



Faculty of Engineering

Monotone Fuzzy Rule Relabeling for the Zero-Order TSK Fuzzy Inference System

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**Doctor of Philosophy
2018**

DECLARATION

The thesis has not been accepted for any degree and is not concurrently submitted in candidature of any other degree.

Name:

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ACKNOWLEDGEMENT

First and foremost, I would like to express my most earnest gratitude to my supervisor, Dr. Tay Kai Meng for his guidance, inspiration, support and patience throughout this study. His advice helped me think rationally about complex problems and solve them in a correct and yet simple manner. His guidance has nurtured the seed of research in me and has motivated me a lot in completing this thesis. His contribution is noted with valued.

Special thanks go to my research mates in the fuzzy group, for sharing the literature and knowledge, offering invaluable assistance, and for their encouragement throughout the study. I also would like to thank the Ministry of Higher Education (MOHE) for their financial support.

My sincere thanks to all my friends and colleagues who never fail to help and encouraged me during my study. My grateful thanks are also extended to Kiew Chan Yong for his constant support and encouragement throughout the entire process of my PhD research journey. Last but not least, I wish my appreciation and thanks to my beloved family for their endless patience, love, motivation, understanding and encouragement along the way aspire me to venture and finally completed my research. Thank you.

ABSTRACT

A monotonically-ordered fuzzy rule base is an essential building block for a monotone fuzzy inference system (FIS). However, fuzzy rules solicited from human experts do not always constitute a monotonically-ordered fuzzy rule base, i.e., noise exists due to errors in human judgments. In this thesis, the idea of monotone fuzzy rule relabeling (MFRR) is proposed to handle the non-monotone fuzzy rules in a fuzzy rule base, for zero-order Takagi-Sugeno-Kang (TSK) FIS modelling. The aim of MFRR is to restore the monotonicity property of the fuzzy rule base in an optimal way, for practical FIS modelling scenarios. MFRR searches for a set of solutions based on the following criteria: 1) satisfying the monotonicity property as the first priority; 2) having the minimum number of fuzzy rules changed as the second priority; and 3) having the minimum loss measure as the third priority. Apart from that, a formal and systematic way is proposed to model and analyze non-monotone fuzzy rules in a fuzzy rule base. A MFRR framework is further devised as a three-stage procedure, for practical implementation. The usefulness of the proposed method is demonstrated using real world applications, and promising results were obtained.

Keywords: Fuzzy inference system, fuzzy rule base, monotone fuzzy rule relabeling, monotonicity property, zero-order TSK fuzzy inference system

Pelabelan Semula Peraturan Kabur secara berekanada untuk Sistem Inferensi Kabur TSK Tertib Sifar

ABSTRAK

Pangkalan peraturan kabur yang disusun secara berekanada merupakan blok bangunan yang penting untuk Sistem Inferensi Kabur (Fuzzy Inference System, FIS) yang berekanada. Walau bagaimanapun, peraturan kabur yang diminta daripada pakar manusia tidak selalu membentuk suatu pangkalan peraturan kabur yang berekanada, iaitu gangguan wujud kerana kesilapan dalam penaksiran manusia. Dalam tesis ini, idea pelabelan semula peraturan kabur yang berekanada (Monotone Fuzzy Rule Relabeling, MFRR) telah dicadangkan untuk mengendalikan peraturan kabur yang tidak berekanada dalam pangkalan peraturan kabur, untuk pemodelan Takagi-Sugeno-Kang (TSK) tertib sifar FIS. Matlamat MFRR adalah untuk memulihkan sifat berekanada pangkalan peraturan kabur dengan cara yang optimum, untuk senario pemodelan FIS secara praktikal. MFRR mencari satu set penyelesaian berdasarkan kriteria berikut: 1) memuaskan sifat berekanada sebagai keutamaan pertama; 2) mempunyai bilangan perubahan peraturan kabur yang minimum sebagai keutamaan kedua; dan 3) mempunyai tahap kerugian yang paling minimum sebagai keutamaan ketiga. Selain itu, cara formal dan sistematik untuk memodelkan dan menganalisis peraturan-peraturan kabur yang tidak berekanada dalam pangkalan peraturan kabur telah dicadangkan. Rangka kerja MFRR telah direka sebagai prosedur tiga peringkat, untuk pelaksanaan praktikal. Kegunaan kaedah yang dicadangkan telah ditunjukkan dengan menggunakan aplikasi dunia nyata dan hasil yang menjanjikan telah diperolehi.

Kata kunci: *Sistem inferensi kabur, pangkalan peraturan kabur, pelabelan semula peraturan kabur yang berekanada, sifat berekanada, sistem inferensi kabur TSK tertib sifar*

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LIST OF ABBREVIATIONS

ASE	Analysis of Structural Elements
CMFCA	Conventional Mamdani Fuzzy Control Algorithms
CRA	Criterion Referenced Assessment
D	Detection
DS	Detailed Sketching
FIS	Fuzzy Inference System
FMEA	Failure Mode and Effect Analysis
JPEG	Joint Photographic Experts Group
MFRR	Monotone Fuzzy Rule Relabeling
MFRR-BF	Monotone Fuzzy Rule Relabeling with Brute Force
MFRR-BFBT	Monotone Fuzzy Rule Relabeling with Brute Force with Backtracking
MFRR-H	Monotone Fuzzy Rule Relabeling with Hybrid
MFRR-NF	Monotone Fuzzy Rule Relabeling with Network Flow
MIS	Maximum Independent Set
MISO	Multi Input Single Output
MOI	Mean of Inversion
ND	Not Detection
O	Occurrence
RPN	Risk Priority Numbers
S	Severity
SKP	Structural Key Plans

SIRM	Single Input Rule Module
SISO	Single Input Single Output
TSK	Takagi-Sugeno-Kang

CHAPTER 1

INTRODUCTION

1.1 Background

Inference process plays an important role in our everyday lives (Evans, Newstead, & Byrne, 1993). Inference is a process of derivation of conclusions from given information or premises by any acceptable form of reasoning ("Inference", 2017). In general, inference is divided into two categories, i.e., deductive inference and inductive inference. In the former, conclusions are derived logically and coherently from given premises and propositions (Johnson-Laird, 2006; Rips, 1990). In the later, as opposed to deductive inference, inductive inference is a process of drawing a general conclusion from specific examples or observations (Rips, 1990; Russell & Norvig, 1995). As an attempt to emulate inference process, various inference techniques have been reported in the literature. These techniques include Bayesian inference (Box & Tiao, 1973), probabilistic inference (Pearl, 1988), automated logical inference (Harrison, 2009) and fuzzy inference (Jang, Sun, & Mizutani, 1997).

The focus of this thesis is on deductive inference, whereby a fuzzy inference systems (FIS) model is employed. FIS adopts an inference mechanism with fuzzy set theory (Zadeh, 1965) and fuzzy logic (Zadeh, 1988), to infer a conclusion. In general, an FIS model consists of three conceptual components, i.e., a fuzzy rule base which consists of a set of fuzzy IF-THEN rules, a dictionary that defines the fuzzy membership functions used in the fuzzy IF-THEN rules, and a reasoning mechanism that performs the inference procedure (Jang et al., 1997). FIS has found successful applications in a wide variety of fields, such as modelling,

decision making, classification, control, and pattern recognition (Jang et al., 1997). Two popular variants of FIS are the Mamdani (Mamdani, 1977) and Takagi-Sugeno-Kang (TSK) (Sugeno & Kang, 1988; Takagi & Sugeno, 1985) FIS models.

Worth mentioning, the importance of the monotonicity property in FIS modelling has gained much research interest in recent years (Alcala-Fdez, Alcala, Gonzalez, Nojima, & Garcia, 2017; Husek, 2016; Kouikoglou & Phillis, 2009; Li, Yi, & Zhang, 2014; Mandal & Jayaram, 2016; Seki, Ishii, & Mizumoto, 2010; Štěpnička & De Baets, 2013; Tay & Lim, 2011b; Teh, Kerk, Tay, & Lim, 2018; Van Broekhoven & De Baets, 2009; Won & Karray, 2014; Won, Park, & Lee, 2002). The key reasons that demand the monotonicity property are as follows:

(i) many real-world systems and control problems obey the monotonicity property between the input and output (Kouikoglou & Phillis, 2009; Won et al., 2002);

(ii) the validity of the FIS output needs to be ensured for undertaking comparison, selection, and decision making problems (Jee, Tay, & Lim, 2015; Kerk, Tay, & Lim, 2017; Kouikoglou & Phillis, 2009; Seki et al., 2010; Tay & Lim, 2008a);

(iii) in the case where the number of data samples is small or the fuzzy rule set is incomplete, it is important to fully exploit the available qualitative information or knowledge pertaining to the system (Jee et al., 2015; Kerk et al., 2017; Van Broekhoven & De Baets, 2009);

(iv) additional qualitative information or knowledge of the system is useful for overcoming issues related to noise and inconsistencies in the data samples, as well as the over-fitting problem (Van Broekhoven & De Baets, 2009).

It is worth noting that most of the real-world problems, but not all, obey monotonicity property between the input and output. In this thesis, only systems that exhibit monotonicity property between its input and output are taken into consideration.

1.2 Problems Statements and Motivations

To maintain the monotonicity property of a fuzzy inference system, a monotonically-ordered and complete set of fuzzy rules is necessary. Despite the popularity of the FIS model, most of the studies in the existing literature assume that fuzzy rules collected from humans are free from judgement errors. This, however, may not be the case in practice. Indeed, when fuzzy rules are gathered from humans, it is possible for the resulting fuzzy rule base to violate the monotonicity property due to judgement errors. It is a challenging task to always ensure that fuzzy rules gathered from humans (even domain experts) are free from judgement errors. In this thesis, such judgment error is defined as noise that leads to the formation of a non-monotone fuzzy rule base.

To handle the non-monotone fuzzy rules in a fuzzy rule base, the idea of monotone fuzzy rule relabeling (MFRR) is proposed in this thesis. The aim of MFRR is to restore the monotonicity property of the fuzzy rule base in an optimal way, for practical FIS modelling scenarios. The rationale of MFRR is inspired by investigations on monotone data relabeling (Daniels & Velikova, 2006; Feelders, Velikova, & Daniels, 2006; Rademaker, De Baets, & De Meyer, 2009, 2012). Indeed, different techniques such as greedy algorithms (Daniels & Velikova, 2006; Feelders et al., 2006) and network flow algorithms (Feelders et al., 2006; Rademaker et al., 2009, 2012) have been developed to facilitate monotone data relabeling.

MFRR searches for a set of solutions based on the following criteria: i) satisfying the monotonicity property as the first priority; ii) having the minimum number of fuzzy rules changed as the second priority; and iii) having the minimum loss measure as the third priority. In other words, MFRR aims to establish a relabeled fuzzy rule base that is monotone with the minimum number of fuzzy rules changed and the minimum loss. Apart from that, up to date, there is no known method yet for analyzing the non-monotone fuzzy rules in a fuzzy rule base. It is important to develop a formal and systematic way to modelling and analyzing non-monotone fuzzy rules in a fuzzy rule base, as it could give better understanding as well as to gain insight into the non-monotonicity relationship of a fuzzy rule base.

1.3 Objectives of the Research

The objectives of this research are set as follows:

- i. To design a formal and systematic way to modelling and analyzing non-monotone fuzzy rules in a fuzzy rule base.
- ii. To formulate MFRR as a systematic method to optimally relabel non-monotone fuzzy rules.
- iii. To develop a practical MFRR framework.
- iv. To demonstrate the applicability of the resulting MFRR framework to real world problems.

1.4 Scopes of the Research

This thesis focuses on the issue of optimal restoration of monotonicity property of a non-monotone fuzzy rule base. In this thesis, the *sufficient conditions* are viewed as a solution to the monotonicity property of FIS model. The zero order TSK FIS model is used.

The effectiveness of the proposed method is demonstrated using FIS-based applications to failure mode and effect analysis (FMEA) and education assessment problems.

1.5 Research Methodology

The research methodology adopted in this research is depicted in Figure 1.1. First, the background and literature related to this research are reviewed. MFRR techniques are formulated and examined with a few simulated examples. A representation of non-monotone fuzzy rules in a fuzzy rule base, using the graph theory (Gibbons, 1985) is proposed to analyzing the non-monotone relationship of a given fuzzy rule base. Network flows method for MFRR are developed. As addition, a practical MFRR framework, suitable for various real-world applications, are devised and analysed.

To illustrate the usefulness of the proposed approaches, three real world case studies are presented. Two of them are directly adopted from the literature, i.e., publicly available information pertaining to FMEA problems for sewage treatment plant (Hsieh, 2006) and hospital purchasing process (Kumru & Kumru, 2013). The remaining is an application to educational assessment in civil engineering course of Universiti Malaysia Sarawak (UNIMAS). Finally, concluding remarks and suggestions for future works are presented.

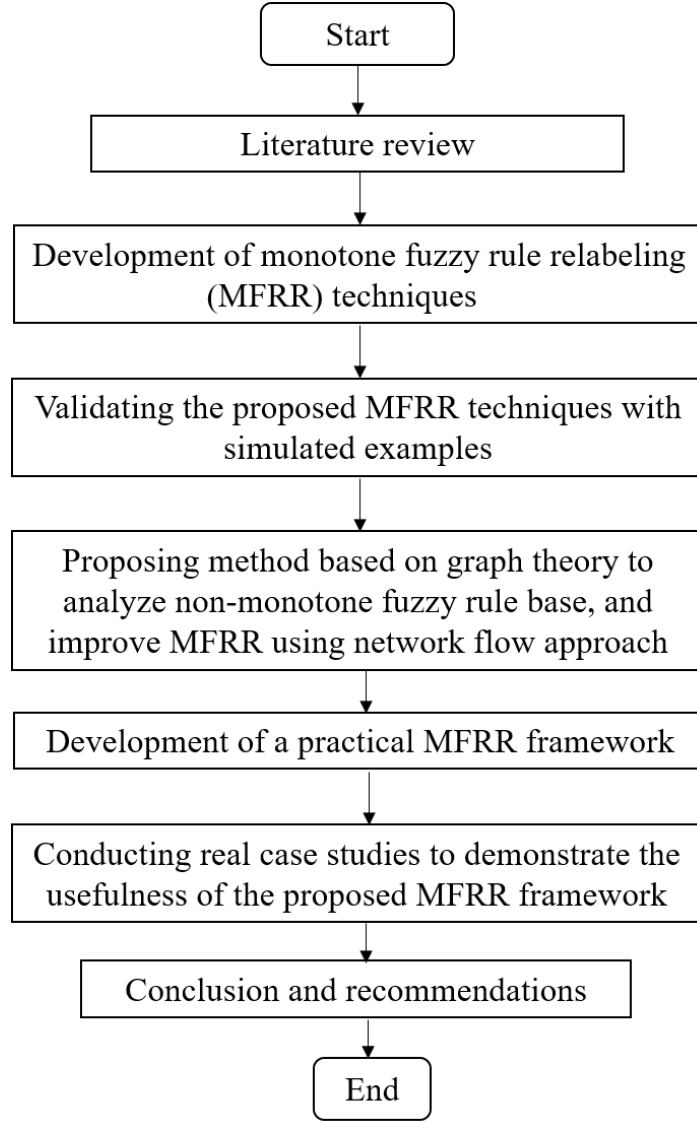


Figure 1.1: Research methodology

1.6 Organization of the Thesis

In this section, the outline of this thesis is summarized as follows.

In Chapter 1, the research background is first described. The problem statement and motivations are explained. The research objectives, methodology, and scope are also presented.

In Chapter 2, the background and literature review on fuzzy sets theory, fuzzy set theoretical operations, fuzzy IF-THEN rules, fuzzy reasoning, FIS models and graph theory are presented. A review on the monotonicity property of FIS, as well as recent advances in monotone data relabeling are also presented in this chapter.

In Chapter 3, the general idea of monotone fuzzy rule relabeling is presented. In addition, two MFRR algorithms, that are based on *brute force search* (i.e., MFRR-BF) and *brute force search with backtracking* (i.e., MFRR-BFBT), respectively, are presented, demonstrated, and analyzed.

In Chapter 4, the use of graph theory to represent the non-monotone relationship of fuzzy rules with a binary relationship is proposed. A formulation of maximum independent set problem is proposed and solved using network flow approach.

In Chapter 5, a MFRR framework for practical implementation is devised as a three-stage procedure. Several challenges emerged during the practical implementation of MFRR is discussed. A number of notions for the three-stage procedure are introduced. Furthermore, the applicability of the three-stage procedure is demonstrated with a simulated example. The results are analyzed and discussed.

In Chapter 6, the importance of MFRR methods is illustrated with an engineering related application. Specifically, the MFRR framework is applied to undertake FMEA problems. The background information for FMEA are included in this chapter too. Two publicly available information pertaining to fuzzy FMEA of a sewage treatment plant (Hsieh, 2006) and a public hospital (Kumru & Kumru, 2013) are used.

In Chapter 7, the use of MFRR as a solution to handle a non-monotone fuzzy rule base for an FIS-based Criterion Referenced Assessment (CRA) model (Tay & Lim, 2011c) is presented. The background information for FIS-based CRA model are introduced in this chapter too. Data pertaining to assessment of students' projects in a structural engineering course is reported.

Finally, concluding remarks and contributions of this research are presented in Chapter 8. Suggestions for future works are also included.

CHAPTER 2

LITERATURE REVIEW

2.1 Introduction

In this chapter, the background and literature relating to fuzzy sets theory, fuzzy set theoretical operations, fuzzy IF-THEN rules, fuzzy reasoning and fuzzy inference systems (FIS) models are reviewed. Recent work relating to monotonicity property of FIS are further presented. A review on monotone data relabeling techniques is further provided. Other related literatures, i.e., graph theory which is useful for developing monotone fuzzy rule relabeling techniques, are also presented in this chapter.

The organization of this chapter is as follows. In Section 2.2, fuzzy sets theory, including the basic definitions and theoretical fuzzy sets operations, are presented. The background of FIS models is presented in Section 2.3. A review on monotone FIS model is presented in Section 2.4. The literature review for monotone data relabeling is presented in Section 2.5. The background for the graph theory is provided in Section 2.6. Finally, concluding remarks are presented in Section 2.7.

2.2 Fuzzy Sets Theory

Fuzzy sets theory was first introduced by Zadeh in 1965, as an extension of the classical set theory (Zadeh, 1965). Let X be a space of objects and x be a generic element of X . A classical set A , is defined as a collection of elements $x \in X$ with sharp boundary, such that each x can either belong to or not belong to the set A . Thus, by defining a characteristic

function for each element x in X , a classical set A can be represented by a set of ordered pairs $(x, 0)$ or $(x, 1)$, which indicates $x \notin A$ or $x \in A$, respectively (Jang et al., 1997).

Unlike the classical set that has a sharp boundary to divide members from non-members in the group, a fuzzy set is a set without a sharp boundary that allows gradual transition from “belong to a set” to “not belong to a set” (Jang et al., 1997). A fuzzy set is characterized by a membership function that allows various degrees of membership for the elements of a given set (Zimmermann, 1996). In what follows, the basic definitions for fuzzy set and its related terminology and operations are presented.

2.2.1 Basic Definitions and Terminology

Definition 2.1. (Jang et al., 1997; Zimmermann, 1996) If X is a collection of objects denoted generically by x , then a fuzzy set A in X is defined as a set of ordered pairs:

$$A = \{(x, \mu_A(x)) \mid x \in X\} \quad (2.1)$$

where $\mu_A(x)$ is called the membership functions for the fuzzy set A . The membership function maps each element of X to a membership grade between 0 and 1. If the value of the membership function $\mu_A(x)$ is restricted to either 0 or 1, then A is reduced to a classical set and $\mu_A(x)$ is the characteristic function of A .

Definition 2.2. (Jang et al., 1997) The **core** of a fuzzy set A is the set of all points x in X such that $\mu_A(x) = 1$:

$$\text{core}(A) = \{x \mid \mu_A(x) = 1\} \quad (2.2)$$

Definition 2.3. (Jang et al., 1997) A fuzzy set A is **normal** if its core is nonempty. In other words, a point $x \in X$ such that $\mu_A(x) = 1$ is always existed.

Definition 2.4. (Jang et al., 1997) The **support** of a fuzzy set A is the set of all points x in X such that $\mu_A(x) > 0$:

$$\text{support}(A) = \{x | \mu_A(x) > 0\} \quad (2.3)$$

Definition 2.5. (Jang et al., 1997) A fuzzy set whose support is a single point in X with $\mu_A(x) = 1$, is called a **fuzzy singleton**.

For a fuzzy set, a fuzzy membership function is a mathematic function that defines how each point, x in the input space, X is mapped to a membership value between 0 and 1. The most commonly used membership functions are triangular, trapezoidal and Gaussian fuzzy membership functions. In this thesis, Gaussian membership function is chosen owing to its smoothness and concise notation (Jang et al., 1997). A Gaussian membership function (as shown in Figure 2.1) is specified by two parameters, i.e., c and σ , as defined in Eq. (2.4):

$$\mu_G(x; c, \sigma) = e^{-(x-c)^2/2\sigma^2} \quad (2.4)$$

where c represents the membership function's centre and σ determines the width of the membership function.

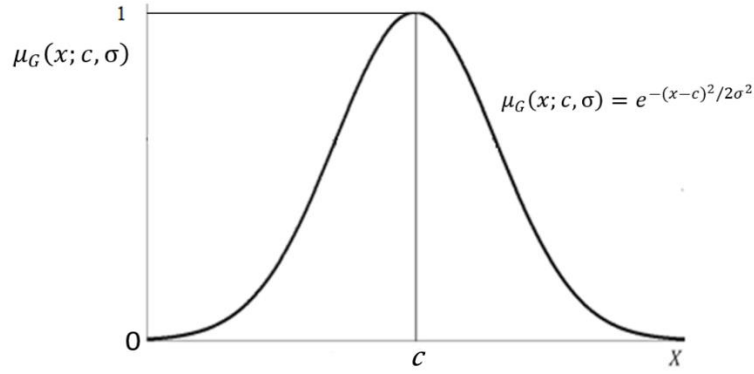


Figure 2.1: A Gaussian membership function

2.2.2 Fuzzy Sets Theoretical Operations

In classical sets, there are three most basic operations, i.e., union, intersection and complement. Corresponding to these three operations, fuzzy sets have similar operations, as defined by Zadeh in his seminal paper in 1965 (Zadeh, 1965). The definitions for these operations are defined as follows.

Definition 2.6. (Jang et al., 1997) The **union** (also known as disjunction) of two fuzzy sets A and B is a fuzzy set C , written as $C = A \cup B$ or $C = A \text{ OR } B$, whose membership function is related to those of A and B by

$$\mu_C(x) = \max(\mu_A(x), \mu_B(x)) = \mu_A(x) \vee \mu_B(x) \quad (2.5)$$

Definition 2.7. (Jang et al., 1997) The **intersection** (also known as conjunction) of two fuzzy sets A and B is a fuzzy set C , written as $C = A \cap B$ or $C = A \text{ AND } B$, whose membership function is related to those of A and B by

$$\mu_C(x) = \min(\mu_A(x), \mu_B(x)) = \mu_A(x) \wedge \mu_B(x) \quad (2.6)$$

Definition 2.8. (Jang et al., 1997) The **complement** (also known as negation) of fuzzy set A , denoted by $\bar{A}$ is defined as

$$\mu_{\bar{A}}(x) = 1 - \mu_A(x) \quad (2.7)$$

Figure 2.2 shows two fuzzy set A and B . Figures 2.3 and 2.4 illustrate the operations for union and intersection on fuzzy set A and B , respectively. Figure 2.5 illustrates the complement operations on fuzzy set A .

