS-based RPN model, exhibit monotonic output (i.e., FRPN score) w.r.t the inputs Sev, Occ and Det at specified conditions.

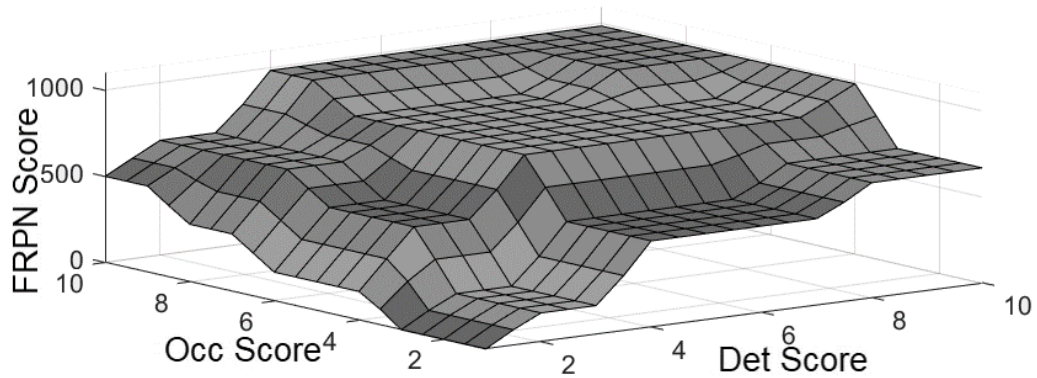


Figure 6.11: Surface plot for FRPN versus Occ and Det with at Sev is 10.

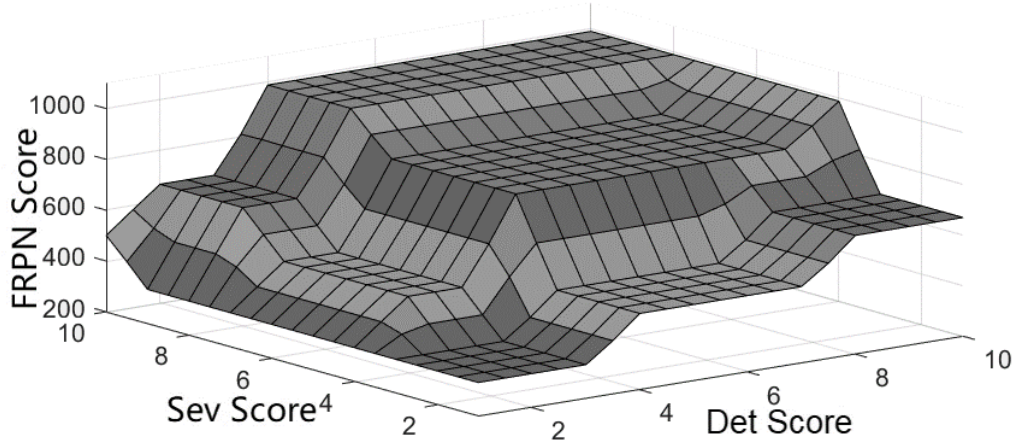


Figure 6.12: Surface plot for FRPN versus Sev and Det with Occ is 10

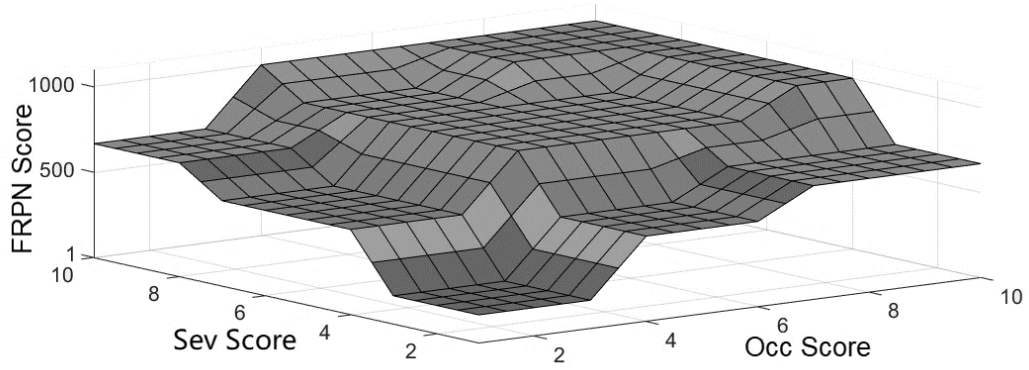


Figure 6.13: Surface plot for FRPN versus Sev and Det with Occ is 10

(B) Monotonicity test

The monotonicity test delineated in Section 4.5.5 of Chapter 4 is adopted in this section. The monotonicity test is instrumental in examining the extent to which the MISO monotone IT2 FIS-based RPN model, denoted $FRPN = f(\mathbf{x})$ comprises $\mathbf{x} = (Sev, Occ, Det)$, adheres to the monotonicity property. In this monotonicity test, input range of $X_{\forall i \in \{Sev, Occ, Det\}} = [1, 10]$ is divided into $n = 900$ subinterval. Let's denote $i = \{Sev, Occ, Det\}$ and $\mathbf{s}$ as subset of $\mathbf{x}$ where x_i is excluded from $\mathbf{s}$ (i.e. $\mathbf{s} \subset \mathbf{x}; x_i \notin \mathbf{s}$).

Totally of $n(n + 1)^2 = 900(900 + 1)^2 = 730620900$ of compare-able pairs i.e., $f(\mathbf{s}, x'_i)$ and $f(\mathbf{s}, x_i)$ for each input x_i are evaluated in the monotonicity tests.

Table 6.2 presents the results of the monotonicity test for $x_{i \in \{\text{Sev}, \text{Occ}, \text{Det}\}}$. The results show that Sev, Occ, and Det have a monotone index of 1. For instance, there are 730620900 compare-able pairs available for x_{Sev} , and all of them satisfy the monotonicity property. Therefore, the monotonicity index of 1 is obtained, which is equal to $730620900/730620900 = 1$.

Table 6.2: Results with the monotonicity test

x_i	Number of compare-able pairs	Number of monotone compare-able pairs	Monotone Index (x_i)
x_{Sev}	730620900	730620900	$\frac{730620900}{730620900} = 1$
x_{Occ}	730620900	730620900	$\frac{730620900}{730620900} = 1$
x_{Det}	730620900	730620900	$\frac{730620900}{730620900} = 1$

6.6.3 MISO monotone IT2 FIS with IT2 Gaussian FMF

Once more, the scale tables for Sev, Occ, and Det, delineated in Table 4.1, Table 4.2, and Table 4.3 of Chapter 4, are invoked in this section. The IT2 Gaussian FMF design for inputs Sev, Occ, and Det, conforming to Remark 6.6 are presented in Figure 6.14, Figure 6.15 and Figure 6.16, respectively. The parameter configurations of the Gaussian FMFs for inputs Sev, Occ, and Det are concisely articulated in Table 6.3. Let's consider $j_i \in \{1 \leq p \leq q \leq m_i\}$ and $i \in \{\text{Sev}, \text{Occ}, \text{Det}\}$. For input Sev, $\underline{\sigma}^{j_{\text{Sev}}}$, $\bar{\sigma}^{j_{\text{Sev}}}$ and $\underline{h}^{j_{\text{Sev}}}$ are set to fix value, as such, $\bar{\sigma}^q = \bar{\sigma}^q = 0.4$, $\underline{\sigma}^q = \underline{\sigma}^q = 0.35$ and $\underline{h}^p = \underline{h}^q = 0.35^2/0.4^2 =$

0.765625, across all values of $1 \leq p \leq q \leq m_i$. Similarly, for input Occ, $\underline{\sigma}^{j_{\text{Occ}}}$, $\bar{\sigma}^{j_{\text{Occ}}}$ and $\underline{h}^{j_{\text{Occ}}}$ are set to fixed at $\bar{\sigma}^q = \bar{\sigma}^q = 0.5$, $\underline{\sigma}^q = \underline{\sigma}^q = 0.5$ and $\underline{h}^p = \underline{h}^q = 0.5^2/0.5^2 = 1$ for all $1 \leq p \leq q \leq m_i$. Lastly, for input Det, $\underline{\sigma}^{j_{\text{Det}}}$, $\bar{\sigma}^{j_{\text{Det}}}$ and $\underline{h}^{j_{\text{Det}}}$ are set at $\bar{\sigma}^q = \bar{\sigma}^q = 0.45$, $\underline{\sigma}^q = \underline{\sigma}^q = 0.4$ and $\underline{h}^p = \underline{h}^q = 0.4^2/0.45^2 \approx 0.7901$ for all $1 \leq p \leq q \leq m_i$. In this example, normalization procedure is employed for calibrating the minimum and maximum FRPN score emanating from the MISO monotone IT2 FIS-based RPN model, to ascertain that the scores of the FRPN lie within a stipulated range, specifically between 1 and 1000, inclusive.

Table 6.3: Gaussian FMFs parameters configuration for input Sev, Occ and Det

Inputs x	Parameters configuration of the Gaussian FMFs $\mu^{j_i}(x_i) = \mu_i^{j_i}(x: \underline{a}_i^{j_i}, \bar{a}_i^{j_i}, \underline{b}_i^{j_i}, \bar{b}_i^{j_i}, \underline{\sigma}_i^{j_i}, \bar{\sigma}_i^{j_i}, \underline{h}_i^{j_i}, 1)$
Sev	• $\tilde{A}_{\text{Sev}}^1 = \mu_{\text{Sev}}^1(x_{\text{Sev}}: 1, 1, 3, 3, 0.35, 0.4, 0.77, 1)$ • $\tilde{A}_{\text{Sev}}^2 = \mu_{\text{Sev}}^2(x_{\text{Sev}}: 4, 4, 7, 7, 0.35, 0.4, 0.77, 1)$ • $\tilde{A}_{\text{Sev}}^3 = \mu_{\text{Sev}}^3(x_{\text{Sev}}: 8, 8, 9, 9, 0.35, 0.4, 0.77, 1)$ • $\tilde{A}_{\text{Sev}}^4 = \mu_{\text{Sev}}^4(x_{\text{Sev}}: 10, 10, 10, 10, 0.35, 0.4, 0.77, 1)$
Occ	• $\tilde{A}_{\text{Occ}}^1 = \mu_{\text{Occ}}^1(x_{\text{Occ}}: 1, 1, 2.5, 3, 0.5, 0.5, 1, 1)$ • $\tilde{A}_{\text{Occ}}^2 = \mu_{\text{Occ}}^2(x_{\text{Occ}}: 4.5, 4.5, 5.5, 6, 0.5, 0.5, 1, 1)$ • $\tilde{A}_{\text{Occ}}^3 = \mu_{\text{Occ}}^3(x_{\text{Occ}}: 7.5, 7.5, 8, 8, 0.5, 0.5, 1, 1)$ • $\tilde{A}_{\text{Occ}}^4 = \mu_{\text{Occ}}^4(x_{\text{Occ}}: 9.5, 9, 10, 10, 0.5, 0.5, 1, 1)$
Det	• $\tilde{A}_{\text{Det}}^1 = \mu_{\text{Det}}^1(x_{\text{Det}}: 1, 1, 1, 1, 0.45, 0.45, 0.79, 1)$ • $\tilde{A}_{\text{Det}}^2 = \mu_{\text{Det}}^2(x_{\text{Det}}: 2.25, 2, 2.75, 3, 0.45, 0.45, 0.79, 1)$ • $\tilde{A}_{\text{Det}}^3 = \mu_{\text{Det}}^3(x_{\text{Det}}: 4.5, 4, 6.5, 7, 0.45, 0.45, 0.79, 1)$ • $\tilde{A}_{\text{Det}}^4 = \mu_{\text{Det}}^4(x_{\text{Det}}: 8.25, 8, 8.75, 9, 0.45, 0.45, 0.79, 1)$ • $\tilde{A}_{\text{Det}}^5 = \mu_{\text{Det}}^5(x_{\text{Det}}: 10, 10, 10, 10, 0.45, 0.45, 0.79, 1)$

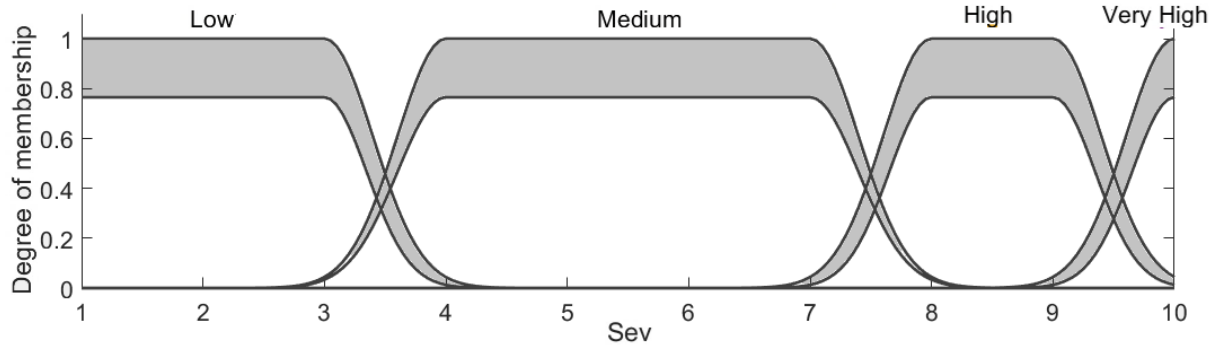


Figure 6.14: IT2 Gaussian FMF design of Sev

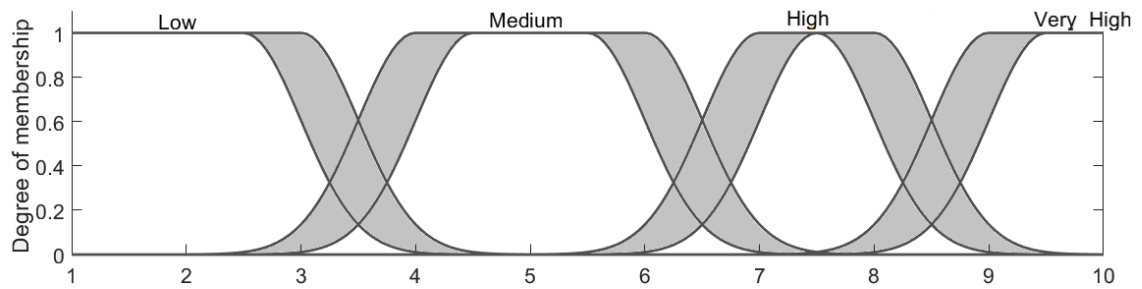


Figure 6.15: IT2 Gaussian FMF design of Occ

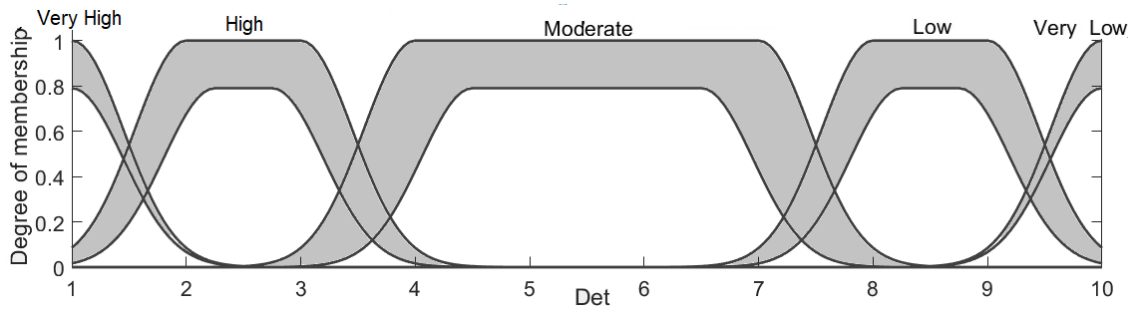


Figure 6.16: IT2 Gaussian FMF design of Det

(A) Surface plots

Figure 6.17 presents a surface plot depicting the relationship between the FRPN score and the variables Occ and Det, while maintaining Sev at 10. Subsequently, Figure 6.18 elucidates the surface plot delineating the FRPN score against Sev and Det, with Occ held constant at 10. Moreover, Figure 6.19 furnishes a surface plot contrasting the FRPN scores with respect to Sev and Occ, concomitant with Det being held a constant at 10. As can be seen in Figure 6.17, Figure 6.18 and Figure 6.19, reveals that the surface plots, representing

the MISO IT2 FIS-based RPN model, exhibit monotonic output (i.e., FRPN score) w.r.t the inputs Sev, Occ and Det at specified conditions.