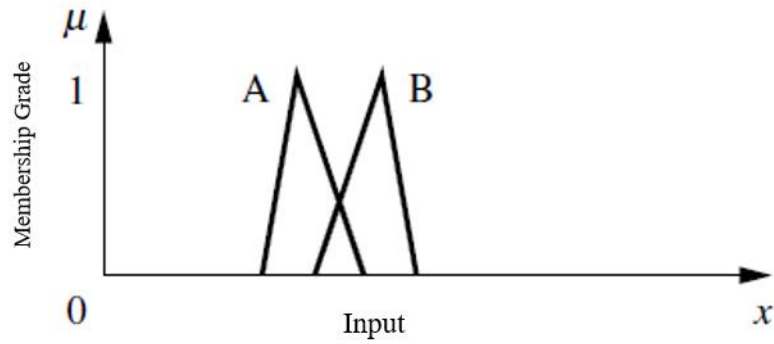


Figure 2.2: Fuzzy set A and B (Ross, 2004)

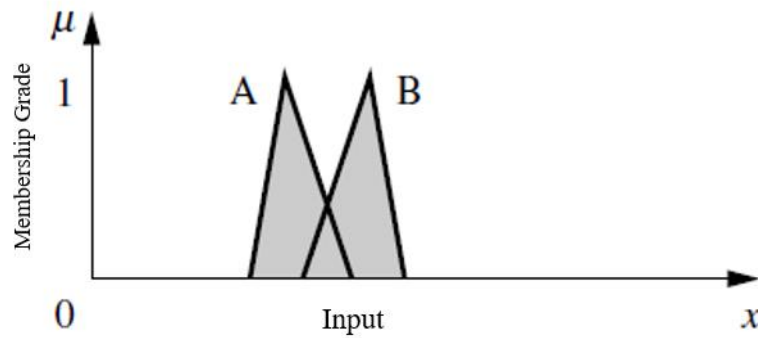


Figure 2.3: Union of fuzzy set A and B (Ross, 2004)

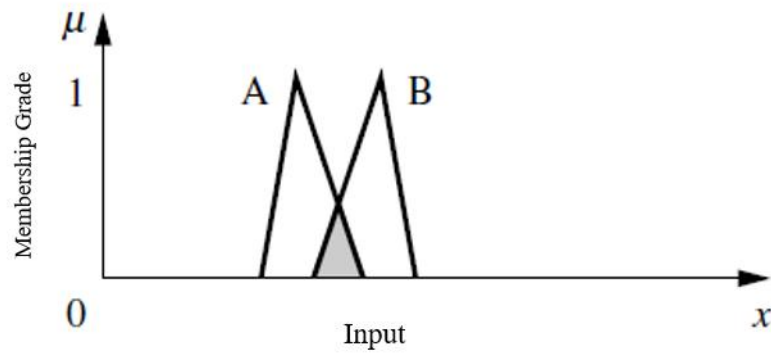


Figure 2.4: Intersection of fuzzy set A and B (Ross, 2004)

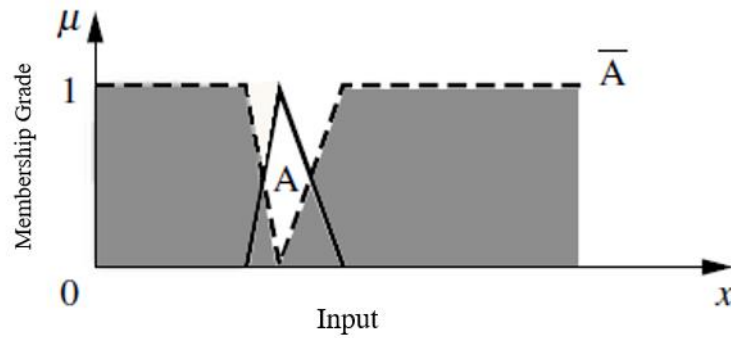


Figure 2.5: Complement of fuzzy set A (Ross, 2004)

2.3 Background on Fuzzy Inference Systems

Fuzzy inference system (FIS) (also known as *fuzzy rule-based system*, *fuzzy expert system*, *fuzzy model*, *fuzzy associative memory*, *fuzzy logic controller* and *fuzzy system*) is one of the most famous applications of fuzzy set theory. FIS implements a nonlinear mapping that derives its output based on fuzzy set theory, fuzzy IF-THEN rules and fuzzy reasoning. The mapping then provides a basis from which decisions or action can be made, and patterns can be discerned through the mapping process. In an FIS model, experts' knowledge is

represented by a knowledge base that comprising a set of fuzzy IF-THEN rules and a database that defines the membership functions used in the fuzzy rules. Thus, in this section, a review on fuzzy IF-THEN rules and fuzzy reasoning for FIS models is presented. Besides that, the types of fuzzy inference models are also described.

2.3.1 Fuzzy IF-THEN Rule

A fuzzy IF-THEN rule (also known as fuzzy production rule or simply fuzzy rule), is expressed as a logical implication in the form of IF-THEN statement. It is a basic unit for capturing knowledge that involved imprecision, vagueness and/or uncertainty in a fuzzy system. Each fuzzy IF-THEN rule comprises two parts, i.e., an antecedent (input) and a consequent (output). An example of a fuzzy rule is *IF x is A Then y is B* , where " x is A " is known as the antecedent or premise, while " y is B " is known as the consequent or conclusion, and A and B are the linguistic variables defined by fuzzy sets on universe of discourse X and Y , respectively (Jang et al., 1997). The antecedent of a fuzzy rule is a fuzzy proposition, whereas the consequent is a fuzzy set or a polynomial function. Fuzzy rules with multiple propositions can be combined or modified via the set-theoretic operations, i.e., AND, OR, and NOT.

Various methods have been proposed to partition the input space to form the antecedent, among them are grid partition, tree partition, and scatter partition (Jang et al., 1997). The grid partition is popular, and it is often chosen for designing FIS models (Jang et al., 1997). With the grid partition, a fuzzy rule with n antecedents has the following form:

$$If (x_1 is A_1) AND (x_2 is A_2) ... AND (x_n is A_n) THEN (y is B)$$

where x_i and y are the inputs and output of the FIS model, A_i and B are linguistic variables/fuzzy sets for the inputs and output, respectively.

2.3.2 Fuzzy Reasoning

Modus ponens is the basic rule of inference in classical propositional logic. The truth of a proposition B is inferred from the truth of A and the implication $A \rightarrow B$. The concept is illustrated as follows:

Premise 1 (fact):	$x \text{ is } A$
Premise 2 (rule):	$\text{If } x \text{ is } A \text{ Then } y \text{ is } B$
Consequent (conclusion):	$y \text{ is } B$

Fuzzy reasoning (also known as approximate reasoning) is an inference procedure that derives conclusion from a set of fuzzy rules. Modus ponens is employed in an approximate manner, thus it is also known as generalized modus ponens (GMP) since modus ponens is its special case. It can be written as

Premise 1 (fact):	$x \text{ is } A'$
Premise 2 (rule):	$\text{If } x \text{ is } A \text{ Then } y \text{ is } B$
Consequent (conclusion):	$y \text{ is } B'$

where A' is close to A , and B' is close to B .

2.3.3 Fuzzy Inference Systems

FISs constitutes a popular computing framework that has been successfully applied to solving different problems (Jang et al., 1997; Mendel, 1995). Figure 2.6 illustrates the general structure of an FIS model, which is composed of five functional blocks, i.e., fuzzification interface, database, rule base, decision making unit and defuzzification interface. The rule base contains a set of fuzzy IF-THEN rules, whereas the database defines the membership functions of the fuzzy sets used in the fuzzy IF-THEN rules. The fuzzification interface maps crisp input into corresponding fuzzy membership of linguistic variables, while the decision making unit performs the inference operations on the rules. The defuzzification interface is then transforms the fuzzy outputs of the inference into a crisp output.

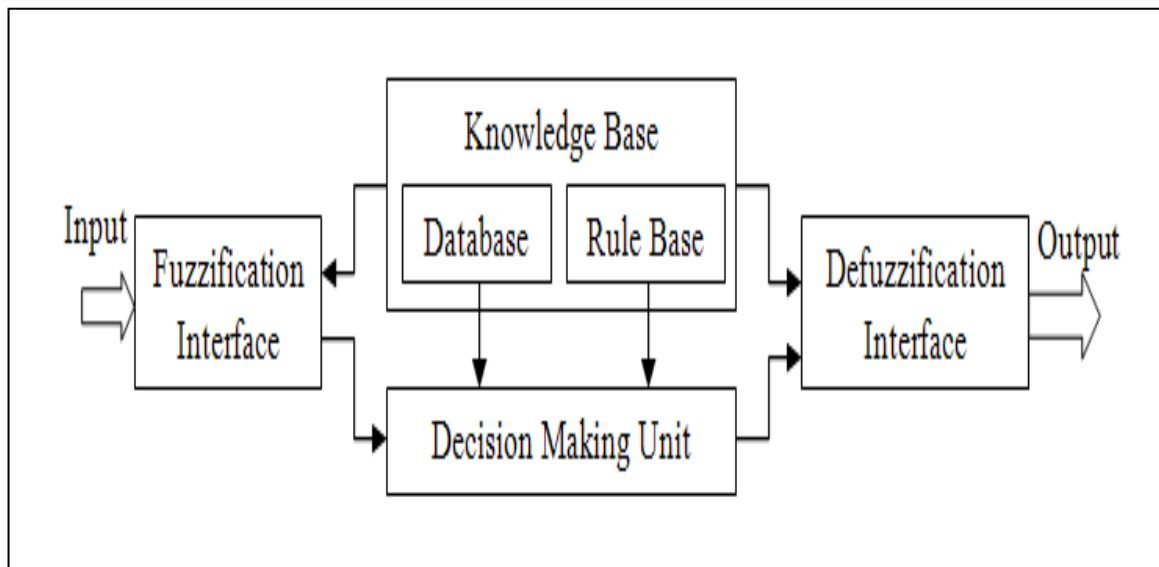


Figure 2.6: A General Fuzzy Inference System Model (Jang, 1993)

Two popular types are the Mamdani-type (Mamdani & Assilian, 1975) and Takagi-Sugeno-Kang (TSK)-type (Sugeno & Kang, 1988; Takagi & Sugeno, 1985) of FIS models.

Both FIS models are characterized by fuzzy IF-THEN rules and have the same antecedent structures (Mendel, 2001). The difference between these two models lie in the consequents structure of their fuzzy rule (Jang et al., 1997). The consequent of a fuzzy rule for Mamdani FIS is a fuzzy set, whereas TSK-FIS has a real-valued function for the consequent part of its fuzzy rules. Thus, their aggregation and defuzzification procedure are different too. Mamdani FIS uses defuzzification technique to obtain crisp output from the aggregated output fuzzy set, while TSK-FIS uses weighted average to compute the output. Since the time consuming defuzzification process is replaced by weighted average operation, TSK-FIS has better processing time (Jang et al., 1997). Thus, it has been widely applied in control system. As for Mamdani FIS, it is suitable and widely applied in decision support applications, due to its expressive power and interpretability of Mamdani FIS output.

2.3.3.1 Mamdani FIS Model

Mamdani FIS model was proposed by Mamdani in 1975, as the first attempt to control a steam engine and boiler combination by a set of linguistic control rules obtained from experienced human operators (Mamdani & Assilian, 1975). Figure 2.7 illustrates the procedure of deriving the overall output z of a two-rule Mamdani FIS model when subjected to two crisp inputs x and y . The fuzzy IF-THEN rules in Mamdani form is described as follows:

$$IF\ x\ is\ A\ and\ y\ is\ B, THEN\ z\ is\ C$$

where A and B are fuzzy sets in the antecedent and C is a fuzzy set in the consequent.

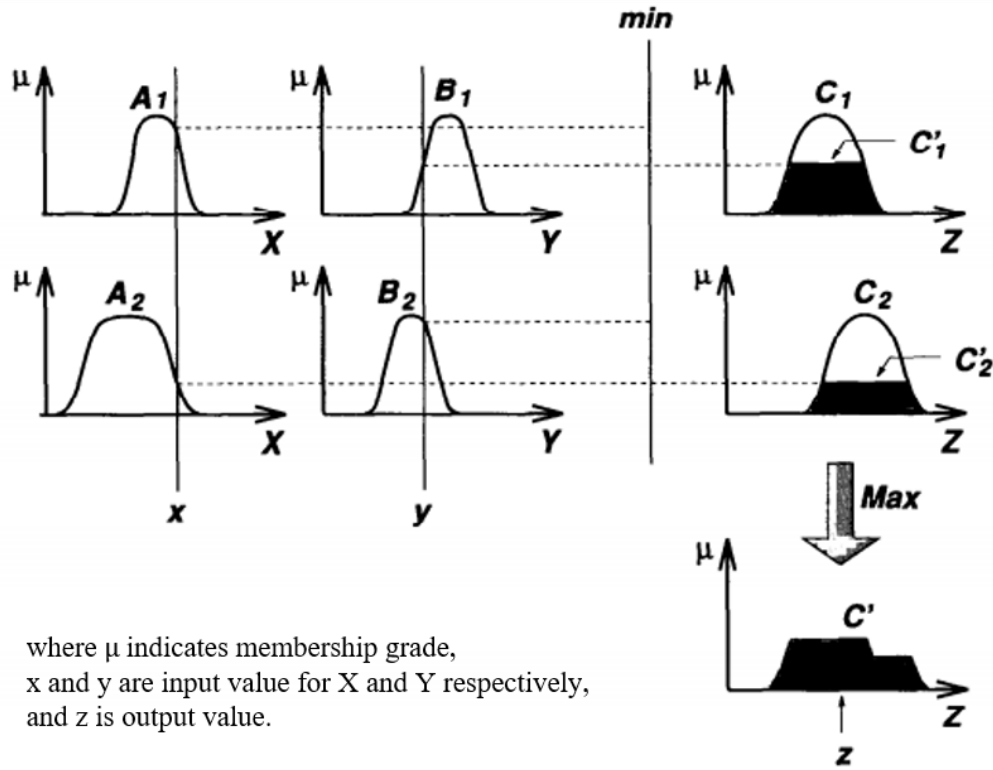


Figure 2.7: Mamdani FIS model (Jang et al., 1997)

In Figure 2.7, min and max are adopted as the T-norm and T-conorm operators, respectively. As a result, max-min composition is obtained. Other variations are possible if different T-norm and T-conorm operators are used. In the case that crisp value is needed as output, defuzzification process is performed to convert the aggregated output fuzzy set into a crisp value. Many defuzzifiers have been proposed in the literature, among them are Maximum Defuzzifier, Mean of Maxima Defuzzifier, Centroid Defuzzifier, Height Defuzzifier and Modified Height Defuzzifier (Mendel, 1995).

2.3.3.2 Takagi-Sugeno-Kang FIS model

TSK-FIS model (also known as Sugeno model) was introduced by Takagi, Sugeno and Kang with the emphasis to develop a systematic approach of generating fuzzy rules from

a given input-output data set (Sugeno & Kang, 1988; Takagi & Sugeno, 1985). A typical fuzzy rule in two inputs TSK model has the following form:

$$IF\ x\ is\ A\ and\ y\ is\ B\ THEN\ z = f(x, y)$$

where A and B are fuzzy sets in the antecedent and $z = f(x, y)$ is a crisp function in the consequent. $f(x, y)$ can be any mathematical function (usually is a polynomial function) as long as it can appropriately describe the output of the model within the fuzzy region specified by the rule antecedent.

When $f(x, y)$ is a first order polynomial (i.e., linear function), the resulting fuzzy inference system is known as first-order TSK-FIS model. Figure 2.8 illustrates the inference procedure for a first-order TSK-FIS model. Since each fuzzy rule has a crisp output given by a function, and thus the final output is obtained via weighted average.

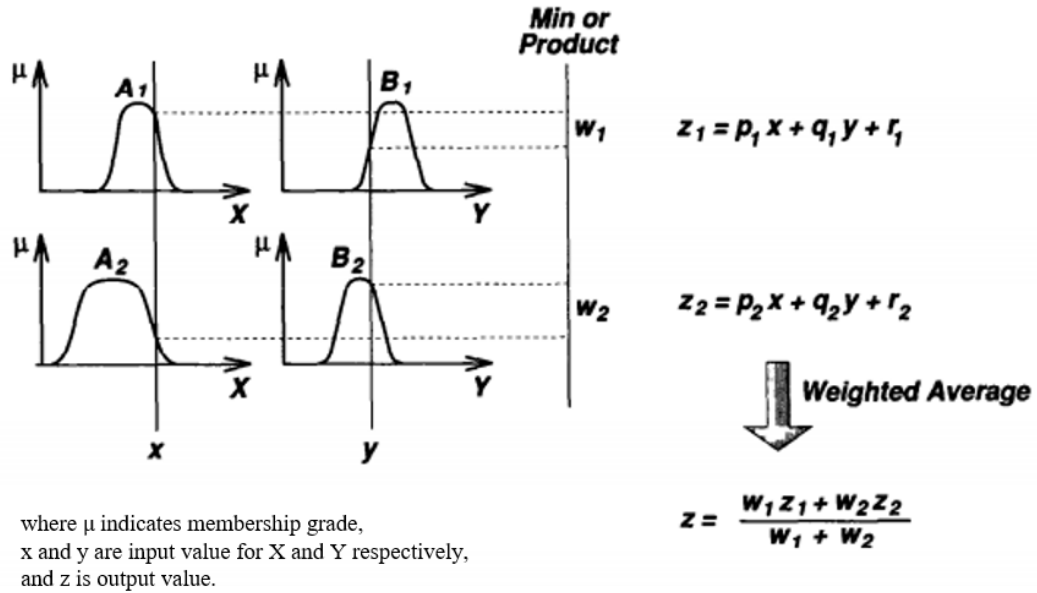


Figure 2.8: Takagi-Sugeno-Kang FIS model (Jang et al., 1997)

When $f(x, y)$ is a constant, the resulting fuzzy inference system is known as zero-order TSK-FIS model. Noted that zero-order TSK-FIS model is a special case of the Mamdani system, in which each fuzzy rule's consequent is specified by a fuzzy singleton. The output of a zero-order TSK-FIS model is a smooth function of its input variable, provided the neighboring membership functions in the antecedent part have enough overlap. This implies that the overlapping of membership functions in the consequent of Mamdani model does not have decisive effect on the smoothness, it is the overlap of the membership functions in antecedents that determines the smoothness of the resulting input-output behavior (Jang et al., 1997).

In this thesis, the focus is on the zero-order TSK-FIS model. The reason why the zero-order TSK-FIS model is important and innovative are due to its good approximation capability, despite its simplicity as compared with other TSK (i.e., first or higher order TSK) models (Sugeno, 1999). Besides that, zero-order TSK-FIS model has good interpretability as compared to higher order TSK-FIS model. Since zero-order TSK-FIS model is a special case of Mamdani FIS model, linguistic labels can be assigned to the singleton value of the rule consequent and allowing human to interpret each rule consequent easily during the fuzzy rule base collection phase. On the other hand, for the first order or higher order TSK-FIS model, it is hard for human experts to assign a linear or polynomial function to the rule consequent. Zero-order TSK-FIS model is suitable and widely applied in decision support applications (Chua, Wong, & Voon, 2005; McIvor, McCloskey, Humphreys, & Maguire, 2004).

2.4 Monotone Fuzzy Inference Systems

Monotone FIS is an FIS model that satisfies the monotonicity property between the

inputs and output spaces. Consider a general function, $y = f(x_1, x_2, \dots, x_n)$ which fulfils the condition of monotonicity between its output, y , with respect to its input i^{th} , x_i variable. The output of the model is either monotonically increases or decreases as x_i increases, as shown in Eq. (2.8) and Eq. (2.9) respectively (Tay & Lim, 2011b).

$$(i) \quad f(x_{1,1}, x_{2,1}, \dots, x_{n,1}) \leq f(x_{1,2}, x_{2,2}, \dots, x_{n,2}) \quad (2.8)$$

$$(ii) \quad f(x_{1,1}, x_{2,1}, \dots, x_{n,1}) \geq f(x_{1,2}, x_{2,2}, \dots, x_{n,2}) \quad (2.9)$$

where, $x_{i,1} < x_{i,2}$ and $i = 1, 2, \dots, n$.

To ease the explanation, in this thesis, only monotonically increasing case is considered.

2.4.1 Related Works

From the literature, a number of studies related to the monotonicity property of FIS models have been reported. In general, these studies focus on two domains: (i) mathematical conditions of an FIS to satisfy the monotonicity property; (ii) a method to construct a monotonicity-preserving FIS model. The focus of this thesis is on the second option. Important findings that are closely related to this research are further discussed.

2.4.1.1 Findings from Wu and Sung (1994, 1996) and Wu (1997)

One of the earlier work related to the monotonicity property in fuzzy controller was due to Wu and Sung (1996) and Wu (1997). They suggested that the monotonicity property is important for the stability analysis of FIS-based control problems. They also stressed the importance of the monotonicity property for JPEG models in image compression (Wu & Sung, 1994, 1996). They proposed a new defuzzification operator, i.e., mean-of-inversion

(MOI) for preserving the monotonicity property of FIS models. The MOI operator defuzzifies each fired rule separately, instead of superimposing all fired rules before defuzzification.

2.4.1.2 Findings from Zhao and Zhu (2000)

Zhao and Zhu (2000) suggested that for most controlled process problems, regardless of single-input single-output, or multi-input multi-output problems, the relationship between the input and output obeys the monotonicity property, i.e., the output of the process can be expressed as a monotonic function of the input variables. They further suggested that the monotonicity property is important to ensure the stability and the steady state error of an FIS-based control problem. The sufficient conditions for single-input single output and two-input single output conventional Mamdani fuzzy control algorithms (CMFCA) to fulfil monotonicity property are derived. Their study considers membership functions that are well partitioned. Their findings suggested that as long as the rule base is monotonically-ordered, the single-input single-output CMFCA is monotone, and the two-input single-output CMFCA is roughly monotone.

2.4.1.3 Findings from Lindskog and Ljung (2000)

Again, the importance of the monotonicity property in FIS-based control problem is highlighted by Lindskog and Ljung (2000). A fuzzy model structure that ensures input–output monotonicity is proposed. It is used to identify the dynamic system whose output is monotonic with respect to its input. The fuzzy model structure is parameterized, whereby constraints for each parameter is developed and imposed in the fuzzy model designing

process. A case study related to a water heating system was reported to show the usefulness of their approach.

2.4.1.4 Findings from Won et al. (2002) and Won and Karray (2014)

Won et al. (2002) reported that many real-world engineering systems satisfy the monotonicity property between its input and output. Two examples, i.e., the cart-pole system and the magnetic crane controller system, are explained. They suggested that FIS models that preserve the monotonicity property are able to better approximate the actual control mechanism.

A set of sufficient conditions for the TSK-FIS by differentiating the output of the FIS with respect to its input(s) is derived. The derived condition suggested that to maintain the monotonicity property of a FIS model, constraints are needed to be imposed on both antecedent parts and consequents parts of the fuzzy rules. These sufficient conditions were later combined with least-square learning (Koo, Won, & Lee, 2004) and evolutionary computation-based learning (Won, Seo, Hwang, & Lee, 2001). Besides, Li et al. (2009) further extended the sufficient conditions to the TSK-FIS with type-two fuzzy sets. Combination of the sufficient conditions with learning algorithms allow a monotonic FIS model to be constructed from data samples, based on a learning theory. Won and Karray (2014) proved that the proposed parametric conditions are necessary and sufficient to build a monotone single-input zero-order TSK FIS with two rules. The proposed parametric conditions are sufficient to build monotone multi-input first- or higher-order TSK FIS model.

2.4.1.5 Findings from Van Broekhoven and De Baets (2008, 2009)

Van Broekhoven and De Baets (2008, 2009) pointed out that it is important to fully exploit the monotonicity property as additional qualitative information, especially in the case whereby the number of data samples is small. The input-output monotonicity of the Mamdani-Assilian FIS model incorporating trapezoidal membership functions and mean of maximum defuzzification is examined.

They also pointed out that a monotone fuzzy rule base only is not sufficient to guarantee the monotonicity property of Mamdani-Assilian FIS model. A smooth and monotone fuzzy rule base is needed for constructing a monotone Mamdani-Assilian FIS model. Broekhoven et al. further investigate the use of three basic AND operators, i.e., minimum, product, and Łukasiewicz, in monotonicity-preserving FIS models. A guideline to design monotonicity-preserving FIS models with minimum, product, and Łukasiewicz operators, is provided. It has been shown that the choices of mathematical operators and the properties of the output membership functions are crucial.

2.4.1.6 Findings from Kouikoglou and Phillis (2009)

Kouikoglou and Phillis (2009) argued that in most application of fuzzy inference, the monotonicity property is a natural requirement such that the output of the fuzzy systems be monotonic with respect to its inputs. They explained the monotone scenario using three control examples, i.e., queuing system, inverted pendulum or cart-pole system, and air conditioner control. In the example of queuing systems, the control actions concerning admission and routing of customers and allocation of resources depend monotonically on the queue sizes and the customer arrival rates. In the fuzzy air conditioner controller, the motor

speed increases as the room temperature deviates from a target value. For the fuzzy cart-pole controller with the objective to maintain the pole in the upright position, the higher the degree that the pole deviates from the upright position, the higher the degree of acceleration from the cart is required. The importance of the monotonicity property in assessment (decision) problems, e.g., assessment of sustainable development and measurement of material recyclability, has also been stressed. They further extended the sufficient conditions to a hierarchical FIS model. The sufficient conditions are incorporated to be part of the design process of a hierarchical FIS model, i.e., design of membership functions and fuzzy rule base.

2.4.1.7 Findings from Seki, Ishii, and Mizumoto (2010)

Seki, Ishii, and Mizumoto (2010) highlighted that the proof of the monotonicity property for an FIS model is difficult. Nevertheless, some mathematical conditions are useful for preserving the monotonicity property in an FIS model. It was pointed out that even if the consequent parts in fuzzy rules are monotone, the inference results are not necessarily monotone when the antecedent parts are noncomparable. The monotonicity of functional type single-input rule module connected (SIRM)-type fuzzy inference methods were proved with the sufficient conditions. Moreover, they also clarified the monotonicity condition for two inputs TSK inference method by using equivalence relation of fuzzy inference.

2.4.1.8 Findings from Li, Yi, and Zhao (2009) and Li, Yi, and Zhang (2014)

Again, Li et al. (2009, 2014) pointed out the importance of monotonicity property in system modelling and control problems. They suggested the usefulness of monotonicity property as qualitative or prior information for FIS modelling. The sufficient conditions for multi-input interval type-2 FIS model subject to monotonicity constraint are then examined.

In Li, Yi, and Zhang (2014), five type-reduction and defuzzification methods (i.e., Karnik-Mendel method, Du-Ying method, Begian-Melek-Mendel method, Wu-Tan method, and Nie-Tan method) are taken into consideration. The derived sufficient conditions are valid for triangular, trapezoidal, and Gaussian interval type-2 fuzzy sets. The presented results are also applicable for type-1 TSK FIS.

2.4.1.9 Findings from Tay and Lim (2011b)

Tay et al. (Tay & Lim, 2011b) examined the sufficient conditions for the zero-order TSK FIS model. The results conclude that two mathematical conditions (at the fuzzy rule antecedent and consequent parts) are important to construct a monotonicity-preserving FIS model. Sufficient conditions for the zero-order TSK FIS model with Gaussian membership functions have been interpreted, whereby a projection of the Gaussian membership functions into a linear function was used to visualize the range of the input variable on which the fuzzy model remains monotonic.

2.4.1.10 Findings from Štěpnička and Jayaram (2016)

Štěpnička and Jayaram (2016) examined and demonstrated the co-existence of the monotonicity and interpolativity properties in the case of fuzzy relational inference system. The monotonicity property suggested that if x, x' are crisp inputs to the system that are ordered $x \leq x'$, then, the corresponding defuzzified output from the system y, y' should also be ordered accordingly, i.e. $y \leq y'$. The interpolativity property suggested that if a fuzzy set equivalent to one of the antecedent fuzzy sets appears on the input of such a system, the inferred output should be equivalent to the respective consequent fuzzy set. Their studies considered the single input single output (SISO) and multi input single output (MISO) cases.

With the proposed of at-least and at-most modifiers, a set of monotone fuzzy rule base is constructed and thereby, ensuring a monotonicity-preserving FIS model. Modified implicative models are then studied with these monotone fuzzy rule base to construct a monotonicity-preserving model with interpolativity property.

2.5 Monotone Data Relabeling Techniques

One of the earliest work relating to monotone data relabeling was due to Dykstra et al. (1999) in which the problem of exact minimization of an empirical loss function in the context of monotone classification is considered. They proposed a monotone relabeling that minimizes absolute error or squared error for obtaining optimal rules via isotonic regression.

Daniels and Velikova (2003) suggested that monotone relationship between the dependent variable and independent variable are existed in many economic classification and regression problems. However, monotone data set is not always available in practice, due to noise. Therefore, they proposed a monotonicity test to measure the degree of monotonicity of a given data set before the dataset is used to construct a decision model. A relabeling algorithm is then proposed to transform a given non-monotone data set into monotone one, by relabel the value of the dependent variable on step by step basis until the data set is monotone. However, their technique does not guarantee a minimum number of label changes.

Rademaker et al. (2009, 2012) further proposed a monotone data relabeling technique based on its formulation as a maximum independent set problem. Considering the monotonicity violation graph that has an edge between two data points whenever they violate the monotonicity property, finding a maximum size of independent set of the monotonicity violation graph is equivalent to the maximum size of monotone subset of the data. A

consistent and optimal relabeled monotone data set is obtained by relabeling the complement of the maximum independent set. Besides that, different loss functions such as absolute loss function, zero-one loss function and square loss function for accommodating monotone data relabeling are also examined.

2.6 Graph Theory

Graph theory is the study of the properties of graph, which consists of vertices connected by edges. In this section, a brief introduction to certain basic concept in graph theory are presented.

2.6.1 Basic Concept and Terminology

A graph $G = (V, E)$ is an ordered pair consisting of a finite set of vertices (also known as nodes) $V = \{v_1, v_2, v_3, \dots\}$ and a set of edges (also known as arcs) $E = \{e_1, e_2, e_3, \dots\}$. In this thesis, a simple directed graph G is considered, i.e., G is finite, having no loops, and without duplicate edges. The set $V(G)$ is called the vertex set of G and the set $E(G)$ is called the edge set of G . The edge is represented in the form of ordered pair of two vertices, i.e., $e = (v_1, v_2) \in E$, for $v_1, v_2 \in V$, with the direction (represented by an arrow in the graph) from v_1 to v_2 . The vertices v_1 are said to be adjacent to v_2 in G and that v_1 and v_2 are joined by the directed edge e . Figure 2.9 shows an example of a simple directed graph G , with $V(G) = \{v_1, v_2, v_3, v_4, v_5, v_6, v_7, v_8\}$ and $E(G) = \{(v_1, v_2), (v_1, v_3), (v_1, v_4), (v_5, v_6), (v_7, v_6), (v_7, v_8)\}$.

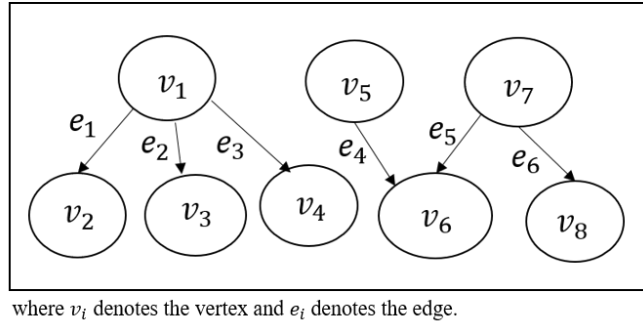


Figure 2.9: A simple directed graph G

A graph whose edge set is empty is called a null graph or a totally disconnected graph. If an edge e has v as an end-point, then e is said to be incident with v . As an example, in Figure 2.9, e_1 is incident with v_1 . The degree of a vertex v_i in a graph G is the number of edges incident with v_i . A vertex v of degree 0 is called an isolated vertex. The indegree of a vertex v in a digraph is the number of edges directed to v ; the outdegree of vertex v is the number of edges directed from v . A subgraph of a graph G is a graph H such that $V(H) \subset V(G)$ and $E(H) \subset E(G)$. Figure 2.10 shows an example of a subgraph H of graph G in Figure 2.9.

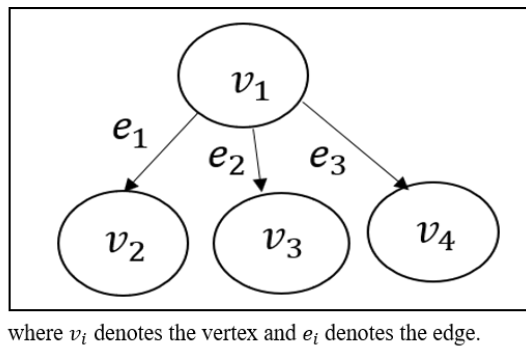


Figure 2.10: An example of subgraph H of graph G

2.6.2 Operations on Graphs

In this section, the primary operations on graph are explained.

Definition 2.8. (Gross & Yellen, 2004) Let G_1 and G_2 be two graphs with disjoint vertex sets. The **union** $G = G_1 \cup G_2$ has $V(G) = V(G_1) \cup V(G_2)$ and $E(G) = E(G_1) \cup E(G_2)$.

Definition 2.9. (Gross & Yellen, 2004) The operation of **adding the vertex** u to a graph $G = (V, E)$, such that $u \notin V$, yields a new graph with vertex set $V \cup \{u\}$ and edge set E , which is denoted $G \cup \{u\}$ (the new vertex u has no neighbors).

Definition 2.10. (Gross & Yellen, 2004) The operation of **deleting the vertex** u from a graph $G = (V, E)$ not only removes the vertex u but also removes every edge of which u is an endpoint.

Definition 2.11. (Gross & Yellen, 2004) The operation of **adding an edge** e to a graph $G = (V, E)$ joining the vertices u and v yields a new graph with vertex set V and edge set $E \cup \{e\}$, which is denoted $G \cup \{e\}$.

Definition 2.12. (Gross & Yellen, 2004) The operation of **deleting an edge** e from a graph $G = (V, E)$ removes only that edge. The resulting graph is denoted $G - e$.

2.7 Summary

In this chapter, the background and literature relating to fuzzy set theory, FIS models, monotone FIS model, monotone data relabeling and graph theory are presented. In the next chapter, the general idea of monotone fuzzy rule relabeling will be presented.

CHAPTER 3

MONOTONE FUZZY RULE RELABELING

3.1 Introduction

In the previous chapter, the literature review was presented. In this chapter, the general idea of monotone fuzzy rule relabeling is presented. Worth mentioning, much work pertaining to relabeling of non-monotone data has been reported in the literature (Daniels & Velikova, 2006; Feelders et al., 2006; Rademaker et al., 2009, 2012). Inspired from the idea of data relabeling, the monotone fuzzy rule relabeling (MFRR) for Takagi-Sugeno-Kang (TSK)- Fuzzy Inference System (FIS), is introduced. To develop a monotone TSK-FIS, a monotone fuzzy rule base is essential. MFRR is important as monotone fuzzy rule base is not always available. MFRR attempts to identify potentially noisy fuzzy rules and relabel them to become a set of monotone fuzzy rules. In this chapter, the general idea of MFRR is introduced. In addition, two MFRR algorithms, that are based on *brute force search* (i.e., MFRR-BF) and *brute force search with backtracking* (i.e., MFRR-BFBT), respectively, are presented, demonstrated, and analyzed. MFRR-BFBT is expected to reduce the computational time than that of MFRR-BF.

In Section 3.2, monotone data relabeling is demonstrated with simulated examples. The background information for the zero order TSK-FIS is explained in Section 3.3. MFRR is formulated in Section 3.4. Simulated examples are illustrated in Section 3.5. In Section 3.6, MFRR-BF is explained and experimented with simulated examples. MFRR-BFBT is proposed and explained in Section 3.7. The computation cost of MFRR-BF and MFRR-

BFBT is further analyzed and discussed. The concluding remarks for Chapter 3 is given in Section 3.8.

3.2 Relabeling of Non-Monotone Data Sets

To illustrate the idea of relabeling for non-monotone data sets, a single input data set with five data points (i.e., data set 1, **D1**) is exemplified, as shown in Table 3.1. In this example, a monotonically increasing relationship between the input (i.e., x) and output (i.e., y) is expected. A monotonically increasing relationship suggests that y for the second data point should be higher than that of the first. However, this is not the case. The second data point is potentially a noisy object, that causes the formation of the non-monotone data set.

Table 3.1: An example of a single input data set, **D1**

Data point	x	y
1	1	3
2	2	2
3	3	4
4	4	6
5	5	6

where x and y are input and output, respectively.

Monotone data relabeling was introduced to restore the monotonicity property of the data set (Daniels & Velikova, 2006; Feelders et al., 2006; Rademaker et al., 2009, 2012). A minimum number of noisy object is identified, and each is relabeled to a value that could result in a monotone data set. A loss function is used to quantify the loss incurred during the relabeling process, by computing the total loss associated with relabeling objects from an old value to a new value (Rademaker, 2009). A relabeled monotone data set with the minimum number of objects change, as well as with the minimum loss incurred is an optimal

solution. As an example, Table 3.2 shows an optimal relabeled monotone data set for **D1**, with one relabeled object and a minimum loss of 1 (a square loss function is assumed).

Table 3.2: Relabeled monotone data set for **D1**

Data point	x	y
1	1	3
2	2	3
3	3	4
4	4	6
5	5	6

where x and y are input and output, respectively.

Table 3.3 shows another example of a two inputs data set, **D2**. To ease the explanation, **D2** is represented in 2-dimensional cells, as shown in Table 3.4. The data set **D2** is not monotone, as it contains non-monotone object(s), i.e., potentially the fourth data point. Using the monotone data relabeling, a relabeled monotone data set is obtained for **D2**, as shown in Table 3.5. The fourth data point is relabeled to a value of 3, with the minimum loss value of 1, and thus a monotone data set is resulted. In this chapter, the idea of monotone data relabeling is extended to monotone fuzzy rule relabeling for TSK-FIS.

Table 3.3: An example of a two inputs data set, **D2**

Data point	x_1	x_2	y
1	1	1	2
2	2	1	2
3	3	1	3
4	1	2	4
5	2	2	3
6	3	2	5
7	1	3	3
8	2	3	5
9	3	3	6

where x_1, x_2 are first input and second input, and y is output.

Table 3.4: Representation of data set **D2** in 2-dimensional cells

	$x_2 = 1$	$x_2 = 2$	$x_2 = 3$
$x_1 = 1$	2	4	3
$x_1 = 2$	2	3	5
$x_1 = 3$	3	5	6

where x_1 and x_2 are first input and second input, respectively.

Table 3.5: Relabeled monotone data set **D2** in 2-dimensional cells

	$x_2 = 1$	$x_2 = 2$	$x_2 = 3$
$x_1 = 1$	2	3	3
$x_1 = 2$	2	3	5
$x_1 = 3$	3	5	6

where x_1 and x_2 are first input and second input, respectively.

3.3 Background and Preliminaries

In this section, the background information pertaining to the monotone zero order TSK-FIS model is presented, as follows.

Definition 3.1. Consider an input space, X , and an output space, Y . Variables $\bar{x} = [x_1, x_2, \dots, x_n] \in X_1 \times X_2 \times \dots \times X_n$ and y are the elements of X and Y , i.e., $\bar{x} \in X$, and $y \in Y$, respectively. The lower and upper bounds of X_i are denoted as $\underline{x}_i$ and $\bar{x}_i$, respectively. Similarly, the lower and upper bounds of Y are denoted by $\underline{y}$ and $\bar{y}$, respectively.

Definition 3.2. Consider a target monotone system $y = f(\bar{x})$, $f: X \rightarrow Y$. For all $\bar{x}$ and y , $f(x_{i,2}, \bar{s}) \geq f(x_{i,1}, \bar{s})$, where $x_{i,2} > x_{i,1} \in X_i$, where $\bar{s}$ is a subset of $\bar{x}$, i.e., $\bar{s} \subset [x_1, x_2, \dots, x_n]$ and $x_i \notin \bar{s}$, is always true.

As an example, consider a monotone system with three inputs, i.e., $n = 3$. Variables $\bar{x} = [x_1, x_2, x_3] \in X_1 \times X_2 \times X_3$. If for any two points $x_{1,1}$ and $x_{1,2}$ such that $x_{1,2} > x_{1,1} \in X_1$, then $f(x_{1,2}, \bar{s}) \geq f(x_{1,1}, \bar{s})$, where $\bar{s} = [x_2, x_3]$.

3.3.1 Zero order Takagi-Sugeno-Kang Fuzzy Inference Systems

A mathematical model, $y = g(\bar{x})$, i.e., $g: X \rightarrow Y$, which attempts to approximate the target monotone system, i.e., $y = f(\bar{x})$, is considered. In this study, the zero-order TSK-FIS model (in this chapter, hereafter abbreviated as FIS) is used to approximate the target system. For an n -input FIS model, where $n > 0$, its fuzzy IF-THEN rule from humans is represented as follows:

$R_{j_1, j_2, \dots, j_n}$:

IF (x_1 is $A_{j_1}^1$) *AND* (x_2 is $A_{j_2}^2$) *AND* $\dots$ *AND* (x_n is $A_{j_n}^n$), *THEN* (y is $B_{j_1, j_2, \dots, j_n}$), where $1 \leq j_i \leq m_i$

For notational convenience, $R_{j_1, j_2, \dots, j_n}$ is denoted as $R_J: A_J \rightarrow B_J$, where $J(j_1, j_2, \dots, j_n) = j_1 + \sum_{k=2}^n [(j_k - 1)(\prod_{p=1}^{k-1} m_p)]$. The AND operator in the rule antecedent part is the product function. Consider that the x_i domain contains m_i membership functions (MFs), whereby the MFs are $\mu_i^1(x_i), \mu_i^2(x_i), \dots, \mu_i^{m_i}(x_i)$ for linguistic terms $A_i^1, A_i^2, \dots, A_i^{m_i}$, respectively. The number of fuzzy rules is $r = \prod_{p=1}^n m_p$. The output is obtained by using the weighted average of a fuzzy singleton, $b_{j_1, j_2, \dots, j_n}$, with respect to its compatibility grade, as shown in Eq. (3.1).

$$y = g(x) = \frac{\sum_{j_n=1}^{j_n=m_n} \dots \sum_{j_2=1}^{j_2=m_2} \sum_{j_1=1}^{j_1=m_1} (\mu_1^{j_1}(x_1) \times \mu_2^{j_2}(x_2) \dots \times \mu_n^{j_n}(x_n) \times b_{j_1, j_2, \dots, j_n})}{\sum_{j_n=1}^{j_n=m_n} \dots \sum_{j_2=1}^{j_2=m_2} \sum_{j_1=1}^{j_1=m_1} (\mu_1^{j_1}(x_1) \times \mu_2^{j_2}(x_2) \dots \times \mu_n^{j_n}(x_n))} \quad (3.1)$$

where $b_{j_1, j_2, \dots, j_n}$ is the fuzzy singleton of $B_{j_1, j_2, \dots, j_n}$. To simplify the explanation, $R_J: A_J \rightarrow B_J$ is written as $R_J: A_J \rightarrow b_J$.

3.3.2 The Sufficient Conditions

Theorem 3.1. (Won, Park, & Lee, 2002) The FIS model fulfils the monotonicity property between y and x_i , if the following conditions are satisfied.

Condition 1:

$$(d\mu_i^q(x_i)/dx_i)/\mu_i^q(x_i) \geq (d\mu_i^p(x_i)/dx_i)/\mu_i^p(x_i).$$

Condition 2: $b_{l,q} \geq b_{l,p}$.

where b is the singleton value for the consequent of a fuzzy rule, the index l is for all combinations of fuzzy membership functions in the hyperplane $(x_1, \dots, x_{i-1}, x_{i+1}, \dots, x_n)^T$ and $1 \leq p < q \leq m_i$. As an example, given a 3-inputs FIS model, i.e., $n = 3$. The consequent value for $R_{1,1,2}$ must greater than or equal to the consequent value for $R_{1,1,1}$, i.e., $b_{1,1,2} \geq b_{1,1,1}$, where $i = 3$, $p = 1$ and $q = 2$. Condition 2 also implies that the fuzzy rule base should be complete and monotone. An extension of Condition 1 is expressed as Corollary 3.1. The proof of Theorem 3.1 is shown in Won, Park & Lee (2002).

Corollary 3.1. (Tay & Lim, 2011b): Let $\mu_i^{j_i}(x_i)$ be a Gaussian MF, i.e., $\mu_i^{j_i}(x_i)(x: c_{x_i}^{j_i}, \sigma_{x_i}^{j_i}) = e^{-[x_i - c_{x_i}^{j_i}]^2 / 2(\sigma_{x_i}^{j_i})^2}$, where $c_{x_i}^{j_i}$ is the centre of the MF and $\sigma_{x_i}^{j_i}$ is its width.

3.1.1 Ratio $(d\mu_i^{j_i}(x_i)/dx_i)/\mu_i^{j_i}(x_i)$ of a Gaussian MF returns a linear function, i.e.,

$$E_{x_i}^{j_i}(x_i) = -\left(1/\sigma_{x_i}^{j_i^2}\right)x_i + \left(c_{x_i}^{j_i}/\sigma_{x_i}^{j_i^2}\right)$$

3.1.2 Condition 1 of Theorem 3.1 is satisfied if $(E_{x_i}^{j_i+1}(\underline{x}_i) \geq E_{x_i}^{j_i}(\underline{x}_i))$ and

$(E_{x_i}^{j_i+1}(\overline{x}_i) \geq E_{x_i}^{j_i}(\overline{x}_i))$ is true.

3.4 The MFRR Formulation

Proposition 3.1. Consider an FIS model with its MFs of each x_i satisfy Condition 1 of Theorem 3.1. Definition 3.3 is further considered.

Definition 3.3. Given a total number of $r_t \leq r$ fuzzy rules collected from domain experts, hereafter defined as $R_{J(j_1, j_2, \dots, j_n)}: A_{J(j_1, j_2, \dots, j_n)} \rightarrow b_{J(j_1, j_2, \dots, j_n)}$, where $J = 1, 2, 3, \dots, r$. The fuzzy rules are denoted as set $\mathbf{R}$. $\mathbf{R}$ can be complete or incomplete.

3.3.1 If $\mathbf{R}$ is *complete*, all $b_{J(j_1, j_2, \dots, j_n)}$ is known.

3.3.2 If $\mathbf{R}$ is *incomplete*, some $b_{J(j_1, j_2, \dots, j_n)}$ is unknown.

3.3.3 $\mathbf{R}$ is *monotone*, if Condition 2 of Theorem 3.1 is satisfied.

3.3.4 $\mathbf{R}$ is *non-monotone*, if Condition 2 of Theorem 3.1 is not satisfied.

3.3.5 $\mathbf{R}$ is said to be *inconsistent* if it contains two or more fuzzy rules that have the same combination of antecedents but different consequent label. $\mathbf{R}$ is consistent if it does not contain such fuzzy rules.

Let $U(B)$ represents a set of unique fuzzy singletons in $\mathbf{R}$, i.e., $U(B) = \{b_{u,1}, b_{u,2}, \dots, b_{u,r_u}\}$. Monotone fuzzy rule relabeling (MFRR) attempts to relabel the consequent part of fuzzy rules with the values in $U(B)$. The relabeled rule of $b_{J(j_1, j_2, \dots, j_n)}$, i.e., $b_{J(j_1, j_2, \dots, j_n)}^{rel}$, must fulfil the following three criteria:

- i) satisfying the monotonicity property as the first priority;
- ii) having the minimum number of relabeled fuzzy rules as the second priority; and
- iii) having the minimum loss measure corresponding to $b_{J(j_1, j_2, \dots, j_n)}$ as the third priority.

The loss measure is computed using the following equation:

$$loss = \sum_{j_n=1}^{j_n=m_n} \dots \sum_{j_2=1}^{j_2=m_2} \sum_{j_1=1}^{j_1=m_1} (b_{j_1,j_2,\dots,j_n} - b_{j_1,j_2,\dots,j_n}^{rel})^2 \quad (3.2)$$

3.5 Simulated Examples

In this section, simulated examples for single input FIS-model and multi-input FIS model are considered. These simulated examples will be used as illustrative examples to demonstrate the monotone fuzzy rule relabeling process. Zero-order TSK fuzzy inference rules are given. A zero-order TSK fuzzy inference rule can be identified by checking the consequent part of the given fuzzy rule. The consequent part of a zero-order TSK fuzzy inference rule is assigned with a constant value (i.e., fuzzy singleton value), instead of fuzzy set or polynomial function.

Given a set of fuzzy IF-THEN rules, the monotonicity of this fuzzy rules can be checked using the Condition 2 of Theorem 3.1 (see Section 3.3.2). The consequent value of the fuzzy rules is compared with each other. If there is a fuzzy rule that do not fulfil Condition 2 of Theorem 3.1, then the fuzzy rule base is not monotone.

3.5.1 Fuzzy Rule Base for Single Input FIS-Model: **R1**

Figure 3.1 shows a fuzzy rule base **R1** with 5 fuzzy rules, for a single input FIS model. The relationship between the input (i.e., x) and output (i.e., y) is described by the fuzzy IF-THEN rules. To ease the explanation, the details for fuzzy rule base **R1** is presented in table form, as shown in Table 3.6.