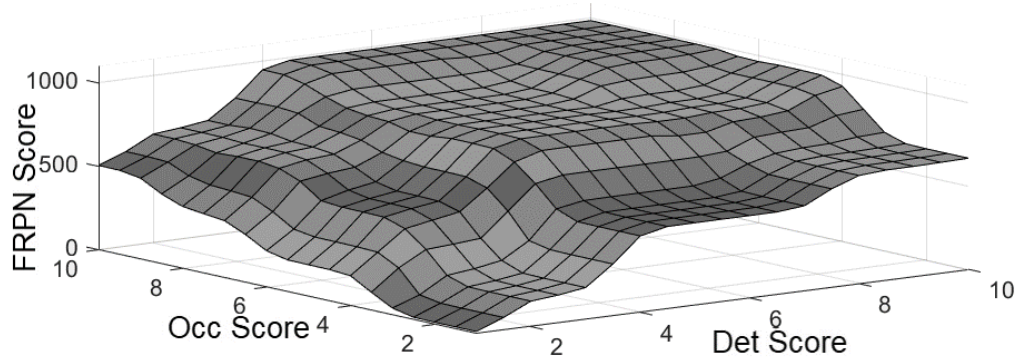


Figure 6.17: Surface plot for FRPN versus Occ and Det with at Sev is 10.

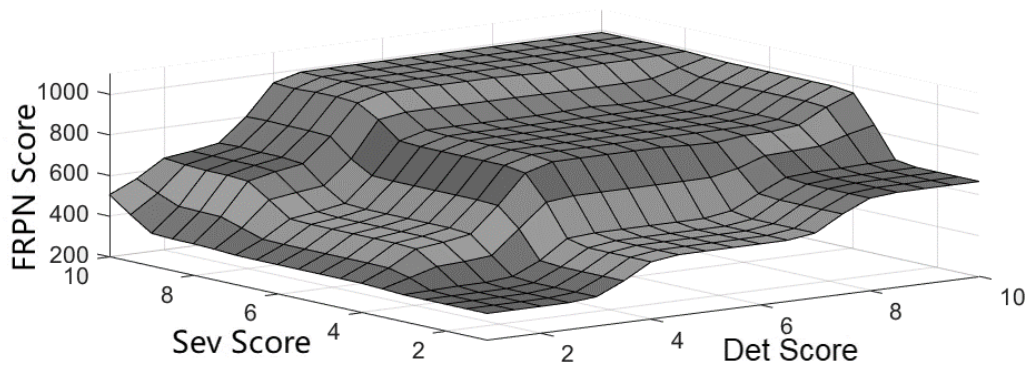


Figure 6.18: Surface plot for FRPN versus Sev and Det with Occ is 10

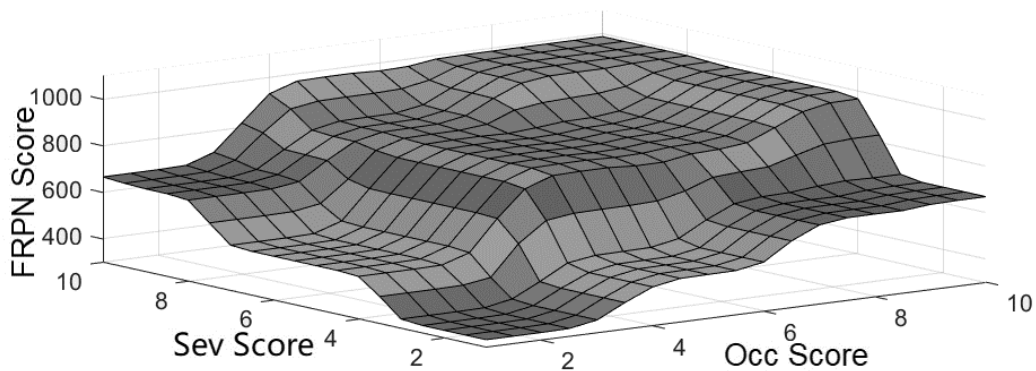


Figure 6.19: Surface plot for FRPN versus Sev and Det with Occ is 10

(B) Monotonicity test

Once more, monotonicity test is undertaken within this section, employing the same setting as delineated in Section 6.3.2(B). Table 6.4 elucidates the outcomes of monotonicity test for $x_{i \in \{Sev, Occ, Det\}}$. The findings indicate that Sev, Occ, and Det have a monotone index of 1. For instance, there are 730620900 compare-able pairs available for x_{Sev} , and all of them satisfy the monotonicity property. Therefore, the monotonicity index of 1 is obtained, which is equal to $730620900/730620900 = 1$.

Table 6.4: Results with the monotonicity test

x_i	Number of compare-able pairs	Number of monotone compare-able pairs	Monotone Index (x_i)
x_{Sev}	730620900	730620900	$\frac{730620900}{730620900} = 1$
x_{Occ}	730620900	730620900	$\frac{730620900}{730620900} = 1$
x_{Det}	730620900	730620900	$\frac{730620900}{730620900} = 1$

6.6.4 Prioritization of TPs using RPN and FRPN Scores

The prioritization of Transmission Potentials (TPs) delineated in the Transmission Chain Elimination Analysis (TCEA) spreadsheet (viz., Table 4.4 and Table 4.5) is employed within the case studies, as recapitulated in Table 6.5 and Table 6.6 in a descending order. The column entitled "Transmission Potentials" elucidates TPs, whereas the column "Area/Room" specifies the locales wherein TPs transpire. The assessments for Severity (Sev), Occurrence (Occ), Detection (Det), RPN and FRPN are allocated within the columns labeled "Sev", "Occ", "Det", "RPN", and "FRPN" respectively. For lucid exposition, monotone T1 FIS-based RPN model is designated as *System A*, MISO monotone IT2 FIS-

based RPN model employing IT2 Trapezoidal FMFs is designated as *System B*, and the MISO monotone IT2 FIS-based RPN model utilizing IT2 Gaussian FMFs is designated as *System C*. As in Table 6.5 and Table 6.6, despite *System A*, *System B* and *System C* calculating marginally distinct FRPN scores for each Transmission Potential (TP), the ordinal ranking of the TPs according to FRPN remains consistent. It is imperative to take into account that *System A*, *System B* and *System C* employ identical fuzzy rules, albeit disparate in fuzzy sets design are employed for the inputs across *System A*, *System B* and *System C*.

Table 6.5: Summary of the TCEA spreadsheet Table 4.4

ID	Transmission Potentials (TPs)	Area/Room	Sev	Occ	Det	RPN	FRPN		
							<i>System A</i>	<i>System B</i>	<i>System C</i>
TP1.5	Necessary entry of contactors to the premise for facilities and utilities maintenance	Lab 1/ Lab 2	3	8	5	120	334.00	334.0	368.92
TP1.10	Necessary entry of contactors to the premise for facilities and utilities maintenance	Discussion Room	3	8	5	120	334.00	334.0	368.92
TP1.14	Necessary entry of contactors to the premise for facilities and utilities maintenance	CTO Office	3	8	5	120	334.00	334.00	368.92
TP1.1	Close distance social interaction which involves 2 and more staff.	Lab 1/ Lab 2	2	4	3	24	167.00	167.50	179.68
TP1.11	Close distance social interaction which involves 2 and more staff.	CTO Office	2	4	2	16	167.00	167.50	156.82
TP1.4	Room and office furniture contamination with infectious agents (e.g., office table and chair)	Lab 1/ Lab 2	1	5	2	10	167.00	167.50	159.38
TP1.9	Room and office furniture contamination with infectious agents (E.g., door handle, floor, meeting table and chair)	Discussion Room	1	5	2	10	167.00	167.50	159.38
TP1.13	Room and office furniture contamination with infectious agents (E.g., door handle office desk and chair)	CTO Office	1	5	2	10	167.00	167.50	159.38
TP1.6	Close distance social interaction which involve 2 and more staff.	Discussion Room	2	4	1	8	167.0	167.50	139.28
TP1.2	Sharing of office stationery	Lab 1/ Lab 2	1	2	1	2	1.00	1.000	1.05
TP1.3	Passing papers and document between staffs	Lab 1/ Lab 2	1	2	1	2	1.00	1.000	1.05
TP1.7	Sharing of office stationery	Discussion Room	1	2	1	2	1.00	1.000	1.05

TP1.8	Passing paper and document between staff	Discussion Room	1	2	1	2	1.00	1.000	1.05
TP1.12	Passing paper and document between staff	CTO Office	1	2	1	2	1.00	1.000	1.05
TP1.15	Potential of server room contamination with infectious agents.	Server Room	1	1	1	1	1.00	1.000	1.00
TP1.16	Potential of repair tools and equipment contamination with infectious agents.	Server Room	1	1	1	1	1.00	1.000	1.00

Table 6.6: Summary of the TCEA spreadsheet Table 4.5

ID	Transmission Potentials (TPs)	Area/Room	Sev	Occ	Det	RPN	FRPN		
							System (A)	System (B)	System (C)
TP2.6	Necessary entry of contractors to the premise for facilities and utilities maintenance	Pantry	3	8	5	120	334.00	334.00	368.92
TP2.8	Necessary entry of contractors to the premise for facilities and utilities maintenance	Restroom	3	8	5	120	334.00	334.00	368.92
TP2.1	Infected non-employees/personnel's using shared lift	Shared-Lift	3	9	3	81	334.00	334.00	357.71
TP2.5	Room and kitchen furniture contamination with infectious agents (e.g., dining table and chair)	Pantry	1	6	3	18	167.00	167.50	196.86
TP2.2	Close distance social interaction which involve 2 and more staff.	Main entrance	2	4	1	8	167.00	167.50	139.28
TP2.7	Room contamination (e.g., toilet, faucet handle and door hander) with infectious agents	Restroom	1	4	2	8	167.00	167.50	156.82
TP2.3	Contamination of the area with infectious agents (door hander of the entrance, floor, etc.)	Main entrance	1	3	1	3	1.00	1.00	27.27
TP2.4	Potential of contamination of the tableware and kitchenware with infectious agents	Pantry	1	1	1	1	1.00	1.00	1.00
TP2.9	Potential of contamination of the document and files with infectious agents	Document room	1	1	1	1	1.00	1.00	1.00
TP2.10	Potential of room contamination (e.g., door hander, wall, floor) with infectious agents.	Document room	1	1	1	1	1.00	1.00	1.00

6.7 Summary

In Section 6.3, the sufficient conditions SISO monotone IT2 FIS model and MISO monotone IT2 FIS model is introduced. To preserve the monotonicity property of the IT2

FIS model, the works of Li et al. (2014) and Li et al. (2011) are streamlined into Remark 6.1 and Remark 6.2, respectively, and are employed as the governing equations for constructing IT2 FIS models. Section 6.4 details the parametric conditions necessary to design trapezoidal and Gaussian IT2 Fuzzy Membership Functions (FMFs) within monotone IT2 TSK FIS models. Section 6.5 provides insights into simulated experiments using these models with trapezoidal and Gaussian FMFs, confirming the theoretical principles through practical examples. Finally, Section 6.6 employs case studies to illustrate the practical application of these designs in MISO monotone IT2 FIS models, highlighting their efficacy and real-world relevance.

These sections collectively contribute to the achievement of RO4, which aims to analyze the parametric conditions of monotone IT2 FIS and apply these conditions systematically for COVID-19 prevention. Section 6.3 identifies and simplifies the necessary conditions (Remark 6.1 and Remark 6.2) to ensure monotonicity in IT2 FIS models. Section 6.4 offers detailed parametric conditions for designing trapezoidal and Gaussian IT2 FMFs that meet these conditions. Section 6.5 validates these theoretical principles through simulated experiments, while Section 6.6 demonstrates their practical application in real-world scenarios, specifically for COVID-19 prevention. This systematic approach ensures that the models are not only theoretically sound but also practically effective in addressing the challenges posed by the pandemic.

In Chapter 7, an investigative case study concerning image edge detection utilizing a monotone MISO IT2 FIS is undertaken, with Remark 6.2 serving as the governing parameters for the construction of the MISO IT2 FIS model.

CHAPTER 7

MONOTONE IT2 FIS MODELLING FOR EDGE DETECTION

7.1 Introduction

In the preceding chapter, the monotone IT2 FIS is outlined. At chapter 5 edge detection framework with monotone TSK FIS was reported. The focus of this chapter is on the use of monotone IT2 FIS for edge detection. The sequence of discussion in this chapter is as follows.

Section 7.2 elucidates the foundational theory and applicational background of the monotone IT2 FIS in the domain of edge detection. The proposed fuzzy blending function, integrated with a monotone IT2 FIS Model, is thoroughly outlined in Section 7.3. Following this, Section 7.4 showcases two distinct manifestations of the IT2 FMF Design in relation to the IT2 Fuzzy Blending function, specifically, the IT2 Trapezoidal FMF and the IT2 Gaussian FMF. The experimental application of the IT2 Fuzzy Blending function, utilizing a digitized mammogram from a standard patient procured from the Mammographic Image Analysis Society's (MIAS) database, followed by a transformation of the mammogram into edge images, is undertaken in Section 7.5. Lastly, a summary of chapter is presented at Section 7.6

7.2 Background

In recent years, there has been a marked surge in investigations into advanced variants of fuzzy logic, specifically focusing on IT2 fuzzy logic (Wagner and Hagrais, 2010; Chen et al., 2013; Hsu and Juang, 2013; Zhou and Ying, 2013; Li and Yang, 2013) and the

newer generalized type-2 fuzzy logic (Melin et al., 2014). This progression towards higher-order forms aims to fabricate more precise models for uncertainty management, with generalized type-2 fuzzy logic theoretically offering enhanced control over uncertainty (Wagner and Hagraas, 2010; Melin et al., 2014).