R_1 : IF x is <i>very small</i> , THEN y is <i>small</i> R_2 : IF x is <i>small</i> , THEN y is <i>small</i> . R_3 : IF x is <i>medium</i> , THEN y is <i>medium</i> R_4 : IF x is <i>large</i> , THEN y is <i>small</i> R_5 : IF x is <i>very large</i> , THEN y is <i>large</i>

Figure 3.1: A single input fuzzy rule base, **R1**

Table 3.6: Fuzzy rule base, **R1**

Fuzzy rule, R_j	A_j		b_j
	c	σ	
R_1	1	0.5	1
R_2	2	0.5	1
R_3	3	0.5	3
R_4	4	0.5	1
R_5	5	0.5	5

where c and σ are center and width of Gaussian MF, A_j and b_j are the antecedent MF and fuzzy singletons for the consequent.

The input domain is associated with five linguistic terms, i.e., *very small*, *small*, *medium*, *large* and *very large*. The linguistic terms are represented by Gaussian MFs, as depicted in Figure 3.2. The centers (i.e., c) and widths (i.e., σ) used to parameterize each Gaussian MF are given in Table 3.6. Note that the Gaussian MFs have been designed to satisfy **Corollary 3.1.1, Condition 1** of **Theorem 3.1**. As an example, the Gaussian MFs for x in Figure 3.2 can be projected as linear lines, as illustrated in Figure 3.3. The linear line label with $E_x^{very\ large}$ is always larger in magnitude than that of E_x^{large} . Therefore, $E_x^{very\ large} \geq E_x^{large} \geq E_x^{medium} \geq E_x^{small} \geq E_x^{very\ small}$ is always satisfied for $x \in [1,5]$.

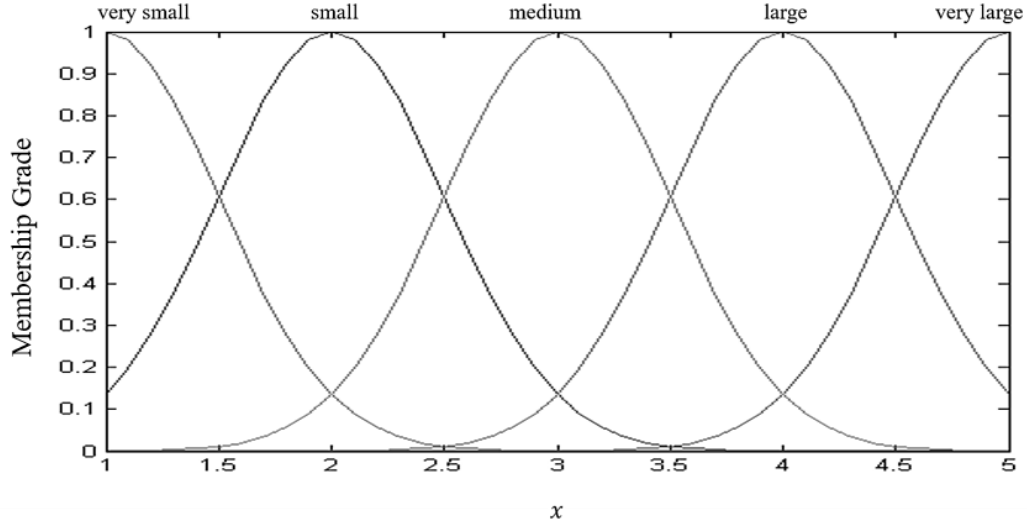


Figure 3.2: Gaussian membership functions for the antecedent's linguistic terms of **R1**

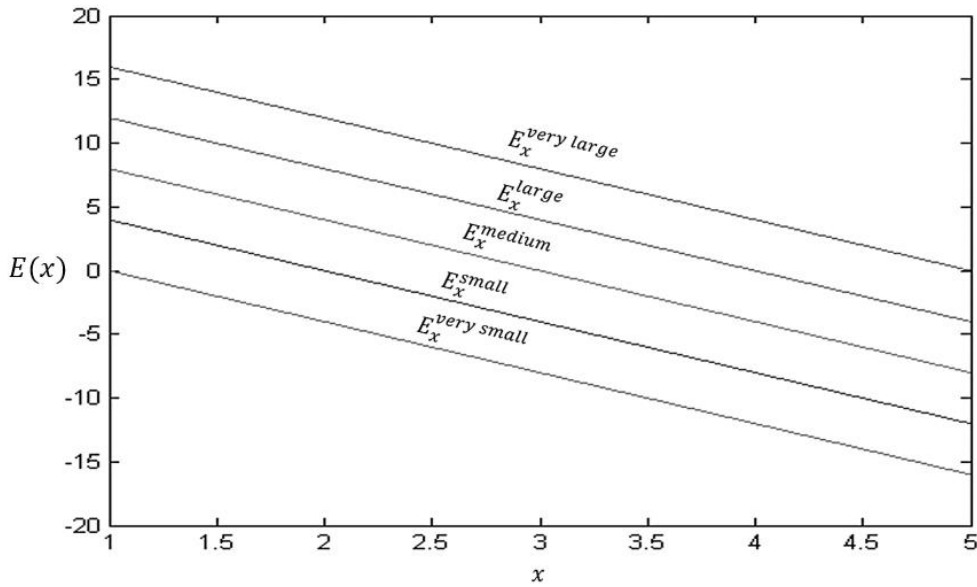


Figure 3.3: Projection of the Gaussian MFs for the antecedent's linguistic terms of **R1**

The output domain is assigned with three linguistic terms, i.e., *small*, *medium* and *large*. The linguistic terms for the consequent of the fuzzy rules are specified by three fuzzy singletons, i.e., 1, 3, and 5, for *small*, *medium* and *large*, respectively. Noted that the fuzzy rules for **R1** are non-monotone, i.e., b_f does not fulfil **Condition 2** of **Theorem 3.1**. As an example, the linguistic terms for x in R_3 and R_4 are *medium* and *large*, respectively. The fuzzy singleton for fuzzy consequent of R_4 is expected to be equal or higher than that of R_3

(i.e., $b_3 \leq b_4$), but, this is not the case.

3.5.2 Fuzzy Rule Base for a 2-Input FIS-Model: **R2** and **R3**

3.5.2.1 Monotone Fuzzy Rule Base, **R2**

Figure 3.4 shows an example of a monotone fuzzy rule base **R2**, for an FIS model with two inputs, i.e., $n = 2$. To ease the explanation, the fuzzy rule base **R2** is presented in a lookup table, as shown in Table 3.7.

R_1 : IF x_1 is very low and x_2 is very low THEN y is very slow.
R_2 : IF x_1 is low and x_2 is very low THEN y is slow.
R_3 : IF x_1 is medium and x_2 is very low THEN y is slow.
R_4 : IF x_1 is high and x_2 is very low THEN y is medium.
R_5 : IF x_1 is very high and x_2 is very low THEN y is medium.
R_6 : IF x_1 is very low and x_2 is low THEN y is very slow.
R_7 : IF x_1 is low and x_2 is low THEN y is slow.
R_8 : IF x_1 is medium and x_2 is low THEN y is medium.
R_9 : IF x_1 is high and x_2 is low THEN y is medium.
R_{10} : IF x_1 is very high and x_2 is low THEN y is fast.
R_{11} : IF x_1 is very low and x_2 is medium THEN y is slow.
R_{12} : IF x_1 is low and x_2 is medium THEN y is slow.
R_{13} : IF x_1 is medium and x_2 is medium THEN y is medium.
R_{14} : IF x_1 is high and x_2 is medium THEN y is fast.
R_{15} : IF x_1 is very high and x_2 is medium THEN y is fast.
R_{16} : IF x_1 is very low and x_2 is high THEN y is slow.
R_{17} : IF x_1 is low and x_2 is high THEN y is medium.
R_{18} : IF x_1 is medium and x_2 is high THEN y is fast.
R_{19} : IF x_1 is high and x_2 is high THEN y is fast.
R_{20} : IF x_1 is very high and x_2 is high THEN y is fast.
R_{21} : IF x_1 is very low and x_2 is very high THEN y is medium.
R_{22} : IF x_1 is low and x_2 is very high THEN y is fast.
R_{23} : IF x_1 is medium and x_2 is very high THEN y is fast.
R_{24} : IF x_1 is high and x_2 is very high THEN y is very fast.
R_{25} : IF x_1 is very high and x_2 is very high THEN y is very fast.

Figure 3.4: A monotone fuzzy rule base **R2** for a two input FIS-model

Table 3.7: Look up table for fuzzy rule base **R2**

	x_2				
	μ_2^1	μ_2^2	μ_2^3	μ_2^4	μ_2^5
μ_1^1	$b_{1(1,1)} = 10$	$b_{6(1,2)} = 10$	$b_{11(1,3)} = 20$	$b_{16(1,4)} = 20$	$b_{21(1,5)} = 30$
μ_1^2	$b_{2(2,1)} = 20$	$b_{7(2,2)} = 20$	$b_{12(2,3)} = 20$	$b_{17(2,4)} = 30$	$b_{22(2,5)} = 40$
μ_1^3	$b_{3(3,1)} = 20$	$b_{8(3,2)} = 30$	$b_{13(3,3)} = 30$	$b_{18(3,4)} = 40$	$b_{23(3,5)} = 40$
μ_1^4	$b_{4(4,1)} = 30$	$b_{9(4,2)} = 30$	$b_{14(4,3)} = 40$	$b_{19(4,4)} = 40$	$b_{24(4,5)} = 50$
μ_1^5	$b_{5(5,1)} = 30$	$b_{10(5,2)} = 40$	$b_{15(5,3)} = 40$	$b_{20(5,4)} = 40$	$b_{25(5,5)} = 50$

Both the inputs, i.e., x_1 and x_2 , have five MFs, i.e., $m_1 = 5$ and $m_2 = 5$. There is a total of 25 fuzzy rules, i.e., $r_t = 5 \times 5 = 25$, in **R2**. The two inputs, i.e., x_1 and x_2 are associated with five linguistic terms, i.e., *very low*, *low*, *medium*, *high* and *very high*, respectively, in which $x_1, x_2 \in [1,10]$. The linguistic terms for x_1 and x_2 are represented by Gaussian MFs, as depicted in Figure 3.5 and Figure 3.6, respectively. Again, the Gaussian MFs used in this example are designed to satisfy **Corollary 3.1.1, Condition 1** of **Theorem 3.1**, as shown in Figure 3.7 and Figure 3.8, respectively.

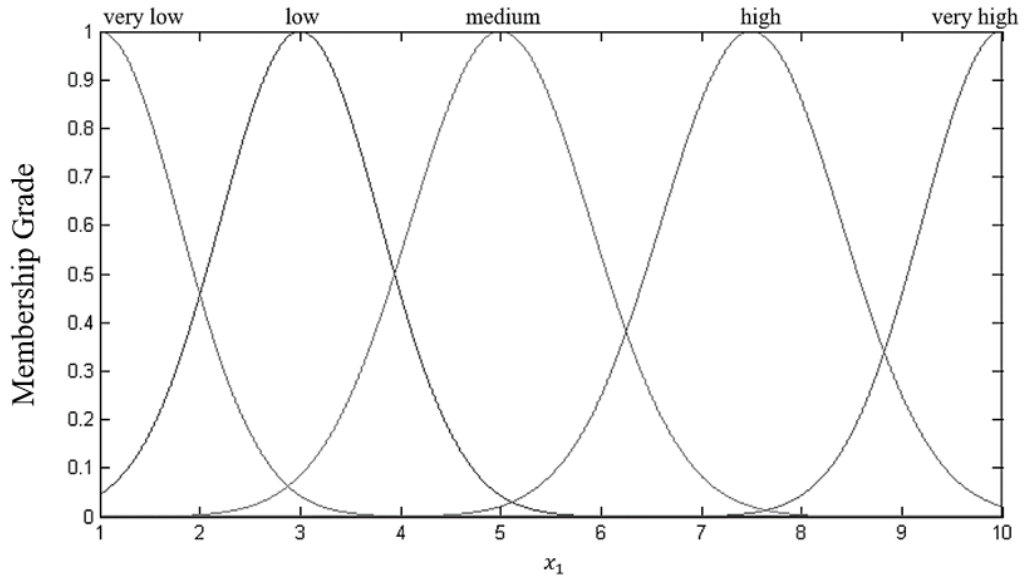


Figure 3.5: Gaussian Membership functions assigned to x_1 in **R2**

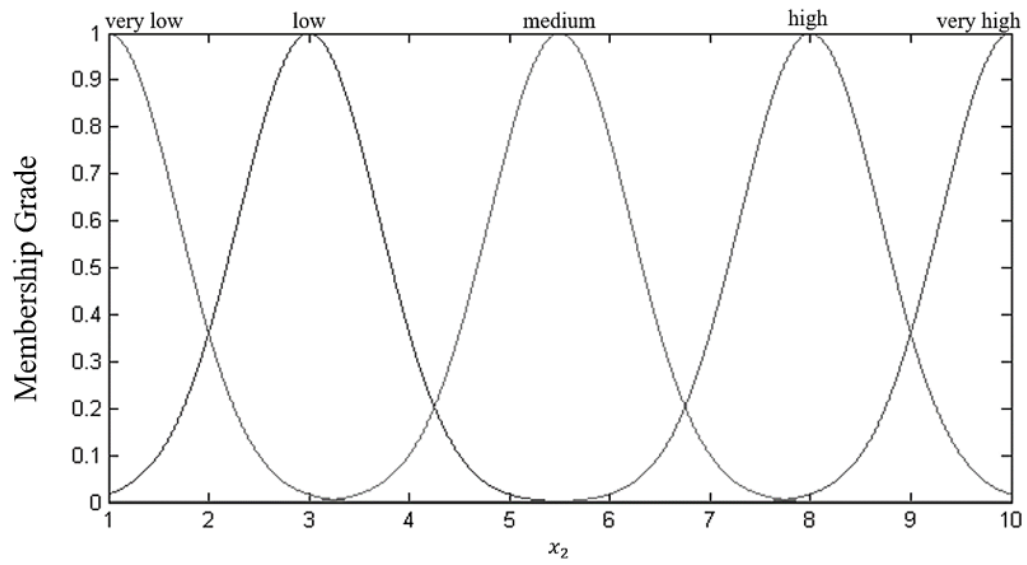


Figure 3.6: Gaussian Membership functions assigned to x_2 in $R2$

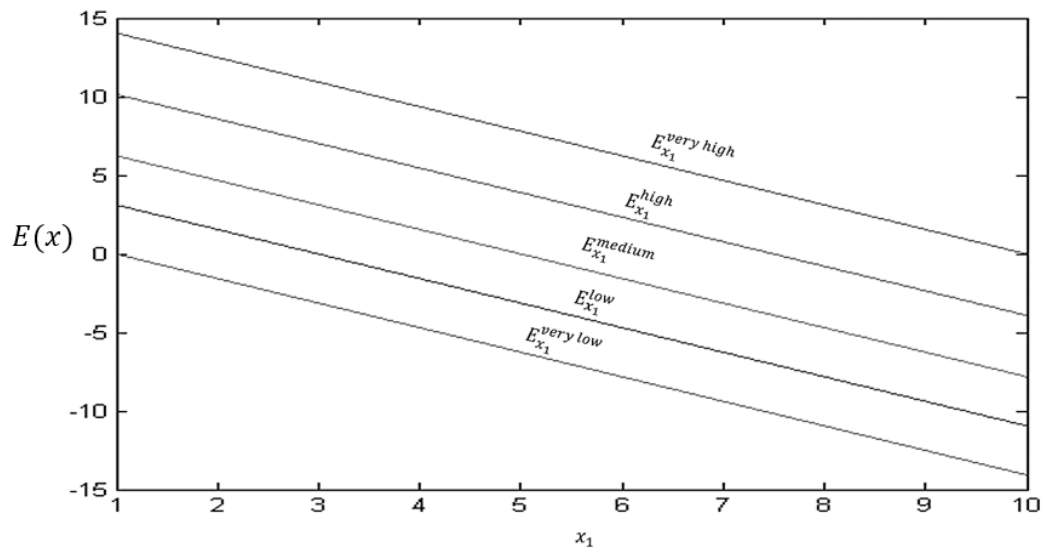


Figure 3.7: Projection of the Gaussian MFs for x_1 in $R2$

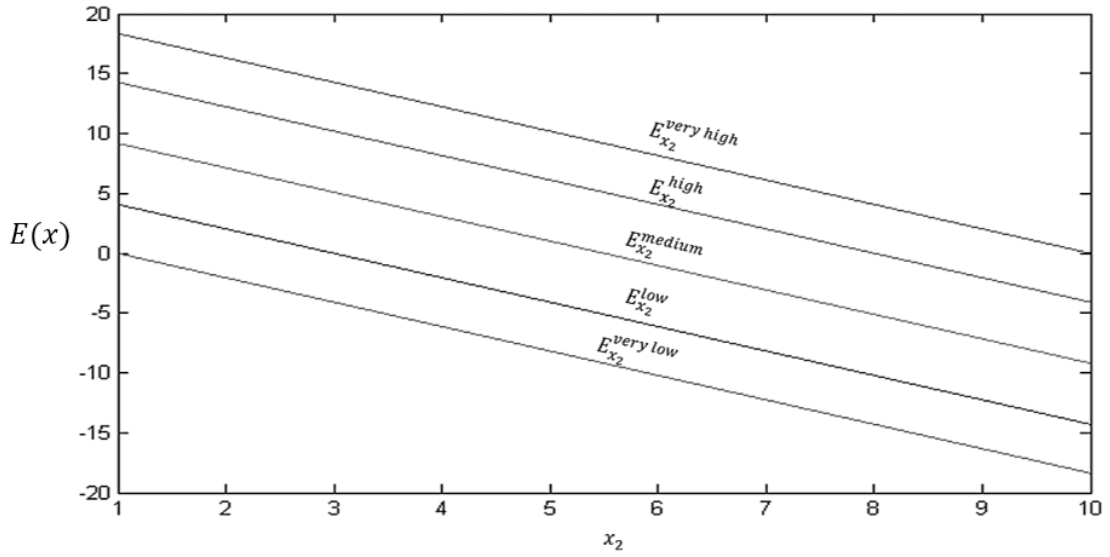


Figure 3.8: Projection of the Gaussian MFs for x_2 in R_2

The fuzzy rule consequents are represented by five fuzzy singleton values, i.e., 10, 20, 30, 40 and 50, for *very slow*, *slow*, *medium*, *fast*, and *very fast*, respectively. R_2 is complete, consistent and monotone.

3.5.2.2 Non-Monotone Fuzzy Rule Base, R_3

The same two-inputs FIS-model (in Section 3.5.2.1) with another set of fuzzy rule base, i.e., R_3 is considered. Figure 3.9 depicts the fuzzy rule base R_3 . Again, to ease the explanation, R_3 is presented in a lookup table as shown in Table 3.8. R_3 is complete, consistent, but not monotone, as the b_j does not fulfils **Condition 2** of **Theorem 3.1**. As an example, R_4 and R_5 have the same linguistic terms for x_2 , i.e., *very low*. The fuzzy linguistic terms assigned for x_1 in R_4 and R_5 are *high* and *very high*, respectively. Therefore, the singleton value for fuzzy consequent of R_5 is expected to be equal or higher than that of R_4 . However, from Table 3.8, the singleton value for fuzzy consequent of R_5 (i.e., 30) is less than that of R_4 (i.e., 50).

Such non-monotone fuzzy rule base could lead to an invalid/unreliable inference outcome produced by the FIS-model. Therefore, MFRR is proposed as a solution to restore the monotonicity property of a non-monotone fuzzy rule base. The details of MFRR is given in next section.

R_1 : IF x_1 is very low and x_2 is very low THEN y is very slow.
 R_2 : IF x_1 is low and x_2 is very low THEN y is slow.
 R_3 : IF x_1 is medium and x_2 is very low THEN y is slow.
 R_4 : IF x_1 is high and x_2 is very low THEN y is very fast.
 R_5 : IF x_1 is very high and x_2 is very low THEN y is medium.
 R_6 : IF x_1 is very low and x_2 is low THEN y is very slow.
 R_7 : IF x_1 is low and x_2 is low THEN y is very slow.
 R_8 : IF x_1 is medium and x_2 is low THEN y is medium.
 R_9 : IF x_1 is high and x_2 is low THEN y is medium.
 R_{10} : IF x_1 is very high and x_2 is low THEN y is fast.
 R_{11} : IF x_1 is very low and x_2 is medium THEN y is slow.
 R_{12} : IF x_1 is low and x_2 is medium THEN y is slow.
 R_{13} : IF x_1 is medium and x_2 is medium THEN y is medium.
 R_{14} : IF x_1 is high and x_2 is medium THEN y is fast.
 R_{15} : IF x_1 is very high and x_2 is medium THEN y is fast.
 R_{16} : IF x_1 is very low and x_2 is high THEN y is slow.
 R_{17} : IF x_1 is low and x_2 is high THEN y is medium.
 R_{18} : IF x_1 is medium and x_2 is high THEN y is fast.
 R_{19} : IF x_1 is high and x_2 is high THEN y is medium.
 R_{20} : IF x_1 is very high and x_2 is high THEN y is fast.
 R_{21} : IF x_1 is very low and x_2 is very high THEN y is medium.
 R_{22} : IF x_1 is low and x_2 is very high THEN y is fast.
 R_{23} : IF x_1 is medium and x_2 is very high THEN y is fast.
 R_{24} : IF x_1 is high and x_2 is very high THEN y is very fast.
 R_{25} : IF x_1 is very high and x_2 is very high THEN y is very fast.

Figure 3.9: A non-monotone fuzzy rule base **R3** for a two input FIS-model

Table 3.8: Look up table for fuzzy rule base **R3**

	x_2				
	μ_2^1	μ_2^2	μ_2^3	μ_2^4	μ_2^5
μ_1^1	$b_{1(1,1)} = 10$	$b_{6(1,2)} = 10$	$b_{11(1,3)} = 20$	$b_{16(1,4)} = 20$	$b_{21(1,5)} = 30$
μ_1^2	$b_{2(2,1)} = 20$	$b_{7(2,2)} = 10$	$b_{12(2,3)} = 20$	$b_{17(2,4)} = 30$	$b_{22(2,5)} = 40$
μ_1^3	$b_{3(3,1)} = 20$	$b_{8(3,2)} = 30$	$b_{13(3,3)} = 30$	$b_{18(3,4)} = 40$	$b_{23(3,5)} = 40$
μ_1^4	$b_{4(4,1)} = 50$	$b_{9(4,2)} = 30$	$b_{14(4,3)} = 40$	$b_{19(4,4)} = 30$	$b_{24(4,5)} = 50$
μ_1^5	$b_{5(5,1)} = 30$	$b_{10(5,2)} = 40$	$b_{15(5,3)} = 40$	$b_{20(5,4)} = 40$	$b_{25(5,5)} = 50$

3.6 Monotone Fuzzy Rule Relabeling: Brute Force-Based Approach

In this section, a simple MFRR technique based on a brute-force approach, namely MFRR-BF is explained. MFRR-BF starts by relabeling a fuzzy rule. If a solution is found by relabeling k fuzzy rules, where $k \leq r_t$, the relabeled k fuzzy rules with the minimum loss measure is considered as an optimal solution. More than one optimal solutions could be found during the relabeling process. The search process is terminated after all potential solutions for the k fuzzy rules are evaluated. No search is required for relabeling of more than k fuzzy rules.

To illustrate the MFRR-BF procedure, **R1** (see Section 3.5.1) is considered. For **R1**, the set of $U(B)$ is $\{1, 3, 5\}$. Table 3.9 shows the list of k -combination of r fuzzy rules from **R1**, for $k = 1$, as well as the values in $U(B)$ that could be arranged as k -combination (i.e., the possible consequent value for relabeling the k -combination of r fuzzy rules).

Table 3.9: k -combination of r fuzzy rules for **R1** and its possible relabeled values

k	k -combination of r fuzzy rules from R1	k -permutation of $U(B)$, with repetition allowed.
1	$\{R_1\}, \{R_2\}, \{R_3\}, \{R_4\}, \{R_5\}$	$\{1\}, \{3\}, \{5\}$

Starting with $k = 1$, MFRR-BF attempts to enumerate all 1-combination of r fuzzy rules from **R1**, i.e., $\{R_1\}, \{R_2\}, \{R_3\}, \{R_4\}, \{R_5\}$. Each enumerated 1-combinations of r fuzzy rules are relabeled with the values of $\{1\}, \{3\}, \{5\}$, respectively. The loss incurred is computed using Eq. (3.2) and the relabeled fuzzy rules are evaluated whether it is satisfying **Condition 2 of Theorem 3.1**.

Table 3.10 illustrates the sequence of relabeling using MFRR-BF. As an example, during first iteration, R_1 is relabeled with the value of 1 (i.e., $b_1^{rel} = 1$). The loss incurred is $(b_1 - b_1^{rel})^2 = (1 - 1)^2 = 0$. Relabeling R_1 with value of 1 do not result a monotone fuzzy rule base. In the second iteration, R_1 is then relabeled to value of 3 (i.e., $b_1^{rel} = 3$), with a loss of 4 obtained. Again, a monotone fuzzy rule base is not obtained. A solution is obtained at the 7th iteration, with relabeling of R_3 with value of 1, and with loss of 4. All possible solutions with relabeling a fuzzy rule were obtained.

Table 3.10: MFRR-BF relabeling process for **R1**

Iteration	k -combination of r fuzzy rules from R1	Relabeled value	Satisfy Condition 2?	Loss
1	$\{R_1\}$	1	No	0
2	$\{R_1\}$	3	No	4
3	$\{R_1\}$	5	No	16
4	$\{R_2\}$	1	No	0
5	$\{R_2\}$	3	No	4
6	$\{R_2\}$	5	No	16
7	$\{R_3\}$	1	Yes	4
8	$\{R_3\}$	3	No	0
9	$\{R_3\}$	5	No	4
10	$\{R_4\}$	1	No	0
11	$\{R_4\}$	3	Yes	4
12	$\{R_4\}$	5	Yes	16
13	$\{R_5\}$	1	No	16
14	$\{R_5\}$	3	No	4
15	$\{R_5\}$	5	No	0

In this example, the search process is terminated after all 1-combinations of r fuzzy rules are evaluated. A minimum of 1 fuzzy rules are required to be relabeled for obtaining a

set of monotone fuzzy rule base. There are 2 optimal relabeled fuzzy rule bases obtained for **R1** (as shown in Tables 3.11 and 3.12, respectively), with a minimum loss measure of 4.

Table 3.11: First optimal relabeled fuzzy rule base, for **R1**

Fuzzy rule, R_j	A_j		b_j
	c	σ	
R_1	1	0.5	1
R_2	2	0.5	1
R_3	3	0.5	1
R_4	4	0.5	1
R_5	5	0.5	5

Table 3.12: Second optimal relabeled fuzzy rule base, for **R1**

Fuzzy rule, R_j	A_j		b_j
	c	σ	
R_1	1	0.5	1
R_2	2	0.5	1
R_3	3	0.5	3
R_4	4	0.5	3
R_5	5	0.5	5

The same process could be applied to multi-input fuzzy rule base. **R3** (see section 3.5.2.2) is exemplified. Table 3.13 shows the list of k -combination of r fuzzy rules from **R3**, for $k = 1, 2, 3$, as well as the value in $U(B)$ that could be arranged as k -combination (i.e., the possible consequent value for relabeling the k -combination of r fuzzy rules).

Table 3.13: k -combination of r fuzzy rules for **R3** and its possible relabeled values

k	k -combination of r fuzzy rules from R3 , without repetition	k -permutation of $U(B)$, with repetition allowed.
1	$\{\{R_1\}, \{R_2\}, \{R_3\}, \{R_4\}, \{R_5\}, \{R_6\}, \{R_7\}, \{R_8\}, \{R_9\}, \{R_{10}\}, \{R_{11}\}, \{R_{12}\}, \{R_{13}\}, \{R_{14}\}, \{R_{15}\}, \{R_{16}\}, \{R_{17}\}, \{R_{18}\}, \{R_{19}\}, \{R_{20}\}, \{R_{21}\}, \{R_{22}\}, \{R_{23}\}, \{R_{24}\}, \{R_{25}\}\}$	$\{10\}, \{20\}, \{30\}, \{40\}, \{50\}$
2	$\{\{R_1, R_2\}, \{R_1, R_3\}, \{R_1, R_4\}, \{R_1, R_5\}, \{R_1, R_6\}, \{R_1, R_7\}, \{R_1, R_8\}, \{R_1, R_9\}, \{R_1, R_{10}\}, \{R_1, R_{11}\}, \{R_1, R_{12}\}, \{R_1, R_{13}\}, \{R_1, R_{14}\}, \{R_1, R_{15}\}, \{R_1, R_{16}\}, \{R_1, R_{17}\}, \{R_1, R_{18}\}, \{R_1, R_{19}\}, \{R_1, R_{20}\}, \{R_1, R_{21}\}, \{R_1, R_{22}\}, \{R_1, R_{23}\}, \{R_1, R_{24}\}, \{R_1, R_{25}\}, \{R_2, R_3\}, \{R_2, R_4\}, \{R_2, R_5\}, \{R_2, R_6\}, \{R_2, R_7\}, \{R_2, R_8\}, \{R_2, R_9\}, \dots, \{R_{24}, R_{25}\}\}$	$\{10,10\}, \{10,20\}, \{10,30\}, \{10,40\}, \{10,50\}, \{20,10\}, \{20,20\}, \{20,30\}, \{20,40\}, \{20,50\}, \{30,10\}, \{30,20\}, \{30,30\}, \{30,40\}, \{30,50\}, \{40,10\}, \{40,20\}, \{40,30\}, \{40,40\}, \{40,50\}, \{50,10\}, \{50,20\}, \{50,30\}, \{50,40\}, \{50,50\}$
3	$\{R_1, R_2, R_3\}, \{R_1, R_2, R_4\}, \{R_1, R_2, R_5\}, \{R_1, R_2, R_6\}, \{R_1, R_2, R_7\}, \{R_1, R_2, R_8\}, \{R_1, R_2, R_9\}, \{R_1, R_2, R_{10}\}, \{R_1, R_2, R_{11}\}, \{R_1, R_2, R_{12}\}, \{R_1, R_2, R_{13}\}, \{R_1, R_2, R_{14}\}, \{R_1, R_2, R_{15}\}, \{R_1, R_2, R_{16}\}, \{R_1, R_2, R_{17}\}, \{R_1, R_2, R_{18}\}, \{R_1, R_2, R_{19}\}, \{R_1, R_2, R_{20}\}, \{R_1, R_2, R_{21}\}, \{R_1, R_2, R_{22}\}, \{R_1, R_2, R_{23}\}, \{R_1, R_2, R_{24}\}, \{R_1, R_2, R_{25}\}, \dots, \{R_{23}, R_{24}, R_{25}\}$	$\{10,10,10\}, \{10,10,20\}, \{10,10,30\}, \{10,10,40\}, \{10,10,50\}, \{10,20,10\}, \{10,20,20\}, \{10,20,30\}, \dots, \{50,50,40\}, \{50,50,50\}.$

In the first iteration, R_1 is relabeled with the value of 10 (i.e., $b_1^{rel} = 10$), and the resulting fuzzy rule base is not monotone. In the second iteration, R_1 is relabeled with value of 20 (i.e., $b_1^{rel} = 20$), with a loss of 100 obtained, and again, the resulting fuzzy rule base is not monotone. All 1-combination of r fuzzy rules are evaluated, and all resulting fuzzy rule

bases are not monotone. The search process is proceeded with setting $k = 2$. All 2-combinations of r fuzzy rules are enumerated, as shown in Table 3.13. Each combination of 2 fuzzy rules are relabeled with all possible consequent values for the 2-combination of r fuzzy rules. As an example, for $\{R_1, R_2\}$ is relabeled with $\{10, 10\}$, i.e., R_1 is relabeled with 10 (i.e., $b_1^{rel} = 10$) and R_2 is relabeled with 10 (i.e., $b_2^{rel} = 10$). The total loss is $((b_1 - b_1^{rel})^2 + (b_2 - b_2^{rel})^2) = ((10 - 10)^2 + (20 - 10)^2) = 100$. However, such relabeling does not produce monotone fuzzy rules. This follows with relabeling $\{R_1, R_2\}$ with others possible sets of values (i.e., $\{10, 20\}$, $\{10, 30\}$, $\{10, 40\}$, $\{10, 50\}$, $\{20, 10\}$, $\{20, 20\}$, $\{20, 30\}$, $\{20, 40\}$, $\{20, 50\}$, $\{30, 10\}$, $\{30, 20\}$, $\{30, 30\}$, $\{30, 40\}$, $\{30, 50\}$, $\{40, 10\}$, $\{40, 20\}$, $\{40, 30\}$, $\{40, 40\}$, $\{40, 50\}$, $\{50, 10\}$, $\{50, 20\}$, $\{50, 30\}$, $\{50, 40\}$, $\{50, 50\}$). All possible 2-combination of r fuzzy rules are evaluated.

In this example, the search process is terminated while $k = 3$. A minimum of 3 fuzzy rules are to be relabeled for obtaining at least a set of monotone fuzzy rules. Two different sets of optimal relabeled fuzzy rule bases were obtained for **R3** (as shown in Table 3.14 and Table 3.15 respectively), with a minimum loss measure of 600. Table 3.14 shows the first set of relabeled fuzzy rules for **R3**, in which fuzzy rule R_2 , R_4 and R_{19} are relabeled with values of 10, 30 and 40 respectively. Table 3.15 shows the second set of relabeled fuzzy rules for **R3**, with fuzzy rule R_4 , R_7 and R_{19} are relabeled with values of 30, 20 and 40, respectively. The two set of relabeled fuzzy rules are the potential solutions.

Table 3.14: First optimal relabeled fuzzy rule base, for **R3**

	x_2				
	μ_2^1	μ_2^2	μ_2^3	μ_2^4	μ_2^5
μ_1^1	$b_{1(1,1)} = 10$	$b_{6(1,2)} = 10$	$b_{11(1,3)} = 20$	$b_{16(1,4)} = 20$	$b_{21(1,5)} = 30$
μ_1^2	$b_{2(2,1)}^{rel} = 10$	$b_{7(2,2)} = 10$	$b_{12(2,3)} = 20$	$b_{17(2,4)} = 30$	$b_{22(2,5)} = 40$
μ_1^3	$b_{3(3,1)} = 20$	$b_{8(3,2)} = 30$	$b_{13(3,3)} = 30$	$b_{18(3,4)} = 40$	$b_{23(3,5)} = 40$
μ_1^4	$b_{4(4,1)}^{rel} = 30$	$b_{9(4,2)} = 30$	$b_{14(4,3)} = 40$	$b_{19(4,4)}^{rel} = 40$	$b_{24(4,5)} = 50$
μ_1^5	$b_{5(5,1)} = 30$	$b_{10(5,2)} = 40$	$b_{15(5,3)} = 40$	$b_{20(5,4)} = 40$	$b_{25(5,5)} = 50$

Table 3.15: Second optimal relabeled fuzzy rule base, for **R3**

	x_2				
	μ_2^1	μ_2^2	μ_2^3	μ_2^4	μ_2^5
μ_1^1	$b_{1(1,1)} = 10$	$b_{6(1,2)} = 10$	$b_{11(1,3)} = 20$	$b_{16(1,4)} = 20$	$b_{21(1,5)} = 30$
μ_1^2	$b_{2(2,1)} = 20$	$b_{7(2,2)}^{rel} = 20$	$b_{12(2,3)} = 20$	$b_{17(2,4)} = 30$	$b_{22(2,5)} = 40$
μ_1^3	$b_{3(3,1)} = 20$	$b_{8(3,2)} = 30$	$b_{13(3,3)} = 30$	$b_{18(3,4)} = 40$	$b_{23(3,5)} = 40$
μ_1^4	$b_{4(4,1)}^{rel} = 30$	$b_{9(4,2)} = 30$	$b_{14(4,3)} = 40$	$b_{19(4,4)}^{rel} = 40$	$b_{24(4,5)} = 50$
μ_1^5	$b_{5(5,1)} = 30$	$b_{10(5,2)} = 40$	$b_{15(5,3)} = 40$	$b_{20(5,4)} = 40$	$b_{25(5,5)} = 50$

3.7 Monotone Fuzzy Rule Relabeling: Brute Force with Backtracking Approach (MFRR-BFBT)

In this section, MFRR-BFBT is devised. MFRR-BFBT reduces the solution search space systematically. Only the partial solutions that fulfil **Condition 2** are considered for rule relabeling. Thus, this reduces the computational time. Definitions 3.4 and 3.5 are considered.

Definition 3.4. A total of k -subsets of **R** is generated in a lexicographic order (Pemmaraju & Skiena, 2003), in such a way that the remaining fuzzy rules (i.e., $R_{remaining}$) that satisfy **Condition 2** are denoted as S_k , where $1 \leq k \leq r$. A combination of fuzzy rules in S_k to be relabeled is denoted as $S_{k,z}$, where $z = 1, 2, \dots, num_{S_k}$. $R_{remaining,z}$ is the remaining fuzzy

rules corresponding to $S_{k,z}$. Note that $R_J = R_{remaining,z} \cup S_{k,z}$ is always true.

The cardinality of S_k (i.e., num_{S_k}) is given by Eq. (3.3).

$$num_{S_k} = |S_k| \quad (3.3)$$

Each $S_{k,z}$ is a combination of fuzzy rules to be relabeled. It can be represented as a k -depth tree, as depicted in Figure 3.10. The k -depth tree represents the solutions with k variables. The partial solution is defined as follows.

Definition 3.5. A partial solution (i.e., $R_{partial}$) of $S_{k,z}$ is a vector with variables fewer than k , obtained by MFRR-BFBT, in such a way that the union of $R_{partial}$ and $R_{remaining,z}$ satisfies **Condition 2**.

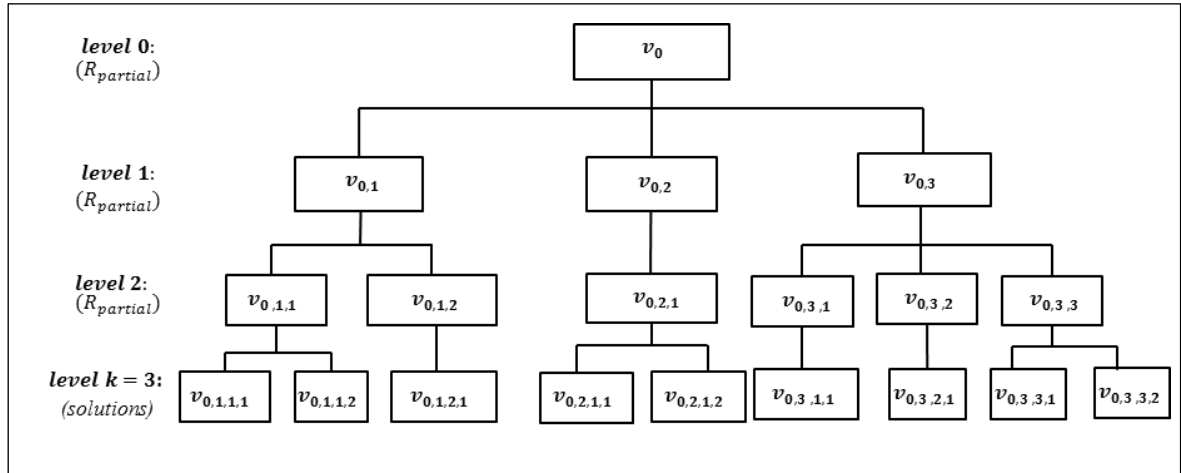


Figure 3.10: A k -depth tree of $S_{k,z}$, with $k = 3$

Instead of using all the values in $U(B)$ to relabel $b_{J(j_1,j_2,\dots,j_n)}$ in $S_{k,z}$, only the values that fall within the lower and upper limits of a particular fuzzy rule are considered. For fuzzy rule $R_{(i_1,i_2,\dots,i_n)}$, its lower and upper limits of $b_{(i_1,i_2,\dots,i_n)}$, are obtained using Eq. (3.4) and Eq. (3.5), as follows.

$$\overline{b_{(i_1, i_2, \dots, i_n)}} = \begin{cases} \min(R_{\text{partial}} \cup R_{\text{remaining}}) & \text{if } b_{j_1 \leq i_1, j_2 \leq i_2, \dots, j_n \leq i_n} \in R_{\text{partial}} \cup R_{\text{remaining}} \text{ is } \emptyset \\ \max(b_{j_1 \leq i_1, j_2 \leq i_2, \dots, j_n \leq i_n} \in (R_{\text{partial}} \cup R_{\text{remaining}})) & \text{Otherwise} \end{cases} \quad (3.4)$$

$$\overline{b_{(i_1, i_2, \dots, i_n)}} = \begin{cases} \max(R_{\text{partial}} \cup R_{\text{remaining}}) & \text{if } b_{j_1 \geq i_1, j_2 \geq i_2, \dots, j_n \geq i_n} \in R_{\text{partial}} \cup R_{\text{remaining}} \text{ is } \emptyset \\ \min(b_{j_1 \geq i_1, j_2 \geq i_2, \dots, j_n \geq i_n} \in R_{\text{partial}} \cup R_{\text{remaining}}) & \text{Otherwise} \end{cases} \quad (3.5)$$

Definition 3.6. For a given fuzzy rule, i.e., $R_{(i_1, i_2, \dots, i_n)}$, the candidate for relabeling $b_{(i_1, i_2, \dots, i_n)}$ is denoted as b_{rel} , and it is obtained using Eq. (3.6):

$$b_{\text{rel}} = \{b_{(j_1, j_2, \dots, j_n)} \in U(B) \mid \underline{b_{(i_1, i_2, \dots, i_n)}} \leq b_{(j_1, j_2, \dots, j_n)} \leq \overline{b_{(i_1, i_2, \dots, i_n)}}\} \quad (3.6)$$

It is useful to consider the fuzzy rules relabeling as a process of traversing a tree (Figure 3.10). Let l denotes the level of the tree. l represents the ordinal label of fuzzy rules to be relabeled. The relabeling process is initialized with an empty vector as a partial solution (i.e., R_{partial}). While $l = 1$, the consequent part of the first fuzzy rule (i.e., $b_{J(j_1, j_2, \dots, j_n)}$ with the smallest index J) in $S_{k,z}$ is relabeled with the candidate values in $b_{\text{rel}, l=1}$. The values in $b_{\text{rel}, l=1}$ are the children of R_{partial} at level $l = 0$. In general, the values in $b_{\text{rel}, l}$ are the children of R_{partial} at level $l - 1$. Then, extend R_{partial} in the depth-first order, i.e., traverse as deep as the $k - \text{th}$ level to search for possible solutions, before backing up to explore other parts of the tree. The solution (i.e., $b_{J(j_1, j_2, \dots, j_n)}^{\text{rel}}$) is obtained when $l = k$.

Note that for a set of $R_{\text{remaining}, z}$ corresponding to $S_{k,z}$, a relabeled $S_{k,z}$ always exists in such a way that the union of $R_{\text{remaining}, z}$ and relabeled $S_{k,z}$ produces a set of fuzzy rules

that satisfy **Condition 2**.

3.7.1 Procedure of MFRR-BFBT

The MFRR-BFBT procedure is summarized in Figure 3.11. The details are explained, as follows.

- (a) Start
- (b) Input set R_J (Definition 3.3).
- (c) Set the number of fuzzy rules to be relabeled to 1, i.e., $k = 1$.
- (d) Generate S_k (Definition 3.4).
- (e) If S_k is an empty set, go to step (f). Otherwise, go to step (g).
- (f) Set $k = k + 1$, i.e., increase the number of fuzzy rules to be relabeled by 1. Then go back to step (d).
- (g) Obtain cardinality of S_k , i.e., $num_{S_k} = |S_k|$.
- (h) Set $z = 1$ to search for a solution in $S_{k,z=1}$.
- (i) Relabel the fuzzy rules in $S_{k,z}$ according to the depth-first order.
 - i(1). For each $b_{J(j_1, j_2, \dots, j_n)}^{rel}$ obtained, compute *loss* using (3.2).
 - i(2). Store $b_{J(j_1, j_2, \dots, j_n)}^{rel}$ with the smallest value of *loss*.
- (j) Update $z = z + 1$.
- (k) If $z \leq num_{S_k}$, go to step (i). Otherwise, go to step (l).
- (l) Obtain the optimal relabeled fuzzy rules (i.e., $b_{J(j_1, j_2, \dots, j_n)}^{rel}$) that fulfil the monotonicity property, with the minimum number of relabeled fuzzy rules and the minimum loss measure.
- (m) End.

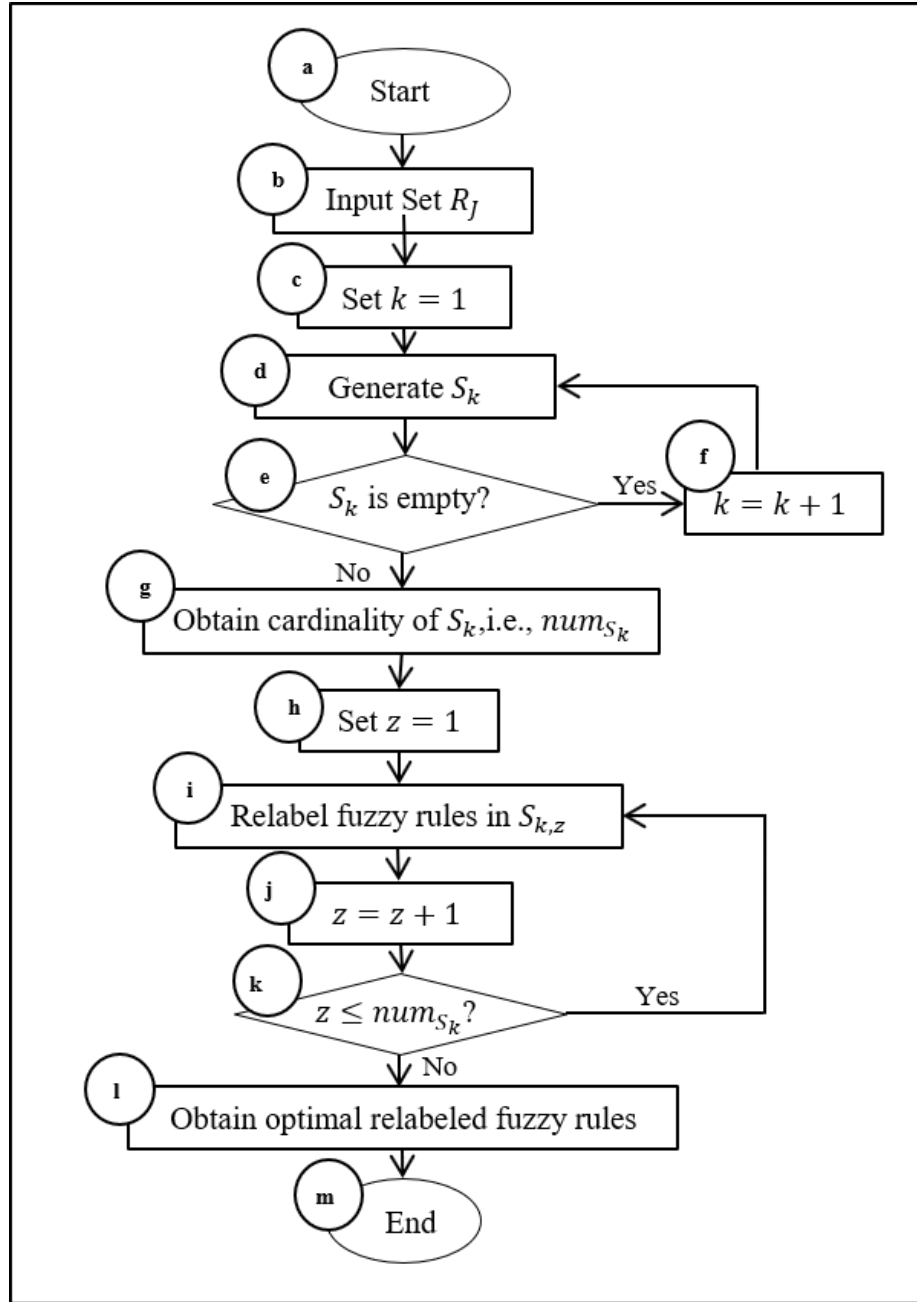


Figure 3.11: The procedure of the MFRR-BFBT method

Theorem 3.2. The number of relabeled fuzzy rule sets using MFRR-BFBT is smaller than those with MFRR-BF.

Proof. The number of relabeled fuzzy rule sets with MFRR-BF is $\sum_{q=1}^k C(r_t, q) (r_u^q) > 0$, where $C(r_t, q)$ is the number of q -combinations given r_t number of fuzzy rules, r_t is the total number of fuzzy rules collected from human expert, and r_u is the number of unique fuzzy singletons. In the worst-case scenario, the maximum fuzzy rule sets to be relabeled with MFRR-BFBT is $C(r_t, k)(r_u^k) > 0$. Using the lower and upper bounds as the values to be relabeled, the number of relabeled fuzzy rule sets is always smaller than $C(r_t, k)(r_u^k)$. Therefore, the number of relabeled fuzzy rule sets can always be reduced by $\sum_{q=1}^k C(r_t, q) (r_u^q) - C(r_t, k)(r_u^k) > 0$.