The present chapter underscores the deployment of the IT2 FIS model in the realm of edge detection, a fundamental component within many computer vision systems (Melin et al., 2014). This technique significantly streamlines image analysis by markedly paring down the data required for processing (Canny, 1986). Edge detection finds varied applications across disciplines such as geography (Shimada et al., 2005), military (Goshtasby, 2005), medicine (Kazerooni et al., 2011), robotics (Siliciano et al., 2010), meteorology (Hocenski et al., 2006), and beyond. In the specific context of image processing, several edge detection methods have been developed, utilizing IT2 fuzzy systems. These methods include combinations with the Sobel operator (Mendoza et al., 2007), standalone IT2 fuzzy systems (Mendoza et al., 2009), the morphological gradient (Melin et al., 2010), and an enhanced version of the Canny method based on IT2 fuzzy logic (Biswas and Sil, 2012).

7.3 Fuzzy Blending Function with Monotone IT2 FIS Model

Research in Bezdek et. al (1998) demonstrated T1 TSK FIS as approximate function to model waterfall blending functions. In this section, an alternative approach utilizing a MISO monotone IT2 FIS model is explored as an approximate function to model waterfall blending functions. This deviation from Bezdek et al. (1998)'s approach, which focused on manipulating the consequent function in T1 TSK FIS to produce output akin to waterfall blending functions. Herein, the proposed strategy emphasizes on the manipulation of the IT2

FMF design within the MISO monotone IT2 FIS model, with an aim to emulate the output results akin to waterfall blending functions.

Reverting to Chapter 5, a Blending function i.e., $b[\mathbf{F}_{IJ}] = B_{IJ}$ aggregates edge information held by $|\mathbf{f}(\mathbf{w})|$, represented as $\mathbf{f}_{ij} \in \mathfrak{R}^1 \times \mathfrak{R}^2$ and mapping into space $\mathfrak{R}$, such that, $b: IJ \times \mathfrak{R}^P \mapsto IJ \times \mathfrak{R}$. $\mathbf{F}_{IJ}$ designates a feature-extracted image comprised of two bands (layers), depicted by $F_{IJ}^1 \in \mathfrak{R}^1$ and $F_{IJ}^2 \in \mathfrak{R}^1$. The intensity of a pixel at spatial location ij at $\mathbf{F}_{IJ}$ is expressed by $\mathbf{f}_{ij} = [f_{ij}^1, f_{ij}^2]$, where $f_{ij}^1 \in F_{IJ}^1$ and $f_{ij}^2 \in F_{IJ}^2$. Here, B_{IJ} signifies the blended image output and b_{ij} denotes intensity of a pixel at spatial location ij in B_{IJ} . The input spaces of the blending function are confined to the $[0,4] \times [0,4]$ space. Hence, remapping of $\mathfrak{R}^1 \times \mathfrak{R}^2 \mapsto [0,4] \times [0,4]$ is necessary, contingent on the type of feature extractor function employed. This chapter examines two feature extractor functions, namely the Sobel operator (Sobel and Feldman, 1968) and the Prewitt operator (Prewitt 1970). A comprehensive discussion regarding their implementation can be found in Section 5.3.2 of Chapter 5. For additional details on the remapping process from $\mathfrak{R}^1 \times \mathfrak{R}^2 \mapsto [0,4] \times [0,4]$, Equations 5.12 and 5.13 in Section 5.3.3 of Chapter 5 should be consulted.

In this chapter, IT2 fuzzy blending function integrated with MISO monotonic IT2 FIS model is presented. The IT2 fuzzy blending function, denoted as $b(\mathbf{x})$, comprises two inputs designated as $\mathbf{x} = [x_1, x_2]$. The IT2 fuzzy blending function is governed by four fuzzy rules, symbolized $R^{(j_1, j_2)}$, as delineated in Equation 7.1. $R^{(j_1, j_2)}$ is explained as mapping $\tilde{A}^{((j_1, j_2))}$ from input space $\mathbf{x} \in \mathfrak{R}^1 \times \mathfrak{R}^2$ to a consequent $\mathcal{B}^{(j_1, j_2)} = [\underline{\mathcal{B}}^{(j_1, j_2)}, \overline{\mathcal{B}}^{(j_1, j_2)}]$ at output space $b_{ij} \in B_{IJ}$. Each input, denoted as $x_{i \in \{1,2\}}$, comprise of two IT2 FMFs, $\tilde{A}_i^{j_i}$, with respective membership functions $\mu_i^{j_i}(x_i) \in \{\underline{\mu}_i^{j_i}(x_i), \overline{\mu}_i^{j_i}(x_i)\}$, where $j_i \in \{1,2\}$ signifies the

j_i -th FMF in input x_i . The consequent of fuzzy rule $R^{(j_1, j_2)}$ is denoted as $\mathcal{B}^{(j_1, j_2)} = [\underline{\mathcal{B}}^{(j_1, j_2)}, \bar{\mathcal{B}}^{(j_1, j_2)}]$. The $[\underline{\mathcal{B}}^{(j_1, j_2)}, \bar{\mathcal{B}}^{(j_1, j_2)}]$ are designed in such a way that *Condition 2* of Remark 6.2 (refer to in Section 6.3.2) is satisfied i.e., $\underline{\mathcal{B}}^{(1,1)} \leq \underline{\mathcal{B}}^{(1,2)} \leq \underline{\mathcal{B}}^{(2,1)} \leq \underline{\mathcal{B}}^{(2,2)}$ and $\bar{\mathcal{B}}^{(1,1)} \leq \bar{\mathcal{B}}^{(1,2)} \leq \bar{\mathcal{B}}^{(2,1)} \leq \bar{\mathcal{B}}^{(2,2)}$, as in Equation 7.1.

$$\begin{aligned}
R^{(1,1)}: & \text{ If } x_1 \text{ is } A_1^1 \text{ and } x_2 \text{ is } A_2^1 \text{ then } \mathcal{B}^{(1,1)} = [0, 0] \\
R^{(1,2)}: & \text{ If } x_1 \text{ is } A_1^1 \text{ and } x_2 \text{ is } A_2^2 \text{ then } \mathcal{B}^{(1,2)} = [220 \ 255]. \\
R^{(2,1)}: & \text{ If } x_1 \text{ is } A_1^2 \text{ and } x_2 \text{ is } A_2^1 \text{ then } \mathcal{B}^{(2,1)} = [220 \ 255]. \\
R^{(2,2)}: & \text{ If } x_1 \text{ is } A_1^2 \text{ and } x_2 \text{ is } A_2^2 \text{ then } \mathcal{B}^{(2,2)} = [255 \ 255].
\end{aligned} \tag{Equation 7.1}$$

The KM type-reducer is utilized for type reduction in blending function. Let's denoted membership function $\mu_i^{j_i}(x_i) \in \{\underline{\mu}_i^{j_i}(x_i), \bar{\mu}_i^{j_i}(x_i)\}$ for $\tilde{A}_i^{j_i}$. The interval firing strength of $R^{(j_1, j_2)}$ i.e., $[\underline{f}^{(j_1, j_2)}(\mathbf{x}), \bar{f}^{(j_1, j_2)}(\mathbf{x})]$ can be calculated by the product operation as in Equation 7.2 and Equation 7.3

$$\underline{f}^{(j_1, j_2)}(\mathbf{x}) = \prod_{i=1}^n \underline{\mu}_i^{j_i}(x_i) \tag{Equation 7.2}$$

$$\bar{f}^{(j_1, j_2)}(\mathbf{x}) = \prod_{i=1}^n \bar{\mu}_i^{j_i}(x_i) \tag{Equation 7.3}$$

Let denote $f^{(j_1, j_2)}(\mathbf{x}) \in \{\underline{f}^{(j_1, j_2)}(\mathbf{x}), \bar{f}^{(j_1, j_2)}(\mathbf{x})\}$, the sequence of $b_l^k(\mathbf{x})$ and $b_r^k(\mathbf{x})$ can be obtained in Equation 7.4 and Equation 7.5, respectively. The number of possible combinations of sequence of $b_l^k(\mathbf{x})$ and $b_r^k(\mathbf{x})$ are denoted by $k = 1, \dots, M$ where $M = 2^4$, consider there are total 4 fuzzy rules in the blending function.

$$b_l^k(\mathbf{x}) = \frac{\sum_{j_1=1}^2 \sum_{j_2=1}^2 (f^{(j_1, j_2)}(\mathbf{x}) \times \underline{\mathcal{B}}^{(j_1, j_2)})}{\sum_{j_1=1}^2 \sum_{j_2=1}^2 \mu^{(j_1, j_2)}(\mathbf{x})} \tag{Equation 7.4}$$

$$b_r^k(\mathbf{x}) = \frac{\sum_{j_1=1}^2 \sum_{j_2=1}^2 (f^{(j_1, j_2)}(\mathbf{x}) \times \bar{\mathcal{B}}^{(j_1, j_2)})}{\sum_{j_1=1}^2 \sum_{j_2=1}^2 \mu^{(j_1, j_2)}(\mathbf{x})} \tag{Equation 7.5}$$

Both $b_l^k(\mathbf{x})$ and $b_r^k(\mathbf{x})$ are then used Equation 7.6 and Equation 7.7 in to compute type-reducer of MISO monotone IT2 FIS-based model i.e., $b_l(\mathbf{x})$ and $b_r(\mathbf{x})$. The output of the model i.e., $b(\mathbf{x})$ is calculated using Equation 7.8. In situations where the $\underline{\mu}_i^{j_i}(x_i) = \bar{\mu}_i^{j_i}(x_i)$ and $\bar{b}^j = \underline{b}^j$, the MISO monotone IT2 FIS-based model IT2 FLS can be interpreted as an MISO monotone T1 TSK FIS.

$$b_l(\mathbf{x}) = \min_{k=1}^M b_l^k(\mathbf{x}) \quad \text{Equation 7.6}$$

$$b_r(\mathbf{x}) = \max_{k=1}^M b_r^k(\mathbf{x}) \quad \text{Equation 7.7}$$

$$b(\mathbf{x}) = \frac{1}{2} [b_l(\mathbf{x}) + b_r(\mathbf{x})] \quad \text{Equation 7.8}$$

7.3.1 Monotonicity test on TSK FIS blending function.

Monotonicity test approach from Tay and Lim (2011b), Tay *et al* (2012a, 2012b) and Teh *et al* (2018) is adopted to evaluate the output of $b(\mathbf{x})$. The procedure is as follows.

Step 1: Divide input range of $x_{\forall i \in \{1,2\}} = [0, 4]$ into $n = 200$ subinterval. As such, fixed

step size as denoted as $\delta_{\forall i \in \{1,2\}} = (4 - 0)/200 = 0.02$ form total spaced points

$\{x_{i,0}, \dots x_{i,m_i}, \dots x_{i,200}\} \in [0, 4]$ where $x_{i,m_i} = 0 + 0.02 \times m_i$.

Step 2: Let's denote $j = \{1,2\}$ and $j \neq i$, evaluate any pair $b(x_j, x_i; t, \chi, \gamma, \omega)$ and

$b(x_j, x'_i; t, \chi, \gamma, \omega)$ as in Equation 5.21, where $[x_i, x'_i] \in \{x_{i,0}, \dots x_{i,m_i}, \dots x_{i,200}\}$.

For a selected $x_i = x_{i,m_i}$, $x' = x_{i,m_i+1}$ should always true where $m_i < 200$.

$$\begin{aligned} & \text{Monotone} \left(b(x_j, x_i), b(x_j, x'_i) \right) \\ &= \begin{cases} 1 & \text{if } b(x_j, x'_i) \geq b(x_j, x_i) \\ 0 & \text{otherwise} \end{cases} \end{aligned} \quad \text{Equation 7.9}$$

Step 3: Evaluate $b(\mathbf{x})$ for all combination of grid points for $\forall i = \{1,2\}$ as in as in Equation

5.21 i.e., total $n(n+1) = 200(200+1) = 40200$ compare-able pairs are available for $n = 200$. Compute the Monotone Index between y and inputs x_i as in Equation 5.22.

$$\begin{aligned} & \text{Monotone_Index}(x_i) \\ &= \frac{\sum_{m_{j \neq i}=0}^{m_{j \neq i}=n} \sum_{m_i=0}^{m_i=n-1} \left(\text{monotone} \left(b(x_j, x_i), b(x_j, x_i) \right) \right)}{n(n+1)} \end{aligned} \quad \text{Equation 7.10}$$

7.4 IT2 FMF Design of the IT2 Fuzzy Blending function

In this section, the IT2 FMF design pertaining to input x_1 and x_2 are established as identical, as such, $\tilde{A}_1^1 = \tilde{A}_2^1 = \mu_{i \in \{1,2\}}^1(x_i)$, and $\tilde{A}_1^2 = \tilde{A}_2^2 = \mu_{i \in \{1,2\}}^2(x_i)$. Four distinct sets of IT2 FMF designs are examined, applied for both IT2 Trapezoidal FMF and IT2 Gaussian FMF.

7.4.1 IT2 Fuzzy Blending function with IT2 Trapezoidal FMF

In this section, an experimental design is presented involving three instances of IT2 Trapezoidal FMF designs, designated as Case (A-1), Case (A-2), Case (A-3) and Case (A-4). For simplicity, $\tilde{A}_1^1 = \tilde{A}_2^1$ is defined as an IT2 Triangular FMF parameterized by, $\tilde{A}_{i \in \{1,2\}}^1 = \mu_{i \in \{1,2\}}^1(x_i: \underline{a}_i^1, \bar{a}_i^1, \underline{b}_i^1, \bar{b}_i^1, \underline{c}_i^1, \bar{c}_i^1, \underline{d}_i^1, \bar{d}_i^1, \underline{h}^1, 1)$ where $\underline{a}_i^1 = \underline{b}_i^1 = \underline{c}_i^1 = \bar{a}_i^1 = \bar{b}_i^1 = \bar{c}_i^1 = 0$. Concurrently, $\tilde{A}_{i \in \{1,2\}}^2$ is identified as an IT2 Trapezoidal FMF parameterized by $\tilde{A}_{i \in \{1,2\}}^2 = \mu_{i \in \{1,2\}}^2(x_i: \underline{a}_i^2, \bar{a}_i^2, \underline{b}_i^2, \bar{b}_i^2, \underline{c}_i^2, \bar{c}_i^2, \underline{d}_i^2, \bar{d}_i^2, \underline{h}^2, 1)$ where $\underline{a}_i^2 = \bar{a}_i^2 = 0$. The experiment investigates four distinct scenarios regarding variations in the area under the membership function of $\tilde{A}_{i \in \{1,2\}}^1$ and $\tilde{A}_{i \in \{1,2\}}^2$, where $\bar{d}_i^1 = \underline{d}_i^1 = \underline{b}_i^2 = \bar{b}_i^2$ are set to be identical. Specifically, in Case (A-1), $\bar{d}_i^1$ is fixed at to 4, as presented in Figure 7.1. In Case

(A-2), $\bar{d}_i^1$ is set to 2, as illustrated in Figure 7.2. In Case (A-3), $\bar{d}_i^1$ is set to 1, as in Figure 7.3. Lastly, in Case (A-4), $\bar{d}_i^1$ is assigned a value of 0.5, as shown in Figure 7.4. Both $\underline{h}^1$ and $\underline{h}^2$ are set to 0.8. The parameters configuration of the IT2 Trapezoidal FMFs for the inputs $x_{i \in \{1,2\}}$ is succinctly encapsulated in Table 7.1