In essence, MFRR searches for all solutions in which the corresponding partial solutions satisfy **Condition 2** of **Theorem 3.1**. The solutions having partial solutions that do not satisfy **Condition 2** of **Theorem 3.1** are rejected directly, and thus the search space was reduced. Meanwhile, the reduced search space consists of all solution to the original problem instance that contain the partial solution that satisfy **Condition 2** of **Theorem 3.1**. Therefore, MFRR-BFBT still produces the same solutions as those from MFRR-BF, but with shorter computation time. This process is further illustrated using a simulated example, as presented in the next section.

3.7.2 Simulated Example

To illustrate the process of MFRR-BFBT, **R3** (see section 3.5.2.2) is exemplified. The set of $U(B)$ obtained is $\{10, 20, 30, 40, 50\}$. By setting $k=1$, MFRR-BFBT generates the 1-subset of **R3** in the lexicographic order to search for $S_{k=1}$. However, $S_{k=1}$ is found to be an

empty set. Therefore, k is increased by 1, and the MFRR-BFBT continues to generate k -subset of **R3** until S_k is no longer an empty set. In this example, the remaining fuzzy rules, i.e., $R_{remaining}$ fulfils **Condition 2** when $k = 3$, with $S_k = \{\{R_2, R_4, R_{19}\}, \{R_4, R_7, R_{19}\}\}$. The cardinality of S_k is $num_{S_k} = 2$. The monotonicity property of **R3** can be achieved by relabeling a minimum of 3 fuzzy rules. Instead of relabeling all combinations of 3 fuzzy rules in **R3**, MFRR-BFBT is able to identify a partial set of all combinations of 3 fuzzy rules to be relabeled that can lead to the formation of a monotone **R3**, as listed in S_k .

For $S_{k,z=1} = \{R_2, R_4, R_{19}\}$, the corresponding $R_{remaining}$ is illustrated in Table 3.16. The relabeling procedure starts with a null vector, v_0 , as the partial solution, i.e., $R_{partial}$, as represented by the shaded region in Table 3.16. The search process is conducted in a depth-first search manner.

Table 3.16: $R_{remaining}$ corresponds to $S_{k,z=1} = \{R_2, R_4, R_{19}\}$

	x_2				
	μ_2^1	μ_2^2	μ_2^3	μ_2^4	μ_2^5
μ_1^1	$b_{1(1,1)} = 10$	$b_{6(1,2)} = 10$	$b_{11(1,3)} = 20$	$b_{16(1,4)} = 20$	$b_{21(1,5)} = 30$
μ_1^2		$b_{7(2,2)} = 10$	$b_{12(2,3)} = 20$	$b_{17(2,4)} = 30$	$b_{22(2,5)} = 40$
μ_1^3	$b_{3(3,1)} = 20$	$b_{8(3,2)} = 30$	$b_{13(3,3)} = 30$	$b_{18(3,4)} = 40$	$b_{23(3,5)} = 40$
μ_1^4		$b_{9(4,2)} = 30$	$b_{14(4,3)} = 40$		$b_{24(4,5)} = 50$
μ_1^5	$b_{5(5,1)} = 30$	$b_{10(5,2)} = 40$	$b_{15(5,3)} = 40$	$b_{20(5,4)} = 40$	$b_{25(5,5)} = 50$

For $S_{k,z=1} = \{R_2, R_4, R_{19}\}$, the first fuzzy rule to be relabeled is R_2 . The lower limit (i.e., $\underline{b_2}$) for b_2 is 10. The upper limit (i.e., $\overline{b_2}$) for b_2 is 10 too. As such, $b_{rel} = \{10\}$. Therefore, b_2 is relabeled to 10, which is denoted as b_2^{rel} . $R_{partial}$ is expanded with b_2^{rel} , as shown in Table 3.17.

Table 3.17: Expanded $R_{partial}$ with b_2^{rel} and the corresponding $R_{remaining}$

	x_2				
	μ_2^1	μ_2^2	μ_2^3	μ_2^4	μ_2^5
μ_1^1	$b_{1(1,1)} = 10$	$b_{6(1,2)} = 10$	$b_{11(1,3)} = 20$	$b_{16(1,4)} = 20$	$b_{21(1,5)} = 30$
μ_1^2	$b_{2(2,1)}^{rel} = 10$	$b_{7(2,2)} = 10$	$b_{12(2,3)} = 20$	$b_{17(2,4)} = 30$	$b_{22(2,5)} = 40$
μ_1^3	$b_{3(3,1)} = 20$	$b_{8(3,2)} = 30$	$b_{13(3,3)} = 30$	$b_{18(3,4)} = 40$	$b_{23(3,5)} = 40$
μ_1^4		$b_{9(4,2)} = 30$	$b_{14(4,3)} = 40$		$b_{24(4,5)} = 50$
μ_1^5	$b_{5(5,1)} = 30$	$b_{10(5,2)} = 40$	$b_{15(5,3)} = 40$	$b_{20(5,4)} = 40$	$b_{25(5,5)} = 50$

The process is repeated for R_4 and R_{19} . The relabeled fuzzy rules for $S_k = \{R_2, R_4, R_{19}\}$, are shown in Table 3.18. The loss measure for the relabeling procedure is computed. As an example, the loss measure for $S_{k,z=1} = \{R_2, R_4, R_{19}\}$ is computed using Eq. (3.2), i.e., $loss = (20 - 10)^2 + (50 - 30)^2 + (30 - 40)^2 = 600$. The same process is repeated for other combinations in S_k , i.e., $\{R_4, R_7, R_{19}\}$. Table 3.19 shows the optimal relabeled fuzzy rules for $S_k = \{R_4, R_7, R_{19}\}$. Noted that the optimal solutions obtained using MFRR-BFBT (see Table 3.18 and Table 3.19) are the same as those obtained using MFRR-BF (see Table 3.14 and Table 3.15).

Table 3.18: Optimal relabeled fuzzy rules for $S_{k,z=1} = \{R_2, R_4, R_{19}\}$

	x_2				
	μ_2^1	μ_2^2	μ_2^3	μ_2^4	μ_2^5
μ_1^1	$b_{1(1,1)} = 10$	$b_{6(1,2)} = 10$	$b_{11(1,3)} = 20$	$b_{16(1,4)} = 20$	$b_{21(1,5)} = 30$
μ_1^2	$b_{2(2,1)}^{rel} = 10$	$b_{7(2,2)} = 10$	$b_{12(2,3)} = 20$	$b_{17(2,4)} = 30$	$b_{22(2,5)} = 40$
μ_1^3	$b_{3(3,1)} = 20$	$b_{8(3,2)} = 30$	$b_{13(3,3)} = 30$	$b_{18(3,4)} = 40$	$b_{23(3,5)} = 40$
μ_1^4	$b_{4(4,1)}^{rel} = 30$	$b_{9(4,2)} = 30$	$b_{14(4,3)} = 40$	$b_{19(4,4)}^{rel} = 40$	$b_{24(4,5)} = 50$
μ_1^5	$b_{5(5,1)} = 30$	$b_{10(5,2)} = 40$	$b_{15(5,3)} = 40$	$b_{20(5,4)} = 40$	$b_{25(5,5)} = 50$

Table 3.19: Optimal relabeled fuzzy rules for $S_{k,z=2} = \{R_4, R_7, R_{19}\}$

	x_2				
	μ_2^1	μ_2^2	μ_2^3	μ_2^4	μ_2^5
x_1 μ_1^1	$b_{1(1,1)} = 10$	$b_{6(1,2)} = 10$	$b_{11(1,3)} = 20$	$b_{16(1,4)} = 20$	$b_{21(1,5)} = 30$
μ_1^2	$b_{2(2,1)} = 20$	$b_{7(2,2)}^{rel} = 20$	$b_{12(2,3)} = 20$	$b_{17(2,4)} = 30$	$b_{22(2,5)} = 40$
μ_1^3	$b_{3(3,1)} = 20$	$b_{8(3,2)} = 30$	$b_{13(3,3)} = 30$	$b_{18(3,4)} = 40$	$b_{23(3,5)} = 40$
μ_1^4	$b_{4(4,1)}^{rel} = 30$	$b_{9(4,2)} = 30$	$b_{14(4,3)} = 40$	$b_{19(4,4)}^{rel} = 40$	$b_{24(4,5)} = 50$
μ_1^5	$b_{5(5,1)} = 30$	$b_{10(5,2)} = 40$	$b_{15(5,3)} = 40$	$b_{20(5,4)} = 40$	$b_{25(5,5)} = 50$

3.7.3 Computational Time Analysis

Table 3.20 summarized the computational times for both MFRR-BF and MFRR-BFBT, using **R1** and **R3**. A computer with Intel (R) Core (TM) i5-3210M CPU @2.50GHz, 6.00 GB of RAM and Microsoft Window 10 Single language was used for the experiment. With Matlab R2012a, both methods, i.e., MFRR-BF and MFRR-BFBT were able to produce the same optimal solutions.

Table 3.20: Computational times of BFRR and MFRR, for **R3**

Information	R1		R3	
	MFRR-BF	MFRR-BFBT	MFRR-BF	MFRR-BFBT
Computational time, $t_{MFRR-BF}, t_{MFRR-BFBT}$ (s)	0.056	0.0017	18.748	0.185
Number of iterations	15	3	295125	4
Number of optimal solution	2	2	2	2
Number of rule changed	1	1	3	3
Minimum loss value	4	4	600	600
Reduced computational time in percentage, $(\frac{t_{MFRR-BF} - t_{MFRR-BFBT}}{t_{MFRR-BF}}) \times 100$	96.96%		99.01%	

For **R1**, the computational time for MFRR-BF and MFRR-BFBT are 0.056s and 0.0017s, respectively. Using MFRR-BF, number of iteration is $C(5,1)(3^1) = 15$. Using MFRR-BFBT, same results could be obtained with 3 iterations, with a reduced computation time of 96.96%. The same observation could be made with **R3** with computation time reduction of 99.01%.

3.8 Summary

In this chapter, MFRR was introduced. Two MFRR, i.e., MFRR-BF and MFRR-BFBT were devised and promising results were obtained. In the next chapter, graphical representation of fuzzy rules will be presented to give a better understanding and visualization of monotonicity relationship of a fuzzy rule base.

CHAPTER 4

A GRAPH REPRESENTATION OF MONOTONE FUZZY RULE BASE

4.1 Introduction

In Chapter 3, the general idea of monotone fuzzy rule relabeling (MFRR) was discussed. MFRR aims to restore the monotonicity property of a set of non-monotone fuzzy rules, in an optimal way. As mentioned in previous chapter, the monotone fuzzy rule base is an essential building block for a monotone fuzzy inference model. To give a better understanding and visualization on the monotonicity relationship of a fuzzy rule base, the use of graph theory to represent the non-monotone relationship of fuzzy rules with a binary relationship is proposed in this chapter. Such representation is useful for providing a formal and systematic way to modelling and analyzing non-monotone fuzzy rules in a fuzzy rule base. The *directed acyclic graph* (Aho, Garey, & Ullman, 1972) is resulted from the representation. It can be reduced to a unique and optimal minimum equivalence subgraph by using a *transitive reduction method*. Furthermore, such graph representation of fuzzy rules allows the maximum number of fuzzy rules, and their combination(s), that are monotone to each other, to be identified. Determination of the fuzzy rules combination(s) constitute to the well-known maximum independent set problems (Pemmaraju & Skiena, 2003). In this chapter, the network flow approach (Ford & Fulkerson, 1962) is proposed to solve the maximum independent set problems.

In Section 4.2, the representation of a non-monotone fuzzy rule base using a graph view is introduced and explained. Maximum independent set problem as well as the

proposed approach are presented in Section 4.3. The results obtained is further analyzed and discussed in Section 4.4. The concluding remarks for Chapter 4 is given in Section 4.5.

4.2 Representation of Non-Monotone Relationship of Fuzzy Rules using Graph Theory

To ease the explanation, two simulated examples (i.e., **R2** and **R3** from Chapter 3) are used for illustration.

4.2.1 Graph Representation of Fuzzy Rules

Definition 4.1. The non-monotone relationship representation of **R** is $G(\mathbf{R})$.

Let $G(\mathbf{R})$ be a finite directed acyclic graph with a vertex set, $V(\mathbf{R}) = \{R_1, R_2, R_3, \dots, R_r \in \mathbf{R}\}$, and an edge set, $E(\mathbf{R}) = \{e_1, e_2, e_3, \dots, e_g\}$, i.e., $G(\mathbf{R}) = (V(\mathbf{R}), E(\mathbf{R}))$. An edge is represented in the form of $e_z = (R_{J,1}(j_{1,1}, j_{2,1}, \dots, j_{n,1}), R_{J,2}(j_{1,2}, j_{2,2}, \dots, j_{n,2})) \in E$, for $z = 1, 2, 3, \dots, g$ and $R_{J,1}, R_{J,2} \in V(\mathbf{R})$, with the direction from $R_{J,1}$ to $R_{J,2}$, if and only if, $b_{J,1}(j_{1,1}, j_{2,1}, \dots, j_{n,1}) > b_{J,2}(j_{1,2}, j_{2,2}, \dots, j_{n,2})$, in which $j_{1,1} \not\geq j_{1,2}$, $j_{2,1} \not\geq j_{2,2}$, ..., and $j_{n,1} \not\geq j_{n,2}$, and $J, 1(j_{1,1}, j_{2,1}, \dots, j_{n,1}) \neq J, 2(j_{1,2}, j_{2,2}, \dots, j_{n,2})$, are true.

4.1.1. $V(\mathbf{R})$ consists of both connected vertex set $C(\mathbf{R})$ and disconnected vertex set $D(\mathbf{R})$, such that $C(\mathbf{R}) \cup D(\mathbf{R}) = V(\mathbf{R})$, and $C(\mathbf{R}) \cap D(\mathbf{R}) = \emptyset$.

- Cardinality of $C(\mathbf{R})$ is denoted as $|C(\mathbf{R})|$, which is the number of vertex in $C(\mathbf{R})$.

4.1.2. If **R** is monotone,

- $G(\mathbf{R})$ is edgeless,
- $E(\mathbf{R})$ is empty set, i.e., $g = 0$.

- Connected vertex set $C(\mathbf{R})$ is empty, and therefore
- $|C(\mathbf{R})| = 0$.

$\mathbf{R2}$ can be represented by graph, as shown in Figure 4.1. Note that $V(\mathbf{R2}) = \{R_1, R_2, R_3, \dots, R_{25}\}$ and $E(\mathbf{R2}) = \{\}$. It can be written as $G(\mathbf{R2}) = (\{R_1, R_2, R_3, \dots, R_{25}\}, \{\})$. $\mathbf{R2}$ is monotone, therefore, $E(\mathbf{R2})$ is an empty set, i.e., $G(\mathbf{R2})$ is edgeless. $C(\mathbf{R2})$ is empty too, therefore $|C(\mathbf{R2})| = 0$.

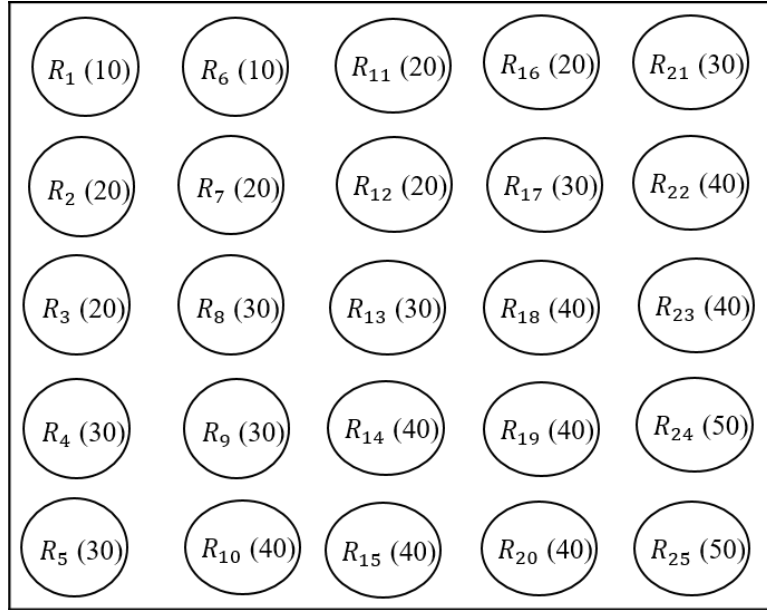
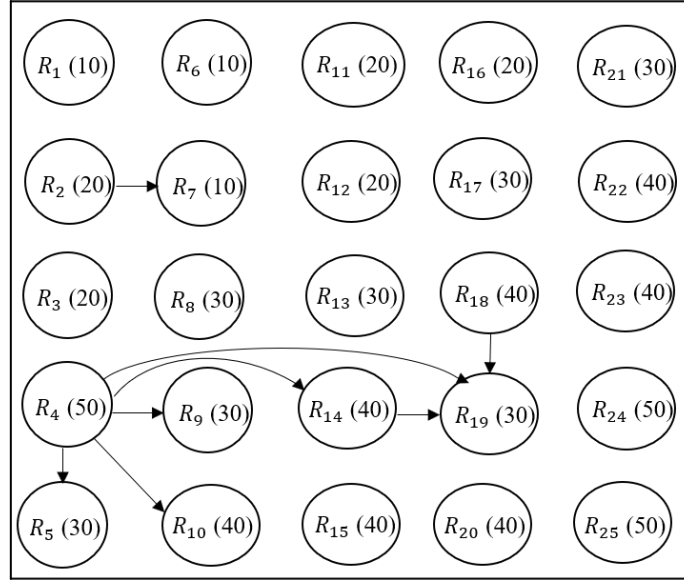


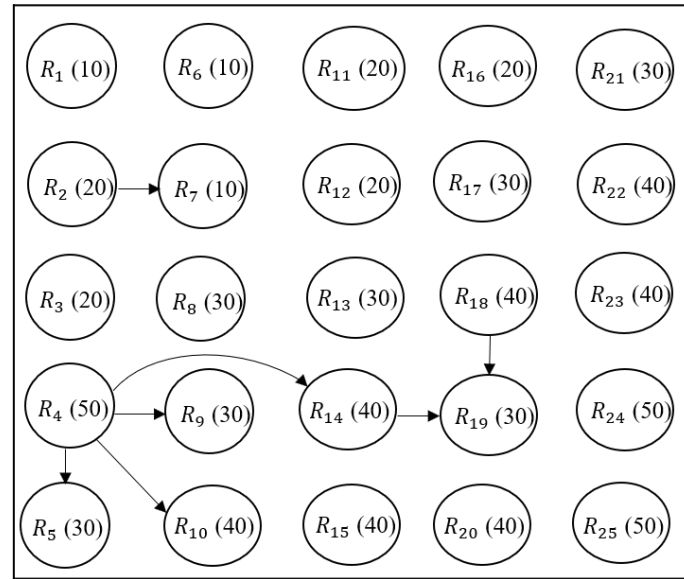
Figure 4.1: A graph representation for $\mathbf{R2}$, i.e., $G(\mathbf{R2})$

$\mathbf{R3}$ can also be represented by graph, as shown in Figures 4.2(a) and 4.2(b). The graph representation in Figure 4.2(a) is written as $G(\mathbf{R3}) = (\{R_1, R_2, R_3, \dots, R_{25}\}, \{(R_2, R_7), (R_4, R_5), (R_4, R_9), (R_4, R_{10}), (R_4, R_{14}), (R_4, R_{19}), (R_{14}, R_{19}), (R_{18}, R_{19})\})$. $\mathbf{R3}$ is non-monotone, the non-monotonicity relationship between the fuzzy rules are represented by an edge set, i.e., $E(\mathbf{R3}) = \{(R_2, R_7), (R_4, R_5), (R_4, R_9), (R_4, R_{10}), (R_4, R_{14}), (R_4, R_{19}), (R_{14}, R_{19}), (R_{18}, R_{19})\}$. The connected vertex set $C(\mathbf{R3}) = \{R_2, R_4, R_5, R_7, R_9, R_{10},$

$R_{14}, R_{18}, R_{19}\}$, and therefore $|C(\mathbf{R3})| = 9$. The disconnected vertex set $D(\mathbf{R3}) = \{R_1, R_3, R_6, R_8, R_{11}, R_{12}, R_{13}, R_{15}, R_{16}, R_{17}, R_{20}, R_{21}, R_{22}, R_{23}, R_{24}, R_{25}\}$, and $|D(\mathbf{R3})| = 16$.



(a)



(b)

Figure 4.2: Graph representation for $\mathbf{R3}$, (a) before transitivity reduction, i.e., $G(\mathbf{R3})$; (b) after its transitivity reduction, i.e., $G'(\mathbf{R3})$

Definition 4.2. $G'(\mathbf{R}) = (V(\mathbf{R}), E'(\mathbf{R}))$ is the minimum equivalence subgraph of $G(\mathbf{R}) = (V(\mathbf{R}), E(\mathbf{R}))$, if the following two conditions are satisfied.

- (i) $\forall z, e_z = (R_{J,1}(j_{1,1}, j_{2,1}, \dots, j_{n,1}), R_{J,2}(j_{1,2}, j_{2,2}, \dots, j_{n,2}))$, there is a directed path from vertex $R_{J,1}(j_{1,1}, j_{2,1}, \dots, j_{n,1})$ to vertex $R_{J,2}(j_{1,2}, j_{2,2}, \dots, j_{n,2})$ in $G'(\mathbf{R})$, if and only if there is a direct path from $R_{J,1}(j_{1,1}, j_{2,1}, \dots, j_{n,1})$ to vertex $R_{J,2}(j_{1,2}, j_{2,2}, \dots, j_{n,2})$ in $G(\mathbf{R})$.
- (ii) There is no other possible graph with fewer edges than $G'(\mathbf{R})$ that satisfies Condition (i), i.e., $E'(\mathbf{R}) \subset E(\mathbf{R})$.

Definition 4.3. The non-monotone relationship graph of $\mathbf{R}$ consists of $C(\mathbf{R})$ and $E'(\mathbf{R})$, and is denoted as $G_{NM}(\mathbf{R}) = (C(\mathbf{R}), E'(\mathbf{R}))$.

4.3.1. $G_{NM}(\mathbf{R})$ is a subset of $G'(\mathbf{R})$.

Remark 4.1. Since $G(\mathbf{R})$ is a finite directed acyclic graph, there is only a unique $G'(\mathbf{R})$ (Aho et al., 1972). The process of obtaining $G'(\mathbf{R})$ from $G(\mathbf{R})$ is known as *transitive reduction* (Aho et al., 1972).

Again, with the same example, $G(\mathbf{R3})$ can be reduced to $G'(\mathbf{R3})$, as shown in Figure 4.2(b), where $G'(\mathbf{R3}) = (\{R_1, R_2, R_3, \dots, R_{25}\}, \{(R_2, R_7), (R_4, R_5), (R_4, R_9), (R_4, R_{10}), (R_4, R_{14}), (R_{14}, R_{19}), (R_{18}, R_{19})\})$. Figure 4.3 shows $G_{NM}(\mathbf{R3})$, which consists of $C(\mathbf{R3})$ and $E'(\mathbf{R3})$. The cardinality of $C(\mathbf{R3})$, i.e., $|C(\mathbf{R3})| = 9$, implies that there are 9 fuzzy rules in $\mathbf{R3}$ that form non-monotone fuzzy rule pairs.

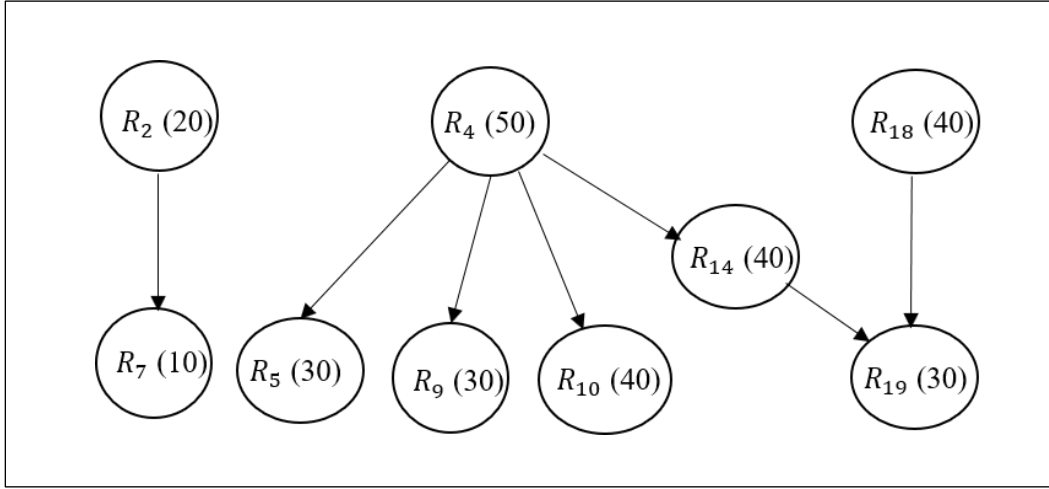


Figure 4.3: $G_{NM}(\mathbf{R3})$

4.3 Determining Maximum Independent Set

Definition 4.4. The maximum independent set of $G_{NM}(\mathbf{R})$.

The maximum independent set, $MIS(\mathbf{R})$, is a subset of $\mathcal{C}(\mathbf{R})$, i.e., $MIS(\mathbf{R}) \subseteq \mathcal{C}(\mathbf{R})$, such that for every vertex in $MIS(\mathbf{R})$, no connecting edges exist (Pemmaraju & Skiena, 2003), and it has the maximum cardinality, i.e., maximum $|MIS(\mathbf{R})|$.

4.4.1. To ease the explanation, hereafter $|MIS(\mathbf{R})|$ is denoted as $\alpha(G_{NM})$.

4.4.2. $MIS(\mathbf{R}) \cup D(\mathbf{R})$ has the largest number of fuzzy rules that are monotone with each other.

4.4.3. More than an $MIS(\mathbf{R})$ for $G_{NM}(\mathbf{R})$ can exist.

Determining an $MIS(\mathbf{R})$ for $G_{NM}(\mathbf{R})$ allows the identification of a maximum subset of fuzzy rules that is internally monotone (Rademaker et al., 2012). From Figure 4.3, an example of maximum independent set that can be obtained from $G_{NM}(\mathbf{R3})$ is $MIS(\mathbf{R3}) =$

$\{R_5, R_7, R_9, R_{10}, R_{14}, R_{18}\}$. No additional vertex can be added to $MIS(\mathbf{R3})$ to make it edgeless.

In general, determining the maximum independent set(s) in an arbitrary graph through a computation approach is an NP-hard problem (Garey & Johnson, 1979), with an exponentially growth of computation time with respect to the problem size. NP-hard (known as Non-deterministic Polynomial-time hardness) problem is a class of decision problems which are at least as hard as the hardest problems in NP, where NP (known as Non-deterministic Polynomial-time) is a class of computational decision problems for which a given solution can be verified as a solution in polynomial time by a deterministic Turing machine (Garey & Johnson, 1979).

However, determining $MIS(\mathbf{R})$ can be conducted in a polynomial growth of computation time by exploiting the associated transitive orientation of $G_{NM}(\mathbf{R})$ (Rademaker et al., 2012), because $G_{NM}(\mathbf{R})$ is a comparability graph (Möhring, 1985) as the non-monotone relationship constitute a strict partial order relationship. Therefore, $MIS(\mathbf{R})$ can be effectively determined by solving a minimum flow problem on a transportation network constructed from $G_{NM}(\mathbf{R})$ in $O(|V(\mathbf{R})|^3)$ time (Möhring, 1985; Rademaker et al., 2012), which is also denoted as the *network flow approach*. The network flow approach is summarized in the following sub-section.

4.3.1 Network Flow Approach

Transformation of $G_{NM}(\mathbf{R}) = (C(\mathbf{R}), E'(\mathbf{R}))$ to its corresponding transportation network, denoted by $G_{NM}''(\mathbf{R}) = (C''(\mathbf{R}), E''(\mathbf{R}))$, is necessary for the network flow approach. The following steps outline the network flow procedure (Rademaker et al., 2012).

Step 1: Adding two new vertices, i.e., the source and sink nodes

Two new vertices, i.e., s and t , are added, where s is the source node and t is the sink node. Both s and t have an *in degree* and an *out degree* of zero, respectively. With the example in Figure 4.3, i.e., $G_{NM}(\mathbf{R3})$, s and t are added, as depicted in Figure 4.4.

Step 2: Performing vertex splitting

In this step, a vertex splitting procedure is adopted to split the vertices in $G_{NM}(\mathbf{R})$. Each vertex, i.e., $R_j \in C(\mathbf{R})$, is split into two vertices, i.e., a lower vertex $R_{\underline{j}}$ and an upper vertex $R_{\overline{j}}$, with an edge directed from $R_{\underline{j}}$ to $R_{\overline{j}}$. Therefore, the resulting transportation network, i.e., $G_{NM}''(\mathbf{R})$ has a new vertex set, denoted as $C''(\mathbf{R}) = \bigcup_{R_j \in C(\mathbf{R})} \{R_{\underline{j}}, R_{\overline{j}}\} \cup \{s, t\}$. In Figure 4.4, for example, R_2 is split to $R_{\underline{2}}$ and $R_{\overline{2}}$.

Step 3: Adding new edges

The transportation network, $G_{NM}''(\mathbf{R})$ has the edge set $E''(\mathbf{R})$ that contains $E'(\mathbf{R})$, with additional edges $(R_{\underline{j}}, R_{\overline{j}})$ for all $R_j \in C(\mathbf{R})$, and additional edges $(R_{\overline{j,1}}, R_{\underline{j,2}})$ for all $(R_{j,1}, R_{j,2}) \in E'(\mathbf{R})$. $E''(\mathbf{R})$ also contains additional edges $(s, R_{\underline{j,1}})$, i.e., directed from s to $R_{\underline{j,1}}$, for all minimal points $R_{j,1}$, as well as, additional edges $(R_{\overline{j,2}}, t)$, directed from $R_{\overline{j,2}}$ to t , for all maximal points $R_{j,2}$. As depicted in Figure 4.4, an additional edge directed from $R_{\underline{2}}$ to $R_{\overline{2}}$ is added. In addition, additional edges directed from s to $R_{\underline{2}}$ as well as, from $R_{\overline{7}}$ to t are added. Examples of minimal points are $R_{\underline{2}}$, $R_{\underline{4}}$ and $R_{\underline{18}}$, while examples of maximal points are $R_{\overline{7}}$, $R_{\overline{5}}$, $R_{\overline{9}}$, $R_{\overline{10}}$ and $R_{\overline{19}}$, respectively.

Step 4: Assigning lower and upper capacities to each edge

Each edge is assigned with a lower and an upper capacities. The $(R_{\underline{J}}, R_{\overline{J}})$ are assigned with a lower and an upper capacities of one and $+\infty$, respectively, i.e., $l(R_{\underline{J}}, R_{\overline{J}}) = 1$ and $u(R_{\underline{J}}, R_{\overline{J}}) = +\infty$. The remaining edges for $E''(\mathbf{R})$ are assigned with a lower and upper capacities of zero and $+\infty$, respectively, i.e., $l(R_{\overline{J,1}}, R_{\underline{J,2}}) = 0$, $u(R_{\overline{J,1}}, R_{\underline{J,2}}) = +\infty$, $l(s, R_{\underline{J,1}}) = 0$, $u(s, R_{\underline{J,1}}) = +\infty$, $l(R_{\overline{J,2}}, t) = 0$ and $u(R_{\overline{J,2}}, t) = +\infty$. $G_{NM}''(\mathbf{R})$ is then obtained.

Figure 4.4 depicts the transportation network $G_{NM}''(\mathbf{R3})$, which is constructed from $G_{NM}(\mathbf{R3})$ with $C''(\mathbf{R3}) = \{s, R_{\underline{2}}, R_{\overline{2}}, R_{\underline{4}}, R_{\overline{4}}, R_{\underline{18}}, R_{\overline{18}}, R_{\underline{14}}, R_{\overline{14}}, R_{\underline{7}}, R_{\overline{7}}, R_{\underline{5}}, R_{\overline{5}}, R_{\underline{9}}, R_{\overline{9}}, R_{\underline{10}}, R_{\overline{10}}, R_{\underline{19}}, R_{\overline{19}}, t\}$ and $E''(\mathbf{R3}) = \{(s, R_{\underline{2}}), (s, R_{\underline{4}}), (s, R_{\underline{18}}), (R_{\underline{2}}, R_{\overline{2}}), (R_{\underline{4}}, R_{\overline{4}}), (R_{\underline{18}}, R_{\overline{18}}), (R_{\overline{2}}, R_{\underline{7}}), (R_{\overline{4}}, R_{\underline{5}}), (R_{\overline{4}}, R_{\underline{9}}), (R_{\overline{4}}, R_{\underline{10}}), (R_{\overline{4}}, R_{\underline{14}}), (R_{\underline{14}}, R_{\overline{14}}), (R_{\overline{14}}, R_{\underline{19}}), (R_{\overline{18}}, R_{\underline{19}}), (R_{\underline{7}}, R_{\overline{7}}), (R_{\underline{5}}, R_{\overline{5}}), (R_{\underline{9}}, R_{\overline{9}}), (R_{\underline{10}}, R_{\overline{10}}), (R_{\underline{19}}, R_{\overline{19}}), (R_{\overline{7}}, t), (R_{\overline{5}}, t), (R_{\overline{9}}, t), (R_{\overline{10}}, t), (R_{\overline{19}}, t)\}$. The lower and upper capacities for each edge is given in Figure 4.4 too.

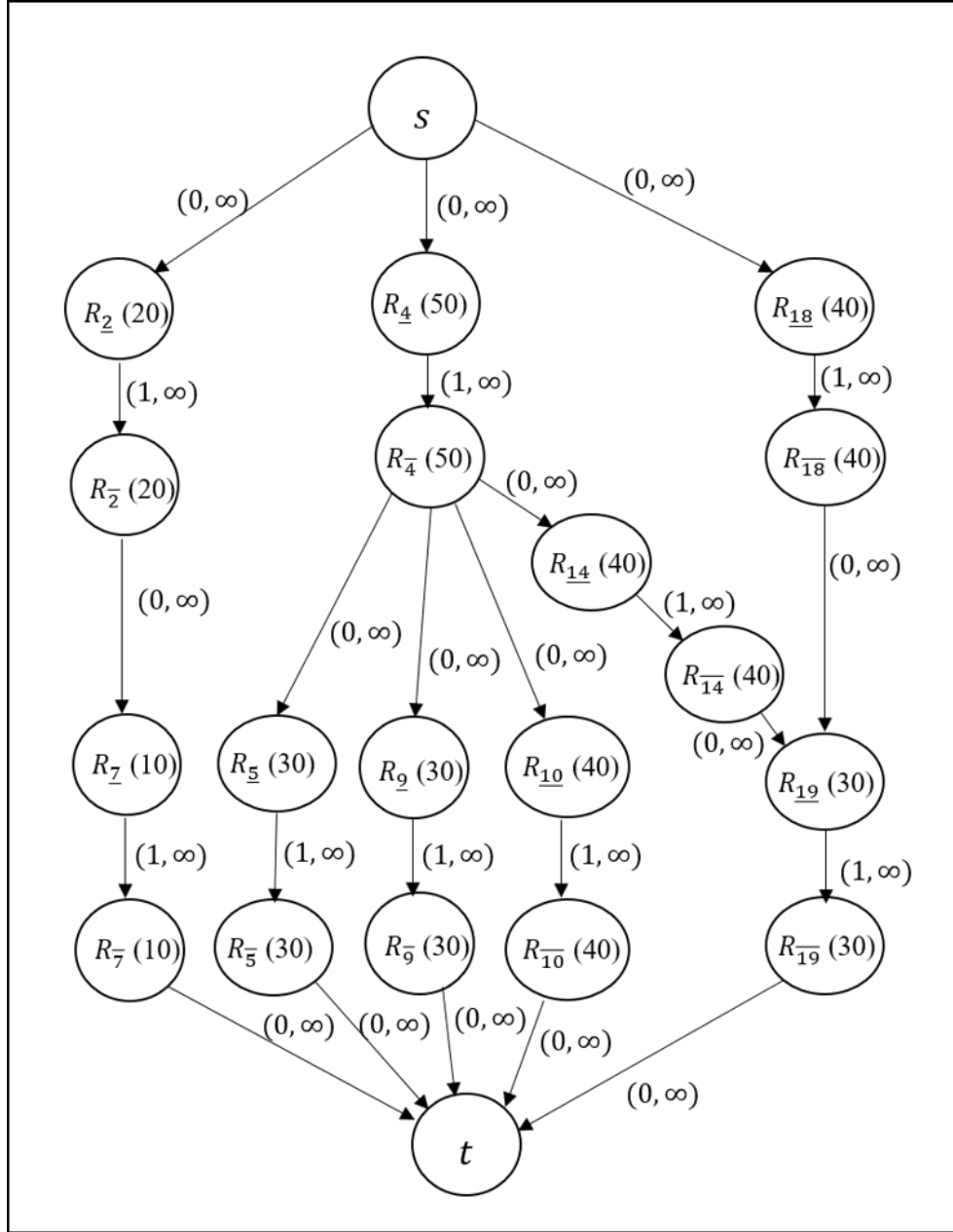


Figure 4.4: Transportation network $G_{NM}''(\mathbf{R3})$ constructed from $G_{NM}(\mathbf{R3})$

Step 5: Determining the minimum flow and the maximum cut

Using $G_{NM}''(\mathbf{R})$, the maximum independent set(s) for $G_{NM}(\mathbf{R})$ can now be obtained by computing the minimum flow value (i.e., f_{value}^{min}) in $G_{NM}''(\mathbf{R})$. The minimum flow

maximum cut Theorem (Ford & Fulkerson, 1962) suggests that f_{value}^{min} equals to the maximum capacity of an $(s - t)$ cut in $G_{NM}''(\mathbf{R})$, as shown in Eq. (4.1).

$$f_{value}^{min} = \max_{S,T} \left[\sum_{\substack{(v,w) \in E''(\mathbf{R}) \\ v \in S, w \in T}} l(v, w) - \sum_{\substack{(v,w) \in E''(\mathbf{R}) \\ v \in T, w \in S}} u(v, w) \right] \quad (4.1)$$

where S, T is an $(s - t)$ cut of $G_{NM}''(\mathbf{R})$. The $(s - t)$ cut partitions the vertices $C''(\mathbf{R})$ of the network into the two disjoint subsets of S and T , where source node s must present in the subset S and sink node t must present in the subset T , i.e., $C''(\mathbf{R}) = S \cup T, S \cap T = \emptyset, s \in S, t \in T$. The technique in (Rademaker et al., 2012) is adopted to compute the minimum flow for $G_{NM}''(\mathbf{R})$, as follows.

Step 5.1: Assigning feasible flow f

A feasible flow f in $G_{NM}''(\mathbf{R})$ is an assignment of a nonnegative integer $f(e)$ to each edge $e = (v, w) \in E''(\mathbf{R})$, for $v, w \in C''(\mathbf{R})$ such that the following three conditions are satisfied:

1. Capacity constraint: $l(e) \leq f(e) \leq u(e)$
2. Skew symmetry: $f(v, w) = -f(w, v)$
3. Flow conservation: $\sum_w f(v, w) - \sum_w f(w, v) = 0$ for all $v \neq s, t$

Note that obtaining a feasible flow is straightforward as the capacity for each edge is unbounded from above (Rademaker et al., 2012). Figure 4.5 illustrate feasible flow that pass into the $G_{NM}''(\mathbf{R3})$, with $f = 300$. The flow assigned to each edge is fulfilling the capacity constraint, skew symmetry as well as the flow conservation.

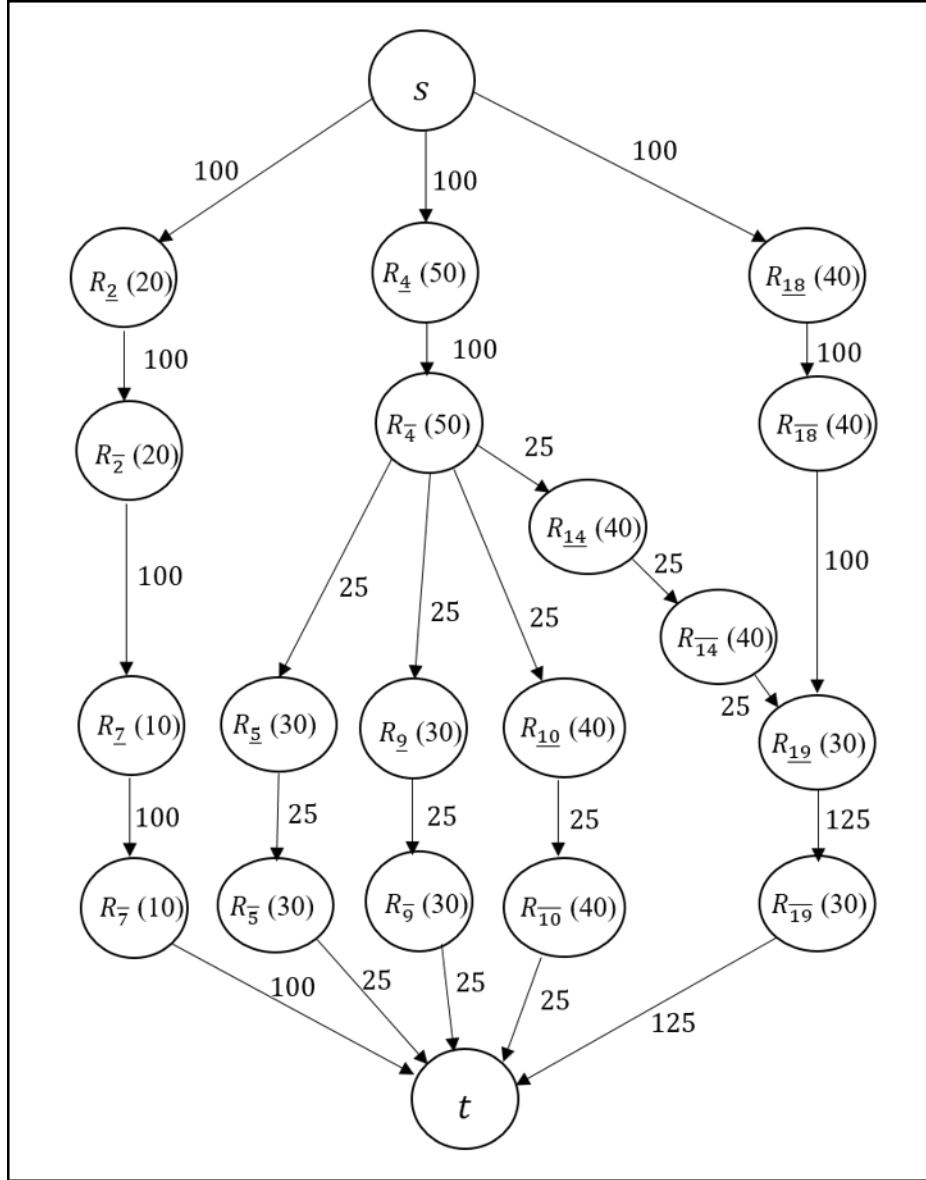


Figure 4.5: Feasible flow assigned to $G_{NM}''(\mathbf{R3})$

Step 5.2: Transforming $G_{NM}''(\mathbf{R})$ into an inverted flow network, i.e., $G_{NM}'''(\mathbf{R})$

$G_{NM}''(\mathbf{R})$ is converted into an inverted flow network, i.e., $G_{NM}'''(\mathbf{R})$ by setting the upper capacity of each backward edge, i.e., $u(w, v)$ equal to the flow in excess of the lower capacity of the corresponding forward edge, i.e., $u(w, v) = f(v, w) - l(v, w)$. As an example, Figure 4.6 shows the inverted network, i.e., $G_{NM}'''(\mathbf{R3})$ that constructed from $G_{NM}''(\mathbf{R3})$.

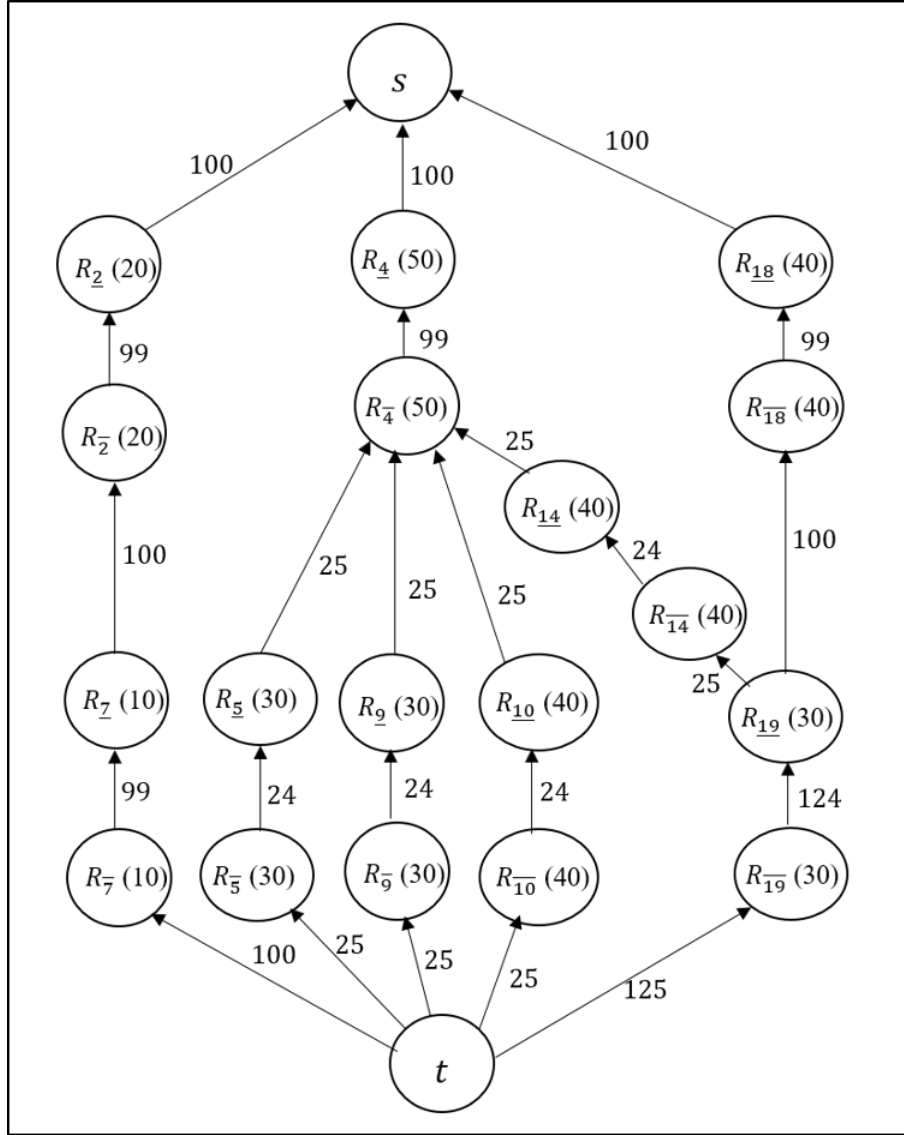


Figure 4.6: Inverted network $G_{NM}'''(\mathbf{R3})$

Step 5.3: Computing the maximum flow, f_{value}^{max} for $G_{NM}'''(\mathbf{R})$, from t to s .

The maximum flow f_{value}^{max} is computed for the inverted flow network $G_{NM}'''(\mathbf{R})$ from the sink node to the source node. The push-relabel algorithm (Goldberg & Tarjan, 1988) is implemented as it is one of the most efficient algorithm to compute the maximum flow (Cherkassky & Goldberg, 1997). As an example, a maximum flow of 294 is obtained for $G_{NM}'''(\mathbf{R3})$, i.e., $f_{value}^{max} = 294$.

Step 5.4: Obtaining the minimum flow, f_{value}^{min} for $G_{NM}''(\mathbf{R})$

The minimum flow f_{value}^{min} is obtained by subtracting the maximum flow from the original flow, i.e., $f_{value}^{min} = f - f_{value}^{max}$. Thus, for $G_{NM}''(\mathbf{R3})$, a minimum flow value of 6 is obtained, i.e., $f_{value}^{min} = 300 - 294 = 6$.