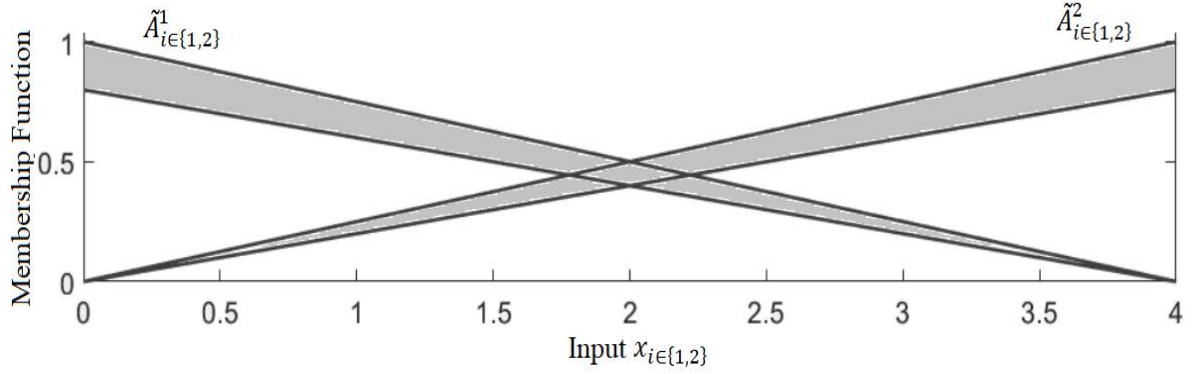


Figure 7.1: IT2 Trapezoidal FMF designs Case (A-1) for $\bar{d}_i^1 = 4$

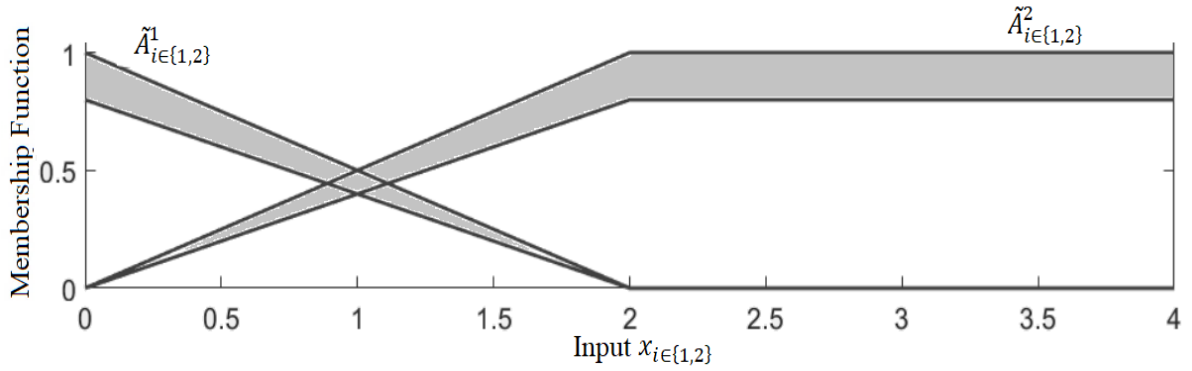


Figure 7.2: IT2 Trapezoidal FMF designs Case (A-2) for $\bar{d}_i^1 = 2$

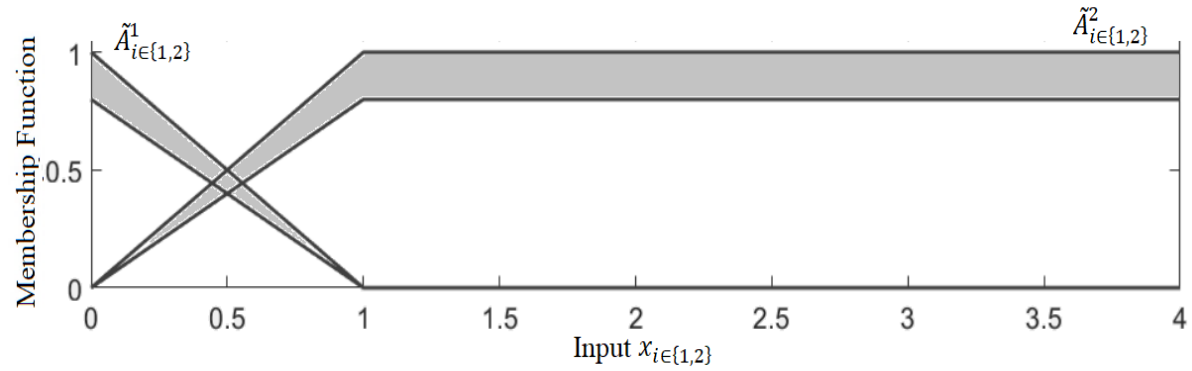


Figure 7.3: IT2 Trapezoidal FMF designs Case (A-3) for $\bar{d}_i^1 = 1$

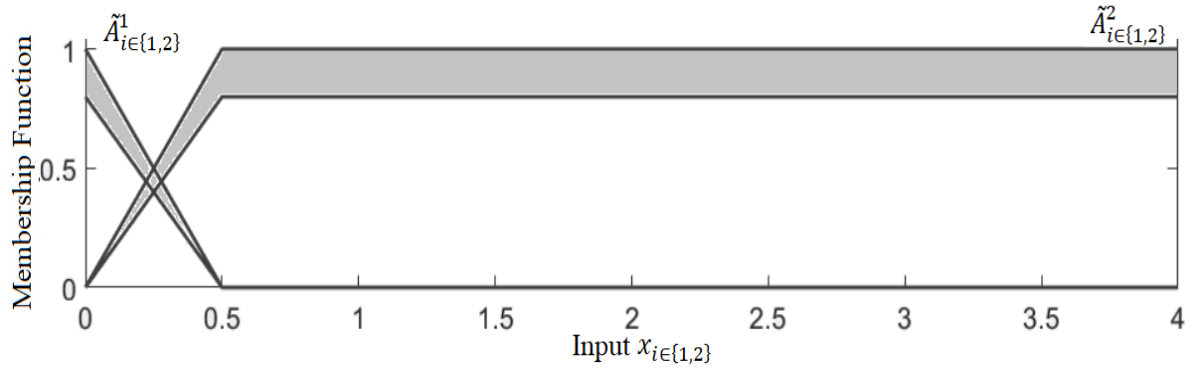


Figure 7.4: IT2 Trapezoidal FMF designs Case (A-4) for $\bar{d}_i^1 = 0.5$

Table 7.1: Trapezoidal FMFs parameters configuration for inputs $x_{i \in \{1,2\}}$

Experiment	Parameters configuration of the IT2 Trapezoidal FMFs $\tilde{A}_{i \in \{1,2\}}^1 = \mu_{i \in \{1,2\}}^1(x_i: \underline{a}_i^1, \bar{a}_i^1, \underline{b}_i^1, \bar{b}_i^1, \underline{c}_i^1, \bar{c}_i^1, \underline{d}_i^1, \bar{d}_i^1, \underline{h}^1, 1)$ $\tilde{A}_{i \in \{1,2\}}^2 = \mu_{i \in \{1,2\}}^2(x_i: \underline{a}_i^2, \bar{a}_i^2, \underline{b}_i^2, \bar{b}_i^2, \underline{c}_i^2, \bar{c}_i^2, \underline{d}_i^2, \bar{d}_i^2, \underline{h}^2, 1)$
Case (A-1) $\bar{d}_i^1 = 4$	$\tilde{A}_{i \in \{1,2\}}^1 = \mu_{i \in \{1,2\}}^1(x_i: 0, 0, 0, 0, 0, 0, 4, 4, 0.8, 1)$ $\tilde{A}_{i \in \{1,2\}}^2 = \mu_{i \in \{1,2\}}^2(x_i: 0, 0, 4, 4, 4, 4, 4, 4, 0.8, 1)$
Case (A-2) $\bar{d}_i^1 = 2$	$\tilde{A}_{i \in \{1,2\}}^1 = \mu_{i \in \{1,2\}}^1(x_i: 0, 0, 0, 0, 0, 0, 2, 20.8, 1)$ $\tilde{A}_{i \in \{1,2\}}^2 = \mu_{i \in \{1,2\}}^2(x_i: 0, 0, 2, 2, 4, 4, 4, 4, 0.8, 1)$
Case (A-3) $\bar{d}_i^1 = 3$	$\tilde{A}_{i \in \{1,2\}}^1 = \mu_{i \in \{1,2\}}^1(x_i: 0, 0, 0, 0, 0, 0, 1, 1, 0.8, 1)$ $\tilde{A}_{i \in \{1,2\}}^2 = \mu_{i \in \{1,2\}}^2(x_i: 0, 0, 1, 1, 4, 4, 4, 4, 0.8, 1)$
Case (A-4) $\bar{d}_i^1 = 0.5$	$\tilde{A}_{i \in \{1,2\}}^1 = \mu_{i \in \{1,2\}}^1(x_i: 0, 0, 0, 0, 0, 0, 0.5, 0.5, 0.8, 1)$ $\tilde{A}_{i \in \{1,2\}}^2 = \mu_{i \in \{1,2\}}^2(x_i: 0, 0, 0.5, 0.5, 4, 4, 4, 4, 0.8, 1)$

The respective output surface plot for the IT2 fuzzy blending function under varying IT2 Trapezoidal FMFs parameters configurations - namely, Case (A-1), Case (A-2), Case (A-3), and Case (A-4) - are shown in Figure 7.5, Figure 7.6, Figure 7.7 and Figure 7.8, respectively.

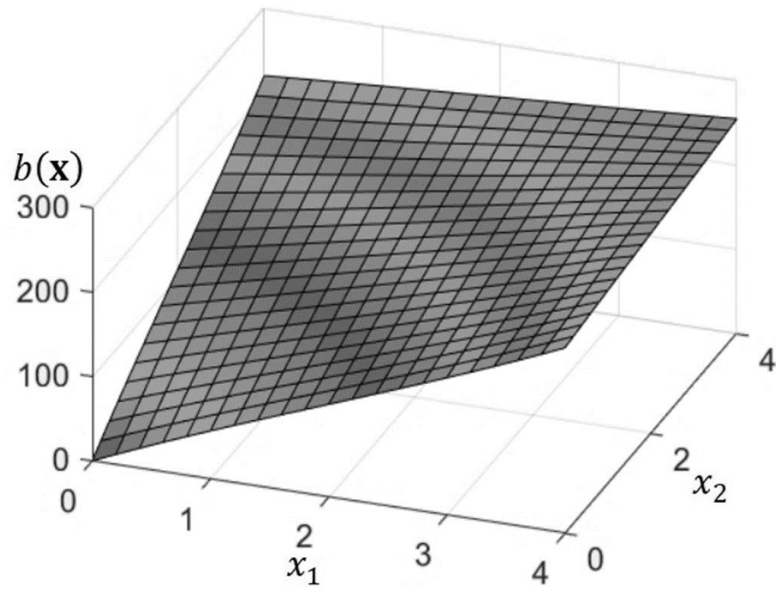


Figure 7.5: Surface plot for IT2 fuzzy blending function for Case (A-1) at $\bar{d}_i^1 = 4$

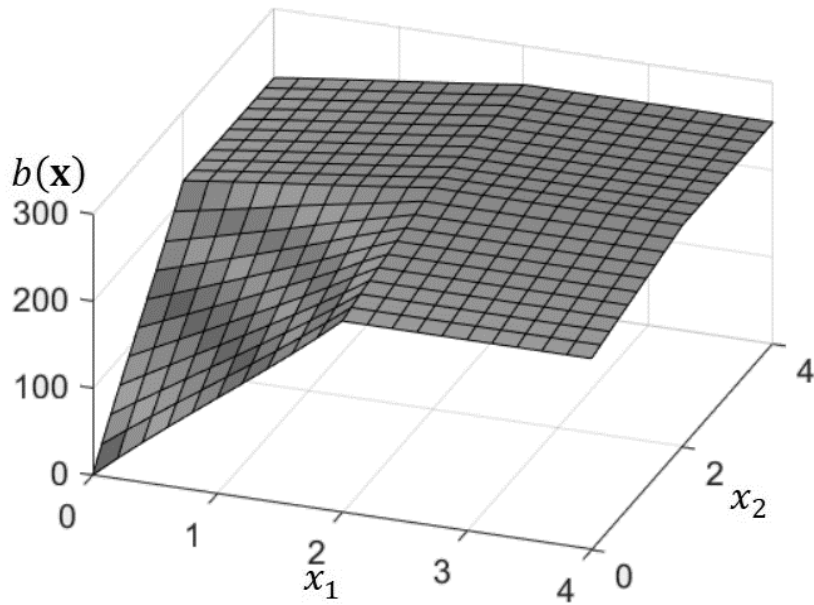


Figure 7.6: Surface plot for IT2 fuzzy blending function for Case (A-2) at $\bar{d}_i^1 = 2$

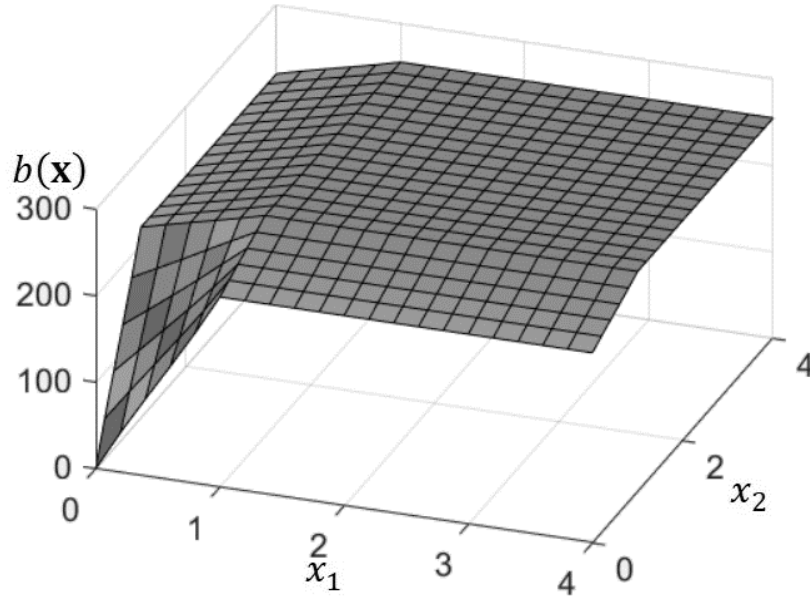


Figure 7.7: Surface plot for IT2 fuzzy blending function for Case (A-3) at $\bar{d}_t^1 = 1$

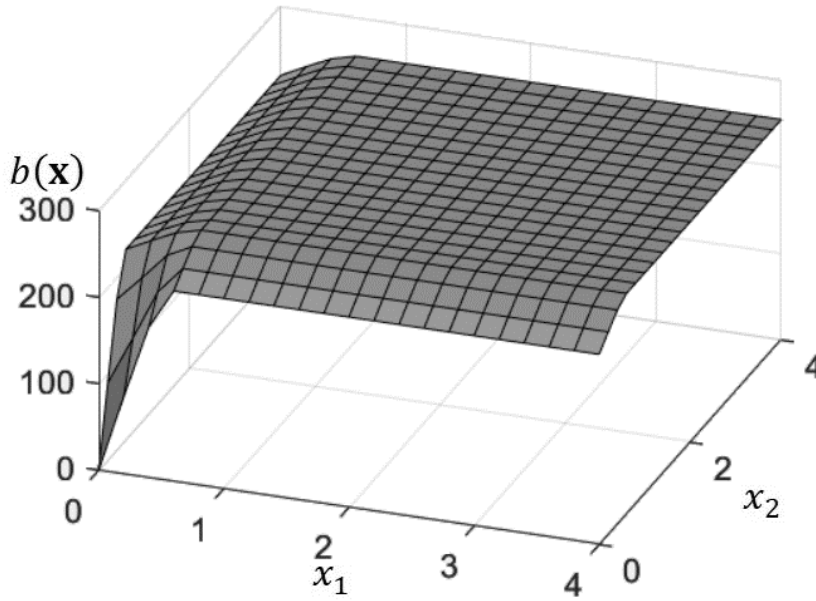


Figure 7.8: Surface plot for IT2 fuzzy blending function for Case (A-4) at $\bar{d}_t^1 = 0.5$

As in Figure 7.5 to Figure 7.8, $b(\mathbf{x})$ exhibits monotone output w.r.t the inputs $x_{i \in \{1,2\}}$ with profile view in analogy to a waterfall blending function (as in Section 5.5.3(A)). Surface plot output shown that $b(\mathbf{x})$ for Case (A-4) as shown in Figure 7.8 has the sharpest change at concave or convex region near origin in feature space $\{x_1, x_2\} \in \Re^1 \times \Re^2$, follows with Case (A-3) in Figure 7.7, Case (A-2) in Figure 7.6 and Case (A-1) in Figure 7.5. The results

of the monotonicity tests performed on the IT2 fuzzy blending function $b(\mathbf{x})$ for experiment Cases (A-1) through (A-4) are consolidated in Table 7.2.

Table 7.2: Monotonicity test for $b(\mathbf{x})$ with IT2 Trapezoidal FMFs

Experiment	Input x_i	Number of monotone compare-able pairs	Monotone_Index (x_i)
Case (A-1) $\bar{d}_i^1 = 4$	x_1	40200	$\frac{40200}{40200} = 1$
	x_2	40200	$\frac{40200}{40200} = 1$
Case (A-2) $\bar{d}_i^1 = 2$	x_1	40200	$\frac{40200}{40200} = 1$
	x_2	40200	$\frac{40200}{40200} = 1$
Case (A-3) $\bar{d}_i^1 = 1$	x_1	40200	$\frac{40200}{40200} = 1$
	x_2	40200	$\frac{40200}{40200} = 1$
Case (A-4) $\bar{d}_i^1 = 0.5$	x_1	40200	$\frac{40200}{40200} = 1$
	x_2	40200	$\frac{40200}{40200} = 1$

7.4.2 Fuzzy Blending function with IT2 Gaussian FMF

In this section, an experimental design is presented involving four instances of IT2 Gaussian FMF designs, designated as Case (B-1), Case (B-2), Case (B-3) and Case (B-4). For simplicity, $\tilde{A}_1^1 = \tilde{A}_2^1$ is defined as an IT2 Gaussian FMF parameterized by, $\tilde{A}_{i \in \{1,2\}}^1 = \mu_{i \in \{1,2\}}^1(x_i; \underline{a}_i^1, \bar{a}_i^1, \underline{b}_i^1, \bar{b}_i^1, \underline{\sigma}_i^1, \bar{\sigma}_i^1, \underline{h}^1, 1)$ where $\underline{a}_i^1 = \underline{b}_i^1 = \bar{a}_i^1 = \bar{b}_i^1 = 0$. Concurrently, $\tilde{A}_{i \in \{1,2\}}^2$ is identified as an IT2 Gaussian FMF parameterized by $\tilde{A}_{i \in \{1,2\}}^2 = \mu_{i \in \{1,2\}}^2(x_i; \underline{a}_i^2, \bar{a}_i^2, \underline{b}_i^2, \bar{b}_i^2, \underline{\sigma}_i^2, \bar{\sigma}_i^2, \underline{h}^2, 1)$. Expounding upon Equation 6.20 detailed in Chapter 6, an intersection point I is computed at the membership function value of 0.5 between $\mu_{i \in \{1,2\}}^1$ and $\mu_{i \in \{1,2\}}^2$, as depicted in Equation 7.11. This intersection point I serves to

recalibrate the area under the membership functions $\mu_{i \in \{1,2\}}^1$ and $\mu_{i \in \{1,2\}}^2$. The configuration of $\tilde{A}_{i \in \{1,2\}}^1$ and $\tilde{A}_{i \in \{1,2\}}^2$ is constraint to Equation 7.11 to Equation 7.14 and adheres to the guidelines outlined in Remark 6.4 from Chapter 6. The value of $\bar{\sigma}_i^1 = \bar{\sigma}_i^2$ is determined with Equation 7.12. The centroid of the lower and upper membership function $\mu_{i \in \{1,2\}}^2$, denoted by $\underline{a}_i^2$ and $\bar{a}_i^2$, is derived using Equation 7.13. Both $\underline{h}^1$ and $\underline{h}^2$ are both set to 0.8 and values of $\underline{\sigma}_i^1$ and $\underline{\sigma}_i^2$ are obtainable through Equation 7.14.

$$I = \bar{a}_i^1 + \sqrt{-2 * (\bar{\sigma}_i^1)^2 * \ln(0.5)}, \text{ where } \bar{a}_i^1 = 0 \quad \text{Equation 7.11}$$

$$\bar{\sigma}_i^1 = \sqrt{\frac{I^2}{-2 * \ln(0.5)}} \quad \text{Equation 7.12}$$

$$\underline{a}_i^2 = \bar{a}_i^2 = 2 * I \quad \text{Equation 7.13}$$

$$\underline{\sigma}_i^1 = \underline{\sigma}_i^2 = \sqrt{\underline{h}^1 * (\bar{\sigma}_i^1)^2} \text{ where } \underline{h}^1 = 0.8 \quad \text{Equation 7.14}$$

Experiment present four instances of IT2 Gaussian FMF designs, designated as Case (B-1), Case (B-2), Case (B-3) and Case (B-4), as shown in Figure 7.9, Figure 7.10, Figure 7.11 and Figure 7.12, in order. In Case (B-1), I is fixed at to 2, as presented in Figure 7.9. In Case (B-2), I is set to 1, as illustrated in Figure 7.10. In Case (B-3), I is set to 1, as in Figure 7.11. Lastly, in Case (B-4), I is assigned a value of 0.25, as shown in Figure 7.12. The parameters configuration of the IT2 Gaussian FMFs for the inputs $x_{i \in \{1,2\}}$ is succinctly encapsulated in Table 7.3.

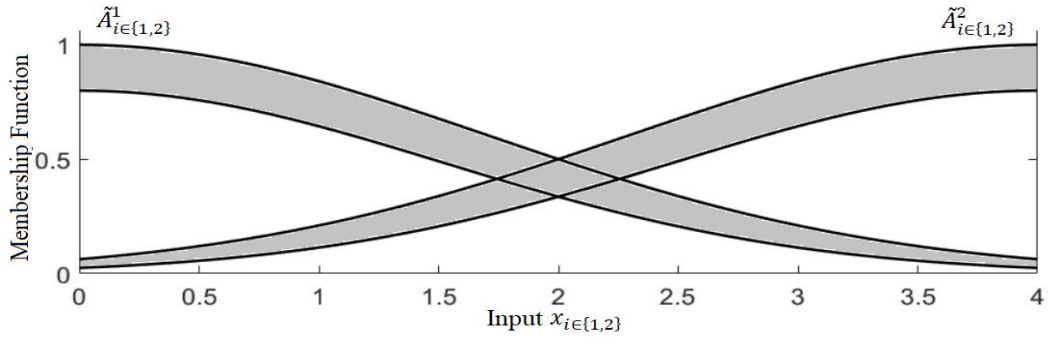


Figure 7.9: IT2 Gaussian FMF designs Case (B-1) for $I = 2$

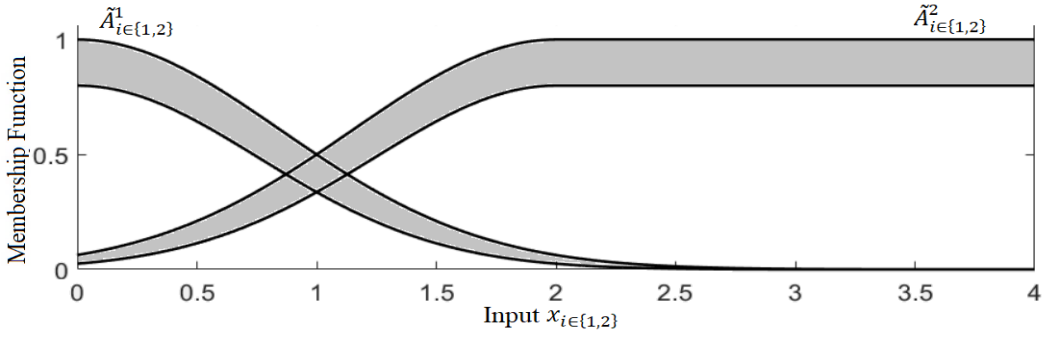


Figure 7.10: IT2 Gaussian FMF designs Case (B-2) for $I = 1$

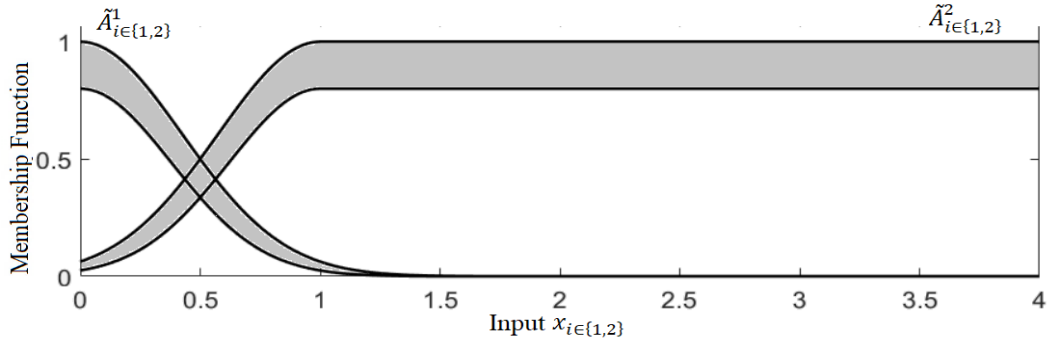


Figure 7.11: IT2 Gaussian FMF designs Case (B-3) for $I = 0.5$

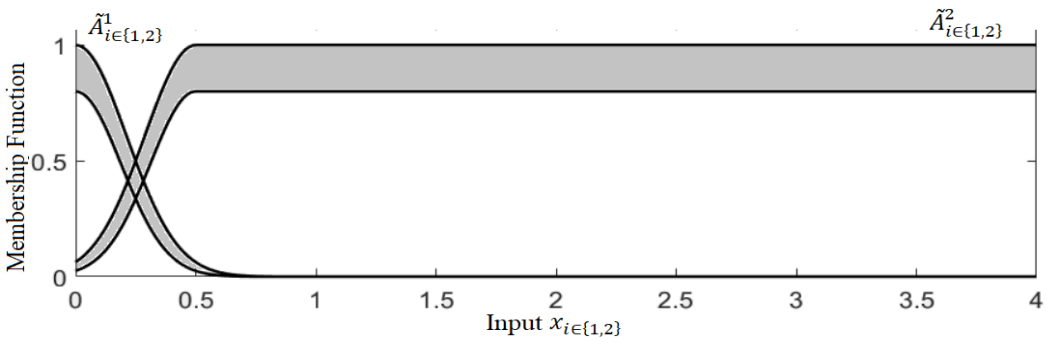


Figure 7.12: IT2 Gaussian FMF designs Case (B-4) for $I = 0.25$

Table 7.3: IT2 Gaussian FMFs parameters configuration for inputs $x_{i \in \{1,2\}}$

Experiment	Parameters configuration of the IT2 Gaussian FMFs $\tilde{A}_{i \in \{1,2\}}^1 = \mu_{i \in \{1,2\}}^1(x_i: \underline{a}_i^1, \bar{a}_i^1, \underline{b}_i^1, \bar{b}_i^1, \underline{\sigma}_i^1, \bar{\sigma}_i^1, \underline{h}^1, 1)$ $\tilde{A}_{i \in \{1,2\}}^2 = \mu_{i \in \{1,2\}}^2(x_i: \underline{a}_i^2, \bar{a}_i^2, \underline{b}_i^2, \bar{b}_i^2, \underline{\sigma}_i^2, \bar{\sigma}_i^2, \underline{h}^2, 1)$
Case (B-1) $I = 2$	• $\tilde{A}_{i \in \{1,2\}}^1 = \mu_{i \in \{1,2\}}^1(x_i: 0, 0, 0, 0, 1.52, 1.70, 0.8, 1)$ • $\tilde{A}_{i \in \{1,2\}}^2 = \mu_{i \in \{1,2\}}^2(x_i: 4, 4, 4, 4, 1.52, 1.70, 0.8, 1)$
Case (B-2) $I = 1$	• $\tilde{A}_{i \in \{1,2\}}^1 = \mu_{i \in \{1,2\}}^1(x_i: 0, 0, 0, 0, 0.76, 0.85, 0.8, 1)$ • $\tilde{A}_{i \in \{1,2\}}^2 = \mu_{i \in \{1,2\}}^2(x_i: 2, 2, 4, 4, 0.76, 0.85, 0.8, 1)$
Case (B-3) $I = 0.5$	• $\tilde{A}_{i \in \{1,2\}}^1 = \mu_{i \in \{1,2\}}^1(x_i: 0, 0, 0, 0, 0.38, 0.42, 0.8, 1)$ • $\tilde{A}_{i \in \{1,2\}}^2 = \mu_{i \in \{1,2\}}^2(x_i: 1, 1, 4, 4, 0.38, 0.42, 0.8, 1)$
Case (B-4) $I = 0.25$	• $\tilde{A}_{i \in \{1,2\}}^1 = \mu_{i \in \{1,2\}}^1(x_i: 0, 0, 0, 0, 0.19, 0.21, 0.8, 1)$ • $\tilde{A}_{i \in \{1,2\}}^2 = \mu_{i \in \{1,2\}}^2(x_i: 0.5, 0.5, 4, 4, 0.19, 0.21, 0.8, 1)$

The respective output surface plot for the IT2 fuzzy blending function under varying IT2 Gaussian FMFs parameters configurations - namely, Case (B-1), Case (B-2), Case (B-3), and Case (B-4) are shown in Figure 7.13, Figure 7.14, Figure 7.15 and Figure 7.16, respectively.

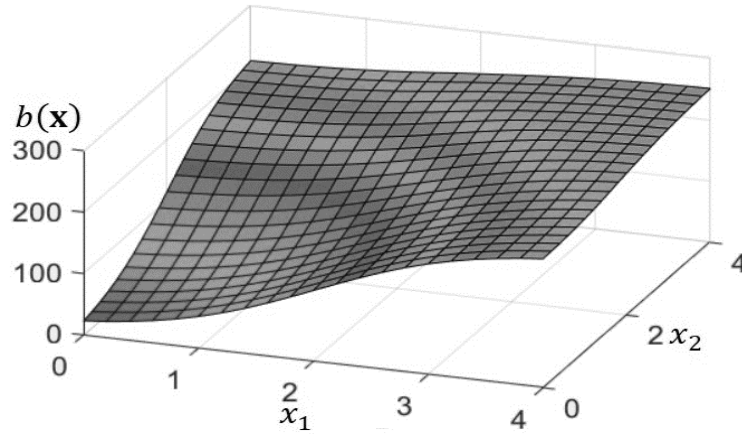


Figure 7.13: Surface plot for IT2 fuzzy blending function for Case (B-1) at $I = 2$

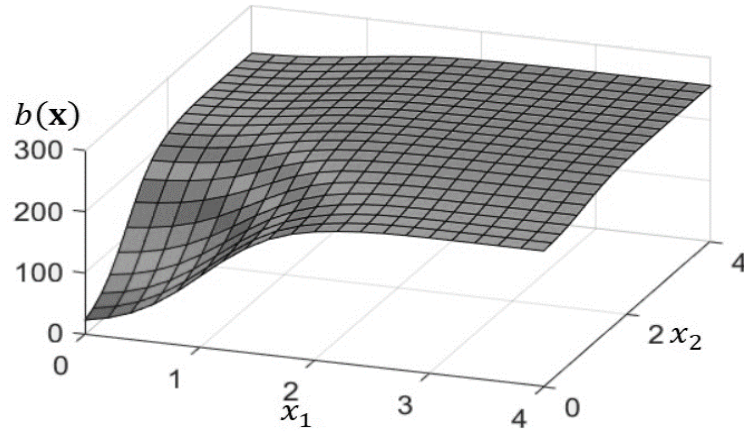


Figure 7.14: Surface plot for IT2 fuzzy blending function for Case (B-2) at $I = 1$

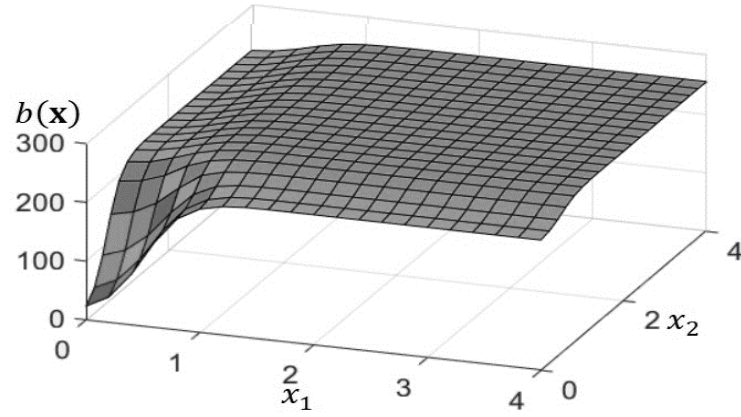


Figure 7.15: Surface plot for IT2 fuzzy blending function for Case (B-3) at $I = 0.5$

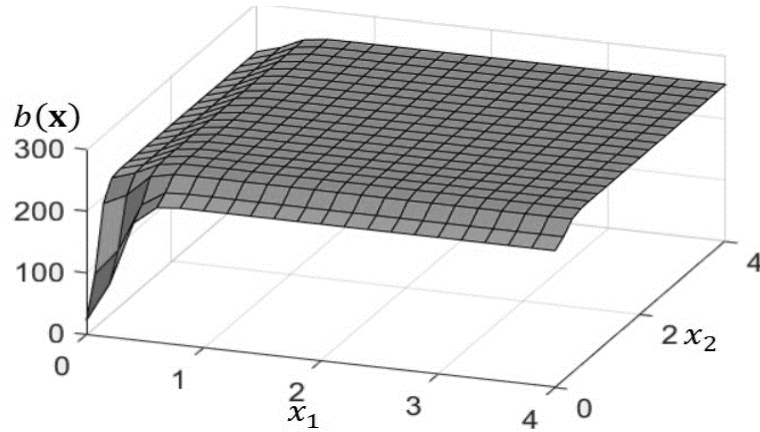


Figure 7.16: Surface plot for IT2 fuzzy blending function for Case (B-4) at $I = 0.25$

Once again, as visualized in Figure 7.13 to Figure 7.16, the function $b(\mathbf{x})$ demonstrates a monotonic output relative to the inputs $x_{i \in \{1,2\}}$, projecting a visual

resemblance to a waterfall blending function as explicated in Section 5.5.3(A). The surface plot for Case (B-4) as depicted in Figure 7.16 exhibits sharpest change at concave or convex region near origin in feature space $\{x_1, x_2\} \in \mathfrak{R}^1 \times \mathfrak{R}^2$, followed by Cases (B-3), (B-2), and (B-1) in Figure 7.15, Figure 7.14, and Figure 7.13, respectively. The results of the monotonicity tests performed on the IT2 fuzzy blending function $b(\mathbf{x})$ for experiment Cases (B-1) through (B-4) are consolidated in Table 7.4.

Table 7.4: Monotonicity test for $b(\mathbf{x})$ with IT2 Gaussian FMFs

Experiment	Input x_i	Number of monotone compare-able pairs	Monotone_Index (x_i)
Case (B-1) $I = 2$	x_1	40200	$\frac{40200}{40200} = 1$
	x_2	40200	$\frac{40200}{40200} = 1$
Case (B-2) $I = 1$	x_1	40200	$\frac{40200}{40200} = 1$
	x_2	40200	$\frac{40200}{40200} = 1$
Case (B-3) $I = 0.5$	x_1	40200	$\frac{40200}{40200} = 1$
	x_2	40200	$\frac{40200}{40200} = 1$
Case (B-4) $I = 0.25$	x_1	40200	$\frac{40200}{40200} = 1$
	x_2	40200	$\frac{40200}{40200} = 1$

7.5 Application Edge Detection Framework on Digitized Mammogram

This section leverages the experimental raw image inputs, denoted P_{IJ} , and augments their contrast via histogram equalization (see Section 5.3.1), yielding conditioned output images denoted $\mathbf{C}_{IJ}$. These images are derived from digitized mammograms - specifically, "mdb107", "mdb108", "mdb279", and "mdb280", as displayed in Figures 5.21, 5.22, 5.23,

and 5.24 of Chapter 5, respectively. These conditioned images serve as the subjects for the experimental study in this section.

The investigation employs both the Sobel operator and the Prewitt operator to generate the feature-extracted image, denoted $\mathbf{F}_{IJ}$. The image histogram analysis algorithm from Section 5.4.1 is utilized for interpreting $\mathbf{F}_{IJ}$. Note that this image histogram analysis algorithm necessitates $\mathbf{F}_{IJ} = [F_{IJ}^1, F_{IJ}^2]$ to be converted into a grayscale spectrum $Q \in [0, 255]$. Recall that $q_{\mathcal{T}}$ is the greatest grayscale intensity corresponding to the first aggregated pixel count at a set threshold approximating 98%. The outcomes for $q_{\mathcal{T}}$ of $\mathbf{F}_{IJ}$ for images "mdb107", "mdb108", "mdb279", and "mdb280" are collated in Table 7.5. As in detailed in Table 7.5, $\mathbf{F}_{IJ}$ for images "mdb107" and "mdb279" is processed with Sobel operator, while "mdb108" and "mdb280" is processed with Prewitt operator,

Table 7.5: $q_{\mathcal{T}}$ of $\mathbf{F}_{IJ}$ at $\mathcal{T} = 0.98$ for images "mdb107", "mdb108", "mdb279" and "mdb280"

Image	Extracted Feature Image	$\mathcal{T}_p$ at $\mathcal{T} = 0.98$	$q_{\mathcal{T}}$ at $\mathcal{T} = 0.98$
$\mathbf{F}_{IJ}$ processed Sobel operator			
"mdb107"	F_{IJ}^1	98.3383%	8
	F_{IJ}^2	98.1891%	7
"mdb279"	F_{IJ}^1	98.6927%	28
	F_{IJ}^2	98.0006%	46
$\mathbf{F}_{IJ}$ processed Prewitt operator			
"mdb108"	F_{IJ}^1	98.2201%	9
	F_{IJ}^2	98.0324%	9
"mdb280"	F_{IJ}^1	98.1120%	23
	F_{IJ}^2	98.0524%	33

7.5.1 IT2 Fuzzy Blending Function with IT2 Trapezoidal FMF

In this section, the IT2 fuzzy blending function $b(\mathbf{x})$ is applied using four different IT2 Trapezoidal FMF designs, specifically Case (A-1), Case (A-2), Case (A-3), and Case (A-4) as discussed in Section 7.4.1. The edge images E_{IJ} processed by the $b(\mathbf{x})$ for these cases, corresponding to images "mdb107" "mdb108" "mdb279" and "mdb280" are illustrated in Figures 7.17 to Figures 7.20, respectively.

Reducing the $\bar{d}_i^1$ value from 4 to 0.5 significantly improves edge detail in the images processed by $b(\mathbf{x})$. At $\bar{d}_i^1 = 4$, the edges are less distinct and more blurred. As $\bar{d}_i^1$ decreases to 2 and 1, edge details become clearer and more pronounced. The greatest enhancement occurs at $\bar{d}_i^1 = 0.5$, where transitions are sharpest and edge features are most distinct, providing a detailed and clear representation of the edges. This reduction effectively sharpens the transition areas in the feature space $\mathbf{x} \in \mathbb{R}^1 \times \mathbb{R}^2$, as observed in the surface plots of the $b(\mathbf{x})$ presented in Figures 7.5 to Figures 7.8 in section in Section 7.4.1. This adjustment sharpens the function's response to input variations, optimizing edge detection. Figures 7.17 to 7.20 illustrate these improvements clearly, showing the processed edge images for the respective cases. This analysis underscores the importance of selecting appropriate parameter settings in fuzzy blending functions to optimize edge detection performance.

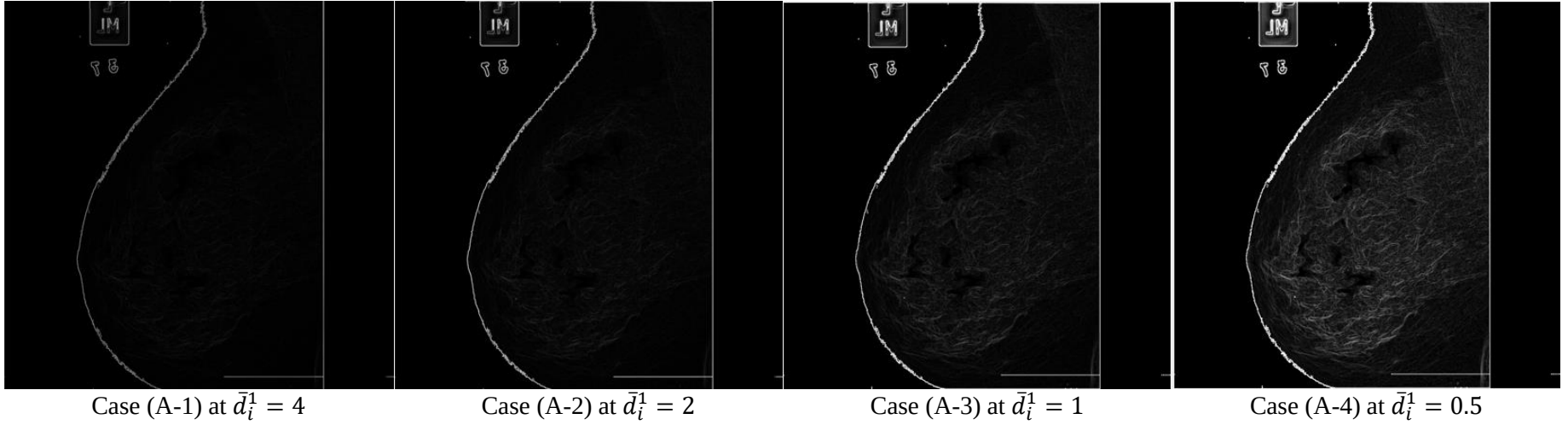


Figure 7.17: E_{IJ} for image “mdb107” processed by IT2 Fuzzy Blending Function with IT2 Trapezoidal FMFs Design

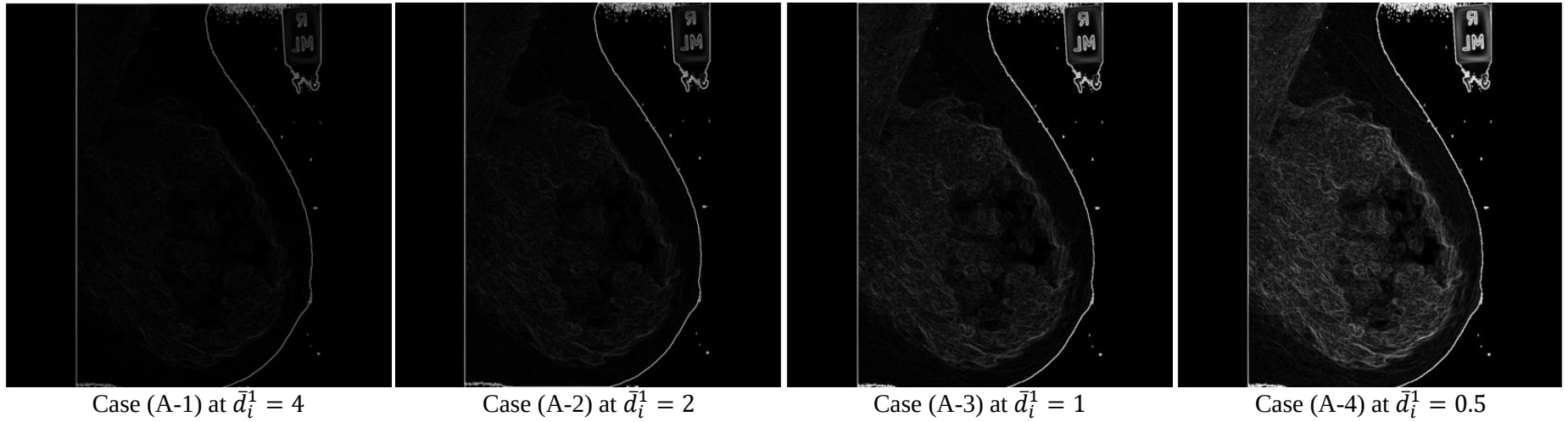


Figure 7.18: E_{IJ} for image “mdb108” processed by IT2 Fuzzy Blending Function with IT2 Trapezoidal FMFs Design

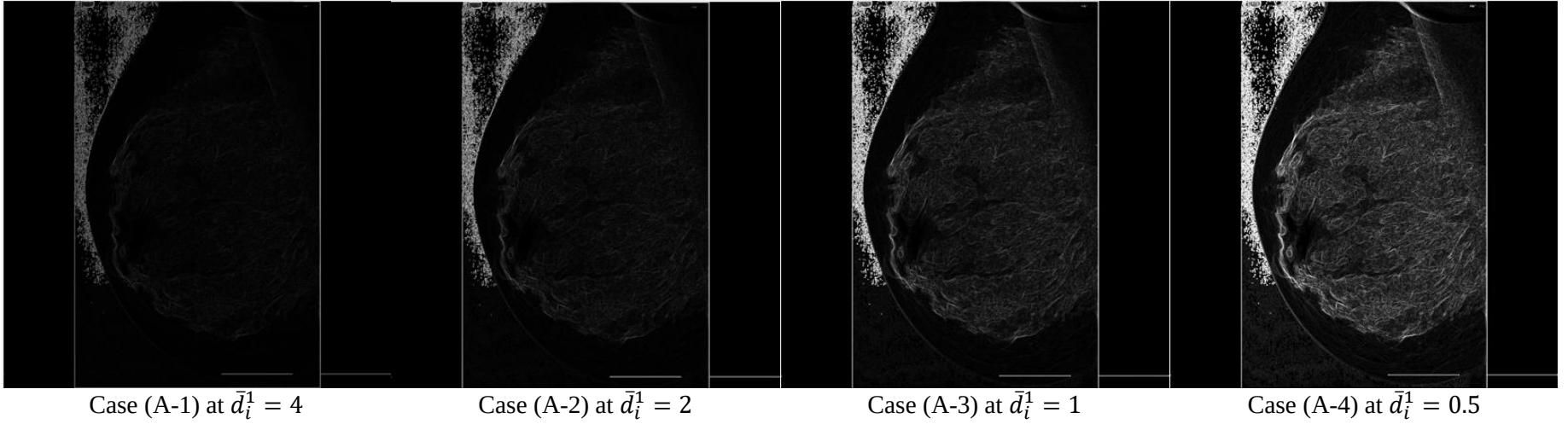


Figure 7.19: E_{IJ} for image “mdb279” processed by IT2 Fuzzy Blending Function with IT2 Trapezoidal FMFs Design

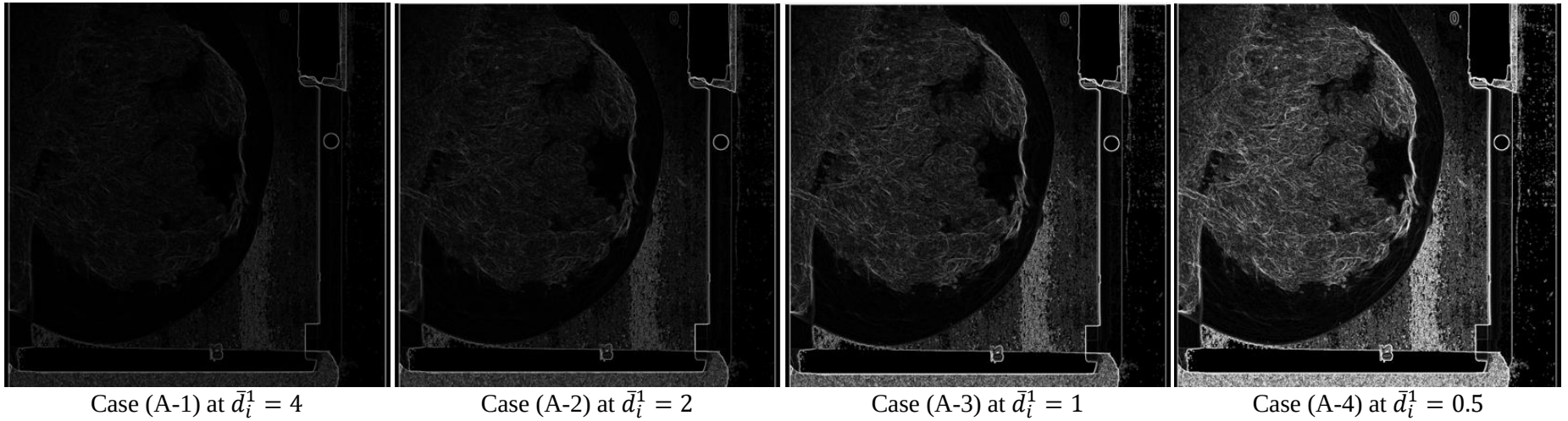
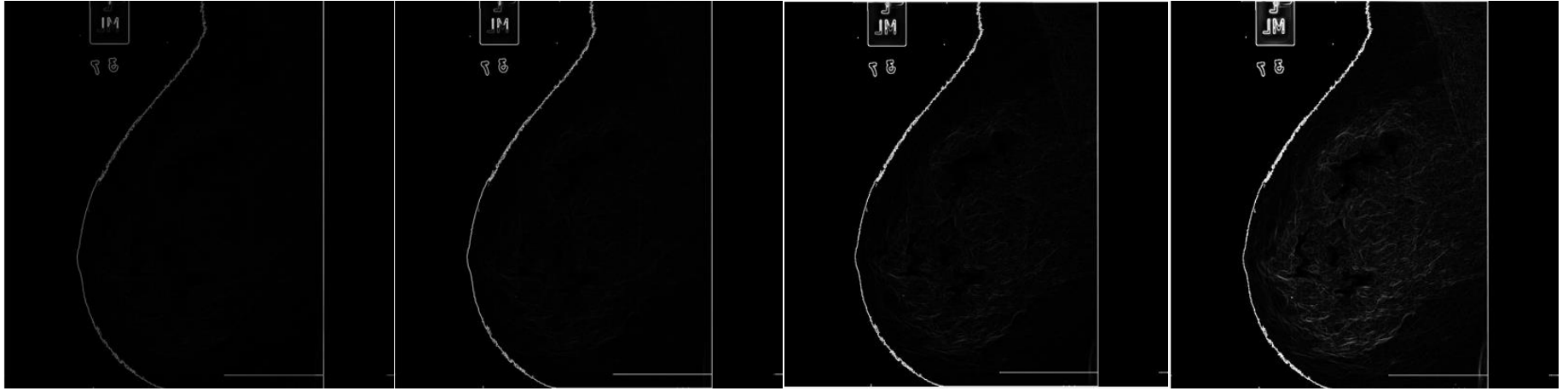


Figure 7.20: E_{IJ} for image “mdb280” processed by IT2 Fuzzy Blending Function with IT2 Trapezoidal FMFs Design

7.5.2 IT2 Fuzzy Blending Function with IT2 Gaussian FMF

In this section, the IT2 fuzzy blending function $b(\mathbf{x})$ is applied using four different IT2 Gaussian FMF designs, specifically Case (B-1), Case (B-2), Case (B-3), and Case (B-4) as discussed in Section 7.4.2. The edge images E_{IJ} processed by $b(\mathbf{x})$ for these cases, corresponding to images "mdb107", "mdb108", "mdb279", and "mdb280", are illustrated in Figures 7.21 to 7.24, respectively.

Reducing the I value from 2 to 0.25 significantly improves edge detail in the images processed by $b(\mathbf{x})$. At $I = 2$, the edges are less distinct and more blurred. As I decreases to 1 and 0.5, edge details become clearer and more pronounced. The greatest enhancement occurs at $I = 0.25$, where transitions are sharpest and edge features are most distinct, providing a detailed and clear representation of the edges. This reduction effectively sharpens the transition areas in the feature space $\mathbf{x} \in \mathbb{R}^1 \times \mathbb{R}^2$ as observed in the surface plots of $b(\mathbf{x})$ presented in Figures 7.13 to 7.16 in Section 7.4.2. This adjustment sharpens the function's response to input variations, optimizing edge detection. Figures 7.21 to 7.24 illustrate these improvements clearly, showing the processed edge images for the respective cases. This analysis underscores the importance of selecting appropriate parameter settings in fuzzy blending functions to optimize edge detection performance.



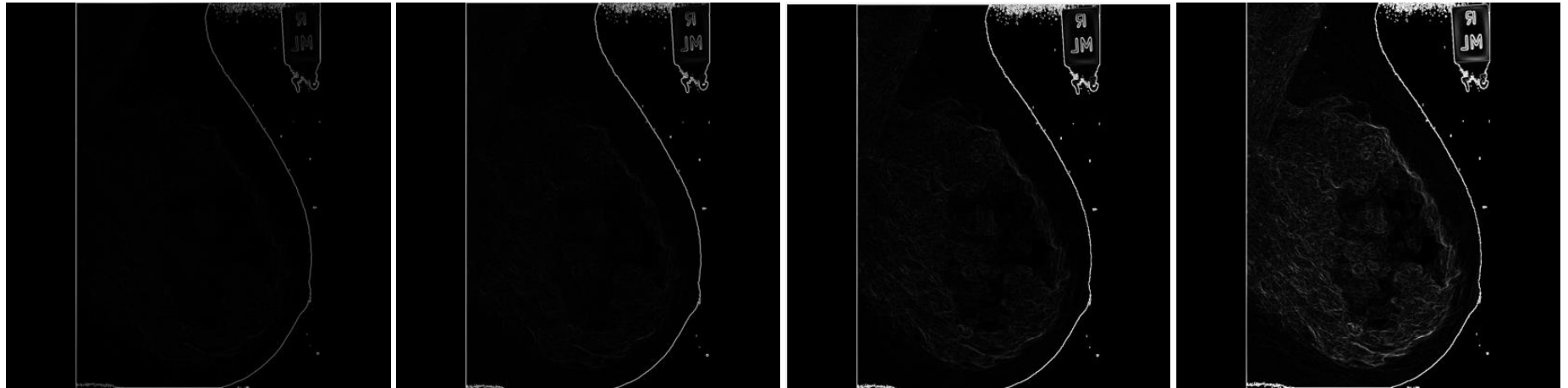
Case (B-1) at $I = 2$

Case (B-2) at $I = 1$

Case (B-3) at $I = 0.5$

Case (B-4) at $I = 0.25$

Figure 7.21: E_{IJ} for image “mdb107” processed by IT2 Fuzzy Blending Function with IT2 Gaussian FMFs Design



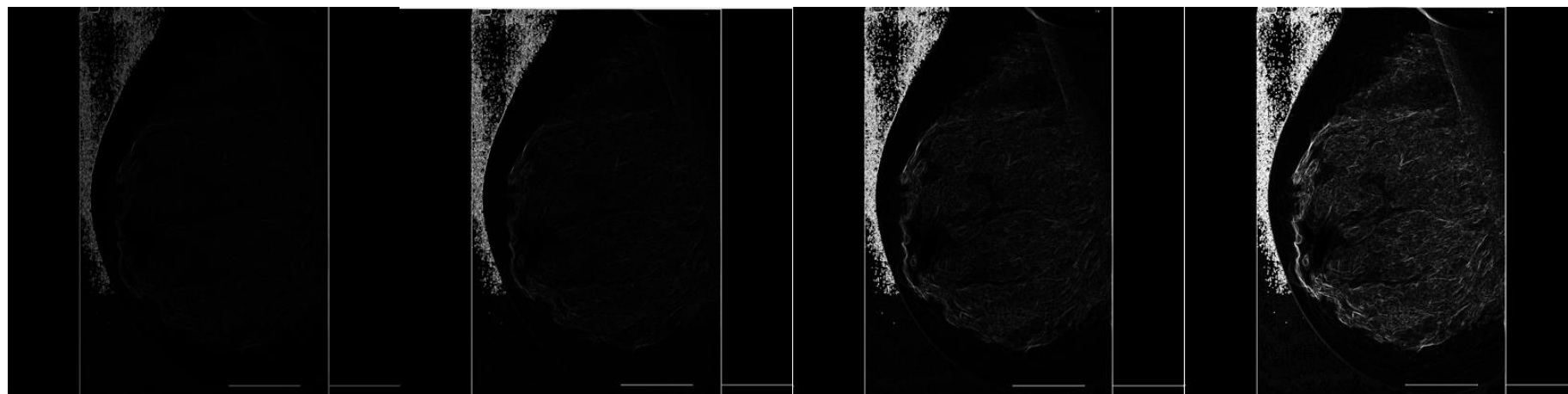
Case (B-1) at $I = 2$

Case (B-2) at $I = 1$

Case (B-3) at $I = 0.5$

Case (B-4) at $I = 0.25$

Figure 7.22: E_{IJ} for image “mdb108” processed by IT2 Fuzzy Blending Function with IT2 Gaussian FMFs Design



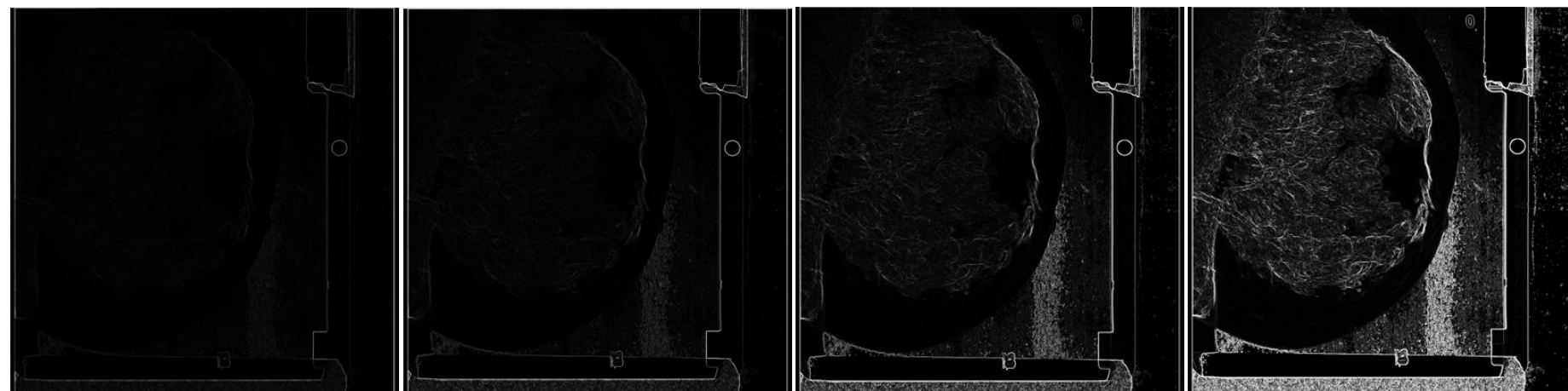
Case (B-1) at $I = 2$

Case (B-2) at $I = 1$

Case (B-3) at $I = 0.5$

Case (B-4) at $I = 0.25$

Figure 7.23: E_{IJ} for image “mdb279” processed by IT2 Fuzzy Blending Function with IT2 Gaussian FMFs Design



Case (B-1) at $I = 2$

Case (B-2) at $I = 1$

Case (B-3) at $I = 0.5$

Case (B-4) at $I = 0.25$

Figure 7.24: E_{IJ} for image “mdb280” processed by IT2 Fuzzy Blending Function with IT2 Gaussian FMFs Design

7.6 Summary

In this chapter, the application of the fuzzy blending function within the edge detection framework (Bezdek et al., 1998) is expanded to include the MISO monotone IT2 FIS Model, referred to as the IT2 Fuzzy Blending function, as detailed in Section 7.3. Section 7.4 presents two variations of the IT2 FMF Design i.e., IT2 Trapezoidal FMF and IT2 Gaussian FMF. Section 7.5 describes an experiment using the IT2 Fuzzy Blending function with digitized mammograms from the Mammographic Image Analysis Society's (MIAS) database, transforming these mammograms into edge images. This chapter contributes to achieving RO5, which focuses on applying the parametric conditions of monotone IT2 FIS to solve image processing problems, specifically enhancing edge detection. The practical application and experiments demonstrate the effectiveness of these parametric conditions in improving edge detection in mammograms. In the next chapter, conclusion remarks are presented.

CHAPTER 8

CONCLUSIONS

8.1 Conclusions and Contributions

This thesis reviews mathematical prerequisites for monotonicity in T1 and IT2 FIS, drawing from key scholars in the field. Theoretical contributions from Won et al. (2002), Tay and Lim (2011a), and Won and Karray (2014) frame the understanding of T1 FIS models, while research by Li et al. (2014) informs the study of IT2 FIS models. Together, these works establish the theoretical groundwork for ensuring FIS model monotonicity. The research then extends its purview to explore the practical applications of these conditions in two distinct applications. Notably, the first application of thesis contribute an innovative Transmission Cause and Effect Analysis (TCEA) procedure is introduced to combat COVID-19, utilizing a FIS-based risk model. The second application uses monotone T1 TSK and IT2 FIS in image processing, specifically for edge detection, transforming digitized mammograms into edge images. The main research findings and contributions of this thesis are summarized as follows.

Chapter 3 significantly deepens the understanding of monotonicity, monotonic functions, and FIS, particularly monotone FIS models. The chapter outlines sufficient conditions for monotone T1 SISO and MISO TSK FIS models, drawing upon the theories proposed by scholars like Won et al. (2002), Tay and Lim (2011a), and Won and Karray (2014), encapsulated in Theorem 3.1 and Theorem 3.2. The chapter also illustrates the design of T1 Trapezoidal and Gaussian FMFs under these theorems and formulates consequent functions for fuzzy rules, contributing to the achievement of RO1, analyzing parametric

conditions required for TSK FIS to exhibit monotonicity, focusing on functional consequents. Specifically:

1. Chapter 3 outlines sufficient conditions for monotone T1 SISO and MISO TSK FIS models, drawing upon the theories proposed by eminent scholars like Won et al. (2002), Tay and Lim (2011a), and Won and Karray (2014). These conditions are simplified and encapsulated in Theorem 3.1 and Theorem 3.2.
2. The chapter illustrates the design of Type-1 Trapezoidal and Gaussian Fuzzy FMF under the guidance of Theorem 3.1.
3. Lastly, the theorems 3.2 are employed to formulate consequent functions of fuzzy rules for monotone TSK FIS, which include both first and second-order polynomial functions.

Chapter 4 applies these parametric conditions to a T1 TSK FIS-based RPN model for COVID-19 prevention, effectively illustrating the application of mathematical conditions governed by Theorem 3.1 and 3.2. It introduces the TCEA methodology, which incorporates these conditions into a novel, systematic approach for managing transmission risks, thus fulfilling RO2, applying the parametric conditions of monotone T1 TSK FIS in a systematic approach for COVID-19 prevention. The main contributions of chapter are as follows:

1. Development of the Transmission Cause and Effect Analysis (TCEA) methodology, which uniquely incorporates the parametric conditions ensuring monotonicity in T1 TSK FIS-based RPN models. This innovative methodology offers a systematic approach for managing transmission risks associated with infectious diseases, specifically COVID-19.
2. The chapter present a detailed framework of the TCEA procedure, including a causal map, severity, occurrence, and detection ratings, and their respective scales. The chapter

thoroughly explains the design elements of the monotone FIS-based RPN model, which forms a critical part of the TCEA approach.

3. Application of the TCEA methodology in a real-world scenario - a case study centered on COVID-19 management in an office environment. The case study exemplifies the efficacy of the TCEA approach in understanding and analysing transmission risks associated with various transmission potentials (TPs), transmission causes (TCs), and transmission effects (TEs).

Chapter 5 translates the theoretical concepts of monotonicity in T1 TSK FIS into actionable insights within image processing. It provides a comprehensive understanding of object edge detection and introduces an image histogram analysis algorithm for designing blending functions. This chapter formulates a T1 monotone TSK FIS blending function and applies it to digitized mammograms, demonstrating practical application in edge detection, thereby achieving RO3, applying the parametric conditions of monotone T1 TSK FIS to solve problems in image processing, focusing on edge detection. The main contributions of chapter are as follows:

1. The chapter provides a comprehensive understanding of object edge detection, focusing on the framework proposed by Bezdek et al. (1998).
2. The chapter introduces an image histogram analysis algorithm to analyse featured image with right-skewed pixel distributions, aiding the design of the blending function for edge detection.
3. By employing the simplified Theorem 3.2 and building upon the work of key scholars, the chapter formulates a T1 monotone TSK FIS blending function. This theoretical advancement integrates the concept of monotonicity into the realm of fuzzy blending functions, bridging a crucial gap in the field.

4. The chapter culminates with the application of the T1 monotone TSK FIS blending function in edge detection, specifically in analysing digitized mammograms. This application not only validates the effectiveness of the formulated blending function but also demonstrates how the theoretical principles can be practically applied to address a real-world problem in image processing.

Chapter 6 extends the understanding of monotonicity in IT2 FIS and their parametric conditions, applying them in TCEA for COVID-19 prevention. It introduces the concepts of monotone IT2 FIS and the associated parametric conditions, validates these through simulations, and applies them to real-world case studies, aligning with RO4, analyzing parametric conditions of monotone IT2 FIS and applying them systematically for COVID-19 prevention. The main contributions of the chapter are as follows:

1. The chapter introduces the concepts of monotone IT2 FIS and the associated parametric conditions, including the design of trapezoidal and Gaussian IT2 Fuzzy Membership Functions (FMFs). The theoretical foundations are drawn from Li et al. (2014) and Li et al. (2011), condensed into key remarks that govern the construction of the IT2 FIS model.
2. Building on the theoretical foundation, chapter conducts simulated experiments on SISO Monotone IT2 FIS, using both trapezoidal and Gaussian FMFs. This experimental approach showcases the effectiveness and versatility of IT2 FIS models in maintaining the monotonicity property.
3. The chapter integrates the insights from the simulations and theoretical foundations to apply Gaussian and trapezoidal IT2 FMF designs to MISO Monotone IT2 FIS models through real-world case studies. These applications extend the innovations of Chapter 4, aligning with the objective of developing new systematic approaches to combat COVID-19 using the IT2 FIS models.

Chapter 7 explores the theory and application of the parametric conditions of monotonicity in IT2 FIS within image processing. It details the integration of the MISO monotone IT2 FIS Model within the edge detection framework, introduces IT2 Trapezoidal and Gaussian FMFs, and demonstrates practical application through experiments with digitized mammograms, achieving RO5, applying the parametric conditions of monotone IT2 FIS to solve problems in image processing, focusing on edge detection. The main contributions of the chapter are as follows:

1. The chapter details the integration of the MISO monotone IT2 FIS Model within the edge detection framework, referring to this as the IT2 Fuzzy Blending function. This integration represents an extension of existing concepts to include the IT2 FIS model.
2. The IT2 Trapezoidal FMF and IT2 Gaussian FMF are introduced as two distinct designs related to the IT2 Fuzzy Blending function. These designs present additional options for the implementation of the parametric conditions of monotonicity in IT2 FIS.
3. The practical application of the IT2 Fuzzy Blending function is demonstrated through an experiment using a digitized mammogram from the Mammographic Image Analysis Society's (MIAS) database. This transformation of the mammogram into edge images provides a real-world example of applying the theoretical concepts discussed in the chapter, showcasing the effectiveness of the IT2 FIS model in enhancing edge detection and improving image feature details.

8.2 Limitations of the Thesis Research

This section discusses the limitations encountered during this research. Despite the significant contributions and findings, several constraints and challenges were identified, which could impact the generalizability and applicability of the results.

1. **Further Investigation of Monotonicity in Various FIS Models:** The research primarily focuses on T1 and IT2 FIS models. However, other types of fuzzy inference systems were not explored. This limits the understanding of how monotonicity principles can be applied across different or more complex FIS models, potentially restricting novel applications in other fields.
2. **Real-world Deployment in Infectious Disease Management:** The thesis outlines a Transmission Cause and Effect Analysis (TCEA) methodology for combating COVID-19. However, the implementation and evaluation of this system in real-world scenarios across various cultures, geographic regions, and healthcare systems were not performed. This limitation restricts the practical assessment of its efficiency, scalability, and adaptability for global infectious disease management.
3. **Expansion to Other Medical Imaging Applications:** The application of IT2 FIS in the thesis is demonstrated through mammogram edge detection. The potential extension to other medical imaging types such as MRI, CT scans, or ultrasound images was not explored, which could have provided a broader validation and application of the proposed methods.
4. **Integration with Machine Learning Techniques:** The research leverages fuzzy logic for image processing and disease prevention but does not integrate these systems with machine learning techniques. This lack of integration may limit the potential for developing more robust, adaptive, and efficient systems that could combine the interpretability of fuzzy logic with the predictive power of machine learning algorithms.

8.3 Suggestions for Future Works

Based on the limitations identified in Section 8.2, the following suggestions are proposed for future research to enhance the findings and address the challenges encountered:

1. **Exploring Monotonicity in Various FIS Models:** Future research should investigate the application of monotonicity in other types of fuzzy inference systems beyond T1 and IT2 models. Adapting the current methodologies for different or more complex FIS models could lead to novel applications in other fields and enhance the understanding of monotonicity principles across a broader spectrum of FIS models.
2. **Implementing TCEA in Real-world Scenarios:** The TCEA methodology should be implemented and evaluated in various real-world scenarios, including different cultures, geographic regions, and healthcare systems. This would assess its efficiency, scalability, and adaptability, providing essential insights for global infectious disease management.
3. **Extending IT2 FIS to Other Medical Imaging Applications:** The IT2 fuzzy blending function approach should be extended to other types of medical imaging, such as MRI, CT scans, and ultrasound images. Developing specialized algorithms for different types of medical images could result in more precise and efficient diagnostic tools, broadening the application and impact of the research findings.
4. **Integrating Fuzzy Systems with Machine Learning:** Future research should explore the integration of fuzzy systems with machine learning techniques. Combining the interpretability of fuzzy logic with the predictive power of machine learning algorithms could lead to the development of more robust, adaptive, and efficient systems. This synergy might open new avenues for research and have far-reaching applications in various domains.

By addressing these areas, future research can build on the current findings, overcome the identified limitations, and further advance the field of medical image processing and infectious disease management using fuzzy systems and machine learning techniques.

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APPENDIX A

Jong Chian Haur

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15 July 2024

Dr. Chai Kok Chin

CEO, NEUON AI Sdn Bhd
Kuching - Samarahan Expressway
94300 Kota Samarahan, Sarawak

Subject: Permission to Publish Research Data from Neuon AI Case Study

Dear Dr. Chai Kok Chin,

I hope this letter finds you well. I am writing to formally seek permission to publish the research data collected from our recent case study conducted at Neuon AI. The data and findings from this study are a crucial part of Chapter 4 of my PhD thesis, titled "Monotone Type-1 FIS-Based RPN Modelling in Transmission Cause and Effect Analysis (TCEA) for Combating COVID-19."

Overview of the Case Study: The case study was undertaken at Neuon AI to evaluate the practical application and effectiveness of the proposed Transmission Cause and Effect Analysis (TCEA) procedure in a real-world setting. The study included:

- Analyzing transmission risks and identifying potential transmission causes and effects within the company's environment.
- Implementing preventive and control measures based on a Monotone Type-1 Fuzzy Inference System (FIS) to prioritize actions aimed at mitigating COVID-19 transmission risks.

Key Findings: The research has provided significant insights into the effectiveness of the TCEA procedure and the Monotone T1-FIS-based RPN model in managing transmission risks. These findings contribute valuable knowledge to the fields of systems engineering and risk management, especially concerning infectious disease prevention.

Request for Permission: To ensure the integrity and authenticity of the research, I kindly request your permission to publish the data collected during the case study at Neuon AI. This data will be used solely for academic purposes and will be included in my thesis, which will be made publicly accessible through the university's repository.

Confidentiality and Anonymity: All data will be anonymized to protect confidentiality, and no sensitive or proprietary information will be disclosed. The publication will focus on the methodologies and overall findings without revealing specific details about Neuon AI that could compromise confidentiality.

Data Involved: The specific data involved includes:

1. **Company Office Floorplan (Figure 4.12) and Basic Company Background:**
 - Detailed floorplan of the company office.
 - Basic background information, including the number of staff and their details as provided on page 94 of the report.
2. **TCEA Spreadsheets (Tables 4.5 and 4.6):**
 - **Table 4.5:** TCEA spreadsheet for workplace, detailing the Transmission Potentials (TPs), Transmission Effects (TEs), Transmission Causes (TCs), Risk Priority Numbers (RPN), and Fuzzy Risk Priority Numbers (FRPN) for various areas within the workplace.
 - **Table 4.6:** TCEA spreadsheet for other facilities, providing similar details for additional areas such as shared lifts, main entrances, pantry, restroom, and document room.
3. **Result Analysis:**
 - **Section 4.6.2:** Analysis of activities with high occurrence ratings, identifying transmission risks related to specific high-risk activities and areas.
 - **Section 4.6.3:** Qualitative analysis of transmission risks, including the creation of a risk matrix based on severity and occurrence ratings.
 - **Section 4.6.4:** Prioritization of Transmission Potentials using RPN and FRPN scores, summarizing the most critical areas and activities requiring intervention.
4. **Discussion Contents (Section 4.7):**
 - Detailed discussion on the implications of the findings.
 - Evaluation of the usefulness of the TCEA methodology in understanding and managing transmission risks.
 - Recommendations for practical applications and future improvements in risk management strategies.

For further details, you may refer to the attached Chapter 4 of the thesis.

I believe that the publication of this research will not only contribute to the academic community but also highlight the proactive measures taken by Neuon AI in combating COVID-19, thereby enhancing the company's reputation for innovation and responsibility.

Thank you for considering my request. I am available to discuss any concerns or questions you may have regarding this matter. I look forward to your positive response.

Best regards,
Jong Chian haur
PHD Student
(Student ID: 15010109)
Faculty of Engineering (FENG)
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Authorization Statement

I, Dr. Chai Kok Chin, CEO at NEUON AI Sdn Bhd, hereby grant permission for the experimental results obtained from the case study conducted at Neuron AI Sdn Bhd to be published in Chapter 4 of Jong Chian Haur's PhD thesis (Student ID: 15010109), titled **"MONOTONE TYPE-1 FIS-BASED RPN MODELLING IN TRANSMISSION CAUSE AND EFFECT ANALYSIS (TCEA) FOR COMBATING COVID-19"**

Neuron AI Sdn Bhd has no objection to the publication of the experimental results as detailed in the attached Chapter 4 of the thesis.

Signatures of Involved Parties

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Date: 15 July 2024

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Date: 15 July 2024