Step 5.5: Obtaining the maximum $(s - t)$ cut

A *residual network* (Ford & Fulkerson, 1962) corresponding to the minimum flow over $G_{NM}'''(\mathbf{R})$ is resulted. Only edges (v, w) for which $l(v, w) < f(v, w)$ are present in the residual network in the backward direction. This means that the edge is directed from w to v in the residual network, if and only if the flow pass through edge (v, w) in $G_{NM}'''(\mathbf{R})$ is larger in value than its lower capacity. Those edges not present in the residual network divide the set of vertices into two disjoint subsets, i.e., S and T , such that vertices that are reachable from the source and sink nodes are included in S and T , respectively. The minimum flow value for the transportation network is equal to the independence number of graph G_{NM} , i.e., $\alpha(G_{NM})$. The set of vertices $\{R_j \in C(\mathbf{R}) \mid (R_{\underline{j}}, R_{\overline{j}}) \in E''(\mathbf{R}), R_{\underline{j}} \in S, R_{\overline{j}} \in T\}$ corresponding to the optimal cut of $G_{NM}''(\mathbf{R})$ is a maximum independent set for $G_{NM}(\mathbf{R})$ (Möhring, 1985).

Figure 4.7 depicts the maximum cut that divides $C''(\mathbf{R3})$ into two disjoint subsets, i.e., $S = \{s, R_{\underline{2}}, R_{\overline{2}}, R_{\underline{4}}, R_{\overline{4}}, R_{\underline{18}}, R_{\underline{7}}, R_{\underline{5}}, R_{\underline{9}}, R_{\underline{10}}, R_{\underline{14}}\}$ and $T = \{R_{\overline{7}}, R_{\overline{5}}, R_{\overline{9}}, R_{\overline{10}}, R_{\overline{14}}, R_{\overline{18}}, R_{\underline{19}}, R_{\overline{19}}, t\}$. Therefore, the maximum independent set for $G_{NM}(\mathbf{R3})$ is $MIS(\mathbf{R3}) = \{R_{\underline{7}}, R_{\underline{5}}, R_{\underline{9}}, R_{\underline{10}}, R_{\underline{14}}, R_{\underline{18}}\}$, with an independence number $\alpha(G_{NM}(\mathbf{R3})) = 6$.

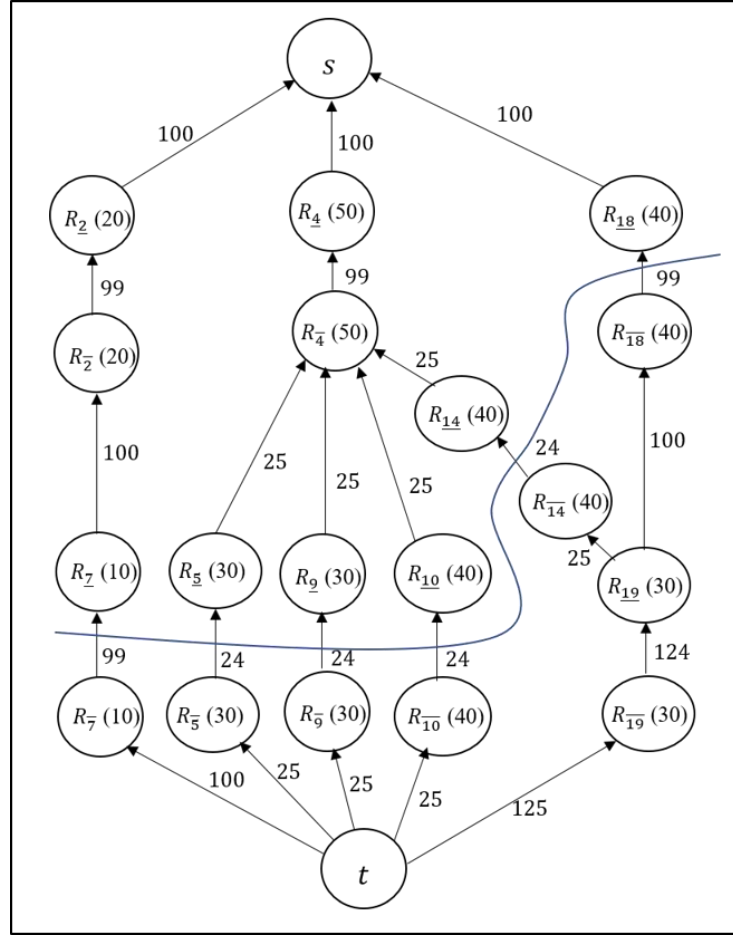


Figure 4.7: A maximum $(s - t)$ cut for $G_{NM}''(\mathbf{R3})$

4.4 Discussion

From Section 4.2, a graph representation of a fuzzy rule base is proposed. The fuzzy rules are represented by a set of vertices and the non-monotonicity relationship between two fuzzy rules are represented by edges. Using the fuzzy rule base **R3**, $G(\mathbf{R3})$ is formed (see Figure 4.2). $G(\mathbf{R3})$ consists of a connected vertex set $C(\mathbf{R3})$ and a disconnected vertex set $D(\mathbf{R3})$. Since $C(\mathbf{R3})$ are those fuzzy rules that forms non-monotone pair with each other, and hence they could be temporarily removed from the fuzzy rule base **R3**, as shown in Table 4.1. In other words, Table 4.1 shows the fuzzy rule base that consists of disconnected vertex set $D(\mathbf{R3})$ only.

Table 4.1: Fuzzy rule base **R3** with disconnected vertex set $D(\mathbf{R3})$ only

	x_2				
	μ_2^1	μ_2^2	μ_2^3	μ_2^4	μ_2^5
μ_1^1	$b_{1(1,1)} = 10$	$b_{6(1,2)} = 10$	$b_{11(1,3)} = 20$	$b_{16(1,4)} = 20$	$b_{21(1,5)} = 30$
μ_1^2			$b_{12(2,3)} = 20$	$b_{17(2,4)} = 30$	$b_{22(2,5)} = 40$
μ_1^3	$b_{3(3,1)} = 20$	$b_{8(3,2)} = 30$	$b_{13(3,3)} = 30$		$b_{23(3,5)} = 40$
μ_1^4					$b_{24(4,5)} = 50$
μ_1^5			$b_{15(5,3)} = 40$	$b_{20(5,4)} = 40$	$b_{25(5,5)} = 50$

In the meantime, a non-monotone relationship graph $G_{NM}(\mathbf{R3})$ (see Figure 4.3) is formed, in which its vertex set consists of connected vertex set $C(\mathbf{R3})$ only. The maximum independent set of $G_{NM}(\mathbf{R3})$ is the maximum subset of fuzzy rules that are monotone to each other as no edges are allowed to present in the maximum independent set. In section 4.3, the network flow approach has shown to be an effective tool for computing a maximum independent set of a non-monotone relationship graph. For $G_{NM}(\mathbf{R3})$, a maximum independent set is obtained using the proposed approach, i.e., $MIS(\mathbf{R3}) = \{R_7, R_5, R_9, R_{10}, R_{14}, R_{18}\}$.

The obtained maximum independent set, i.e., $MIS(\mathbf{R3}) = \{R_7, R_5, R_9, R_{10}, R_{14}, R_{18}\}$ could be combined back with $D(\mathbf{R3})$, as shown in Table 4.2. Notice that these fuzzy rules are monotone to each other, in a maximal way. The fuzzy rules R_2, R_4 , and R_{19} could not be included, as adding any one of these fuzzy rules will render the fuzzy rules to violate **Condition 2** of **Theorem 3.1**. And thus, we could identify the remaining fuzzy rules, i.e., R_2, R_4 , and R_{19} are the minimum fuzzy rules that need to be relabeled.

Table 4.2: Fuzzy rule base **R3** that consists of $MIS(\mathbf{R3})$ and $D(\mathbf{R3})$

	x_2				
	μ_2^1	μ_2^2	μ_2^3	μ_2^4	μ_2^5
μ_1^1	$b_{1(1,1)} = 10$	$b_{6(1,2)} = 10$	$b_{11(1,3)} = 20$	$b_{16(1,4)} = 20$	$b_{21(1,5)} = 30$
μ_1^2		$b_{7(2,2)} = 10$	$b_{12(2,3)} = 20$	$b_{17(2,4)} = 30$	$b_{22(2,5)} = 40$
μ_1^3	$b_{3(3,1)} = 20$	$b_{8(3,2)} = 30$	$b_{13(3,3)} = 30$	$b_{18(3,4)} = 40$	$b_{23(3,5)} = 40$
μ_1^4		$b_{9(4,2)} = 30$	$b_{14(4,3)} = 40$		$b_{24(4,5)} = 50$
μ_1^5	$b_{5(5,1)} = 30$	$b_{10(5,2)} = 40$	$b_{15(5,3)} = 40$	$b_{20(5,4)} = 40$	$b_{25(5,5)} = 50$

Noted that it could be more than one MIS exist in a $G_{NM}(\mathbf{R})$. However, the proposed network approach could only search for one MIS for the given $G_{NM}(\mathbf{R})$. In fact, for $G_{NM}(\mathbf{R3})$, there are 2 different MIS exist, i.e., $MIS_1(\mathbf{R3}) = \{R_7, R_5, R_9, R_{10}, R_{14}, R_{18}\}$ and $MIS_2(\mathbf{R3}) = \{R_2, R_5, R_9, R_{10}, R_{14}, R_{18}\}$. The combinations of fuzzy rules to be relabeled are depending on the MIS obtained by the proposed approach.

4.5 Summary

In this chapter, a representation of fuzzy rule base in graph view is devised. A formulation of maximum independent set problem is proposed, and solved using network flow approach. Such representation is useful and served as an important part for practical implementation of MFRR, which will be detailed in the next chapter. In the next chapter, a practical monotone fuzzy rule relabeling framework for Zero-order TSK Fuzzy inference system will be presented.

CHAPTER 5

A MONOTONE FUZZY RULE RELABELING FRAMEWORK

5.1 Introduction

In the previous chapter, representation of fuzzy rule base using graph view was introduced. In this chapter, a MFRR framework for implementation is devised as a three-stage procedure. Several challenges emerged during the implementation of MFRR is discussed. A number of notions for the three-stage procedure is introduced. The applicability of the three-stage procedure is demonstrated with a simulated example. The three-stage procedure is also analyzed and discussed.

This chapter is organized as follows. In Section 5.2, the three-stage procedure is briefed, and discussed. A number of MFRR methods are discussed with example, in Sections 5.3, 5.4 and 5.5 for Stage-1, Stage-2, and Stage-3, respectively. The concluding remarks for Chapter 5 is given in Section 5.6.

5.2 The Challenges for Implementation of MFRR

5.2.1 The Three-Stage Framework

A MFRR framework is devised as a three-stage procedure, as shown in Figure 5.1. The MFRR framework receives a non-monotone fuzzy rule base as the input. Stage-1 determines the minimum number of noisy fuzzy rules (i.e., k -noisy fuzzy rules) to be relabeled. The minimum number of noisy fuzzy rules to be relabeled (i.e., k number of fuzzy rules) is determined using a search procedure. Analogy to the idea of noise detection in signal

and image processing applications, noisy fuzzy rules can be viewed as an unwanted “signals” that interfere with the fuzzy inference process and produce unreliable/invalid outcomes. Note that the first and second relabeling criteria (See Chapter 3, Section 3.4) can be achieved in Stage-1 of MFRR.

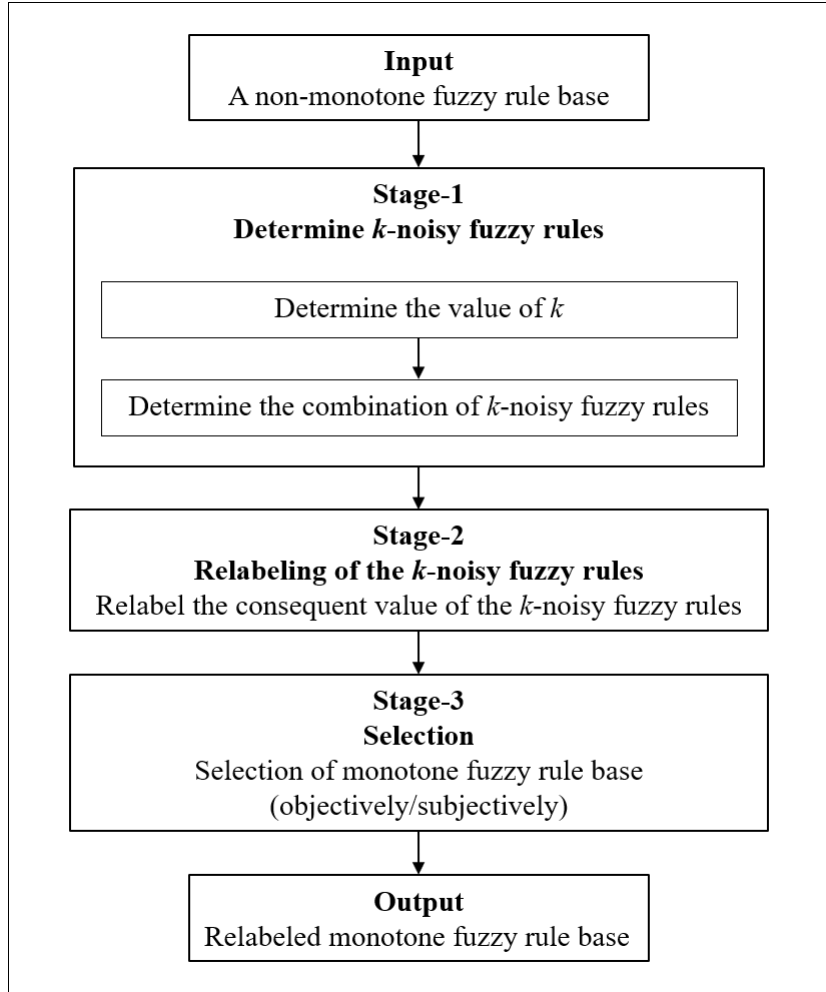


Figure 5.1: A general and practical MFRR framework

Stage-2 relabels the consequent part of k -noisy fuzzy rules obtained in Stage-1. Relabeling the k -noisy fuzzy rules can be explained as a monotonicity restoration process. A loss function is used in this stage to search in a fuzzy rule base with the minimum loss

incurred during the relabeling process. Note that the third relabeling criteria is achieved in Stage-2. More than one relabeled fuzzy rule with the minimum loss can be obtained in Stage-2. Stage-3 aims to select a set of relabeled fuzzy rule, either objectively and/or subjectively, for TSK-FIS modelling. In summary, the MFRR output is a set of monotone and optimal relabeled fuzzy rule base that is useful for practical TSK-FIS modelling.

5.2.2 Challenges

Several challenges emerge during the implementation of MFRR. In Stage-1, the computational time for determining k -noisy fuzzy rules grows exponentially with the increase in the input size and the k setting due to the search process itself is a combinatorial problem. In addition, it is necessary to search for all the combinations of k -noisy fuzzy rules in Stage-1, in order to obtain all global optimal solution(s) in Stage-2. For the completeness and optimality with respect to the outcome of Stage-1, it is important to recognize that MFRR is a multi-extremum problem. In addition, it is not clear which loss function should be used to ensure optimality of MFRR in Stage 2. As such, a variety of loss functions and their properties are examined.

Three strategies are formulated to search for the k -noisy fuzzy rules, and analyze optimality and completeness of each strategy. The value of k is pre-specified, in order to regulate the computational time. A hybrid method consisting of *network flow* and *brute force* for effectively determining k -noisy fuzzy rules is devised. The reasonable properties for the loss functions to be used in MFRR are outlined. A number of commonly used loss functions, which include the absolute loss function, square loss function, and zero-one loss function, are discussed.

5.2.3 New Notions

For implementation of MFRR, the three-stage framework (Figure 5.1) is considered. The definitions for optimality and completeness of Stage-1 and Stage-2 are as follows.

Definition 5.1. Optimality and Completeness

5.1.1. Stage-1 optimality. Stage-1 optimality is satisfied if at least a combination of k -noisy fuzzy rule sets with the minimum number of fuzzy rule change, i.e., k , is obtained.

5.1.2. Stage-1 completeness. Stage-1 completeness is satisfied if the complete list of combinations of k -noisy fuzzy rules sets is obtained.

5.1.3. Stage-2 optimality. Stage-2 optimality is satisfied if at least a relabeled fuzzy rule set with the minimum loss for a combination of k -noisy fuzzy rules set from Stage-1 is obtained.

5.1.4. Stage-2 completeness. Stage-2 completeness is satisfied if the complete list of relabeled fuzzy rule sets with the minimum loss for a combination of k -noisy fuzzy rules from Stage-1 is obtained.

5.1.5. MFRR optimality. MFRR optimality is satisfied if at least a relabeled fuzzy rule set that satisfies the MFRR criterion is obtained.

5.1.6. MFRR completeness. MFRR completeness is satisfied if the complete list of relabeled fuzzy rule sets that satisfies the MFRR criterion is obtained.

5.3 Stage-1: Determination of k -Noisy Fuzzy Rules

Definition 5.2 is considered, and Lemmas 5.1 and 5.2 are presented. Three methods to determine k -noisy fuzzy rules, i.e., a network flow method (i.e, MFRR-NF), a brute-force with backtracking method (MFRR-BFBT), and a hybrid method (i.e., MFRR-H) are

explained in Sections 5.3.1, 5.3.2, and 5.3.3, respectively. Their empirical evaluation is discussed in Section 5.3.4.

Definition 5.2. A k -subset of $\mathbf{R}$ with its remaining fuzzy rules (i.e., $R_{remaining}$) that satisfy Condition 2 of Theorem 3.1 is denoted as K_s , where $1 \leq k \leq r$, in which k should be as minimal as possible. Note that $\mathbf{R} = R_{remaining} \cup K_s$ is always true.

Lemma 5.1. $k = |C(\mathbf{R})| - \alpha(G_{NM}(\mathbf{R}))$ (5.1)

Proof. $\alpha(G_{NM}(\mathbf{R}))$ is the cardinality for the maximum independent set(s) of $G_{NM}(\mathbf{R})$. $|C(\mathbf{R})|$ is the cardinality of $C(\mathbf{R})$ or the connected vertex of $G_{NM}(\mathbf{R})$. Therefore, it is straightforward that $k = |C(\mathbf{R})| - \alpha(G_{NM}(\mathbf{R}))$.

As an example, for $G_{NM}(\mathbf{R3})$, $\alpha(G_{NM}(\mathbf{R3})) = 6$ and $|C(\mathbf{R3})| = 9$; therefore $k = |C(\mathbf{R3})| - \alpha(G_{NM}(\mathbf{R3})) = 3$. In $C(\mathbf{R3})$, there are 9 fuzzy rules having a non-monotonicity relationship, i.e., $|C(\mathbf{R3})| = 9$. Among them, it has been identified that a maximum of 6 fuzzy rules are monotone to each other, i.e., $\alpha(G_{NM}(\mathbf{R3}))$. By subtracting $\alpha(G_{NM}(\mathbf{R3}))$ from $|C(\mathbf{R3})|$, this implies that a minimum of 3 fuzzy rules (i.e., $k = 3$) must be excluded from $G_{NM}(\mathbf{R3})$, in order to make $G_{NM}(\mathbf{R3})$ monotone.

Lemma 5.2. $K_s = (MIS(\mathbf{R}) \cup D(\mathbf{R}))'$

Proof. $D(\mathbf{R})$ contains fuzzy rules that are monotone to each other for a fuzzy rule base $\mathbf{R}$. $C(\mathbf{R})$ contains fuzzy rules that form the non-monotonicity relationship graph $G_{NM}(\mathbf{R})$. $MIS(\mathbf{R})$ is the maximum subset of $C(\mathbf{R})$, such that the fuzzy rules in $MIS(\mathbf{R})$ are monotone to each other. Therefore, the union of $MIS(\mathbf{R})$ and $D(\mathbf{R})$ results in a maximum subset of

monotone fuzzy rules for $\mathbf{R}$, and the complement of the subset contains the k -noisy fuzzy rules.

As an example, a maximum independent set obtained for $G_{NM}(\mathbf{R3})$ is $MIS(\mathbf{R3}) = \{R_7, R_5, R_9, R_{10}, R_{14}, R_{18}\}$ and $D(\mathbf{R3}) = \{\mathbf{R3} \setminus C(\mathbf{R3})\}$. The k -noisy fuzzy rules are the complement of $MIS(\mathbf{R3}) \cup D(\mathbf{R3})$, i.e., $K_s = \{R_2, R_4, R_{19}\}$.

5.3.1 MFRR-NF

MFRR-NF uses the network flow procedure (see Section 4.3.1) to search for a maximum independent set, i.e., $MIS(\mathbf{R})$. By using **Lemma 5.1** and **Lemma 5.2**, the minimum number of fuzzy rules, i.e., k , as well as its combination to be relabeled, i.e., K_s , are obtained. Even though multiple K_s may exist, only one K_s can be obtained by MFRR-NF.

Remark 5.1. Stage-1 optimality is guaranteed, but, not Stage-1 completeness, using MFRR-NF.

5.3.2 MFRR-BFBT

It is observed that Stage-1 completeness is not always satisfied by MFRR-NF, as only one maximum independent set is obtained, and, therefore, only one K_s could be obtained. To achieve Stage-1 completeness, MFRR-BFBT is used to search for all combinations of K_s , in which excluding the fuzzy rules in K_s can lead to a set of monotone fuzzy rule base. In Section 3.7, MFRR-BFBT attempts to enumerate all k -subset of R_j lexicographically, starting with $k=1$, until the minimum k is found. In this section, instead of starting with $k=1$, we enumerate the k -subset of R_j lexicographically using the k value obtained in **Lemma 5.1**.

As an example, in Section 5.3, $k=3$ is obtained for $G_{NM}(\mathbf{R3})$. We enumerate all possible combinations of 3 fuzzy rules from 125 fuzzy rules, i.e., starting from $\{R_1, R_2, R_3\}$, $\{R_1, R_2, R_4\}, \dots, \{R_{123}, R_{124}, R_{125}\}$, and check whether removing any combination of fuzzy rules does not affect the remaining fuzzy rules to fulfil the monotonicity property. Using the brute-force with backtracking technique, two K_s can be obtained, i.e., $K_s = \{R_2, R_4, R_{19}\}$ and $\{R_4, R_7, R_{19}\}$, in which their remaining fuzzy rules are still able to satisfy the monotonicity property. In general, MFRR-BFBT need to enumerate $C(r, k)$ combination for searching the optimal solution, where $C(r, k)$ is the number of k -combinations given r fuzzy rules.

Remark 5.2. Stage-1 optimality and Stage-1 completeness is guaranteed by using MFRR-BFBT.

5.3.3 MFRR-H

Even though MFRR-BFBT satisfies both optimality and completeness of Stage-1, the computational cost of MFRR-BFBT becomes expensive with the growth of the problem size. Therefore, MFRR-H is proposed. MFRR-H aims to integrate the advantages of MFRR-NF and MFRR-BFBT, and to overcome the weakness possessed by each method. MFRR-H is able to provide a complete list of K_s in a shorter time. Inspired by the work of Rademaker et al. (2012), the useful concepts of *core fuzzy rules*, *corona fuzzy rules* and *void fuzzy rules* are summarized in Definition 5.3.

Definition 5.3. *Core fuzzy rules, corona fuzzy rules and void fuzzy rules*

5.3.1. Fuzzy rules that are member of every $MIS(\mathbf{R})$ are *core fuzzy rules*.

5.3.2. Fuzzy rules that are member of some, but not for all, $MIS(\mathbf{R})$ are *corona fuzzy rules*.

5.3.3. Fuzzy rules that are not a member of any $MIS(\mathbf{R})$ are *void fuzzy rules*.

As an $MIS(\mathbf{R})$ contains a maximum subset of the internally monotone fuzzy rules, this implies that *core fuzzy rules* are always monotone, i.e., these rules do not need to be relabeled. In other words, *core fuzzy rules* always are excluded in K_s . Since *void fuzzy rules* do not belong to any $MIS(\mathbf{R})$, they are always non-monotone. In other words, *void fuzzy rules* are always included in K_s . *Corona fuzzy rules* are the fuzzy rules that cause the formation of different combination of K_s . If no *corona fuzzy rules* exist in a non-monotone fuzzy rule base, there is only one K_s . The concept of a residual network (Rademaker et al., 2012) is essential to determine the *void fuzzy rules*, *core fuzzy rules*, and *corona fuzzy rules*, after the implementation of the minimum flow maximum cut algorithm (Rademaker et al., 2012). The procedure to obtain *void fuzzy rules*, *core fuzzy rules*, and *corona fuzzy rules* is summarized in Figure 5.2.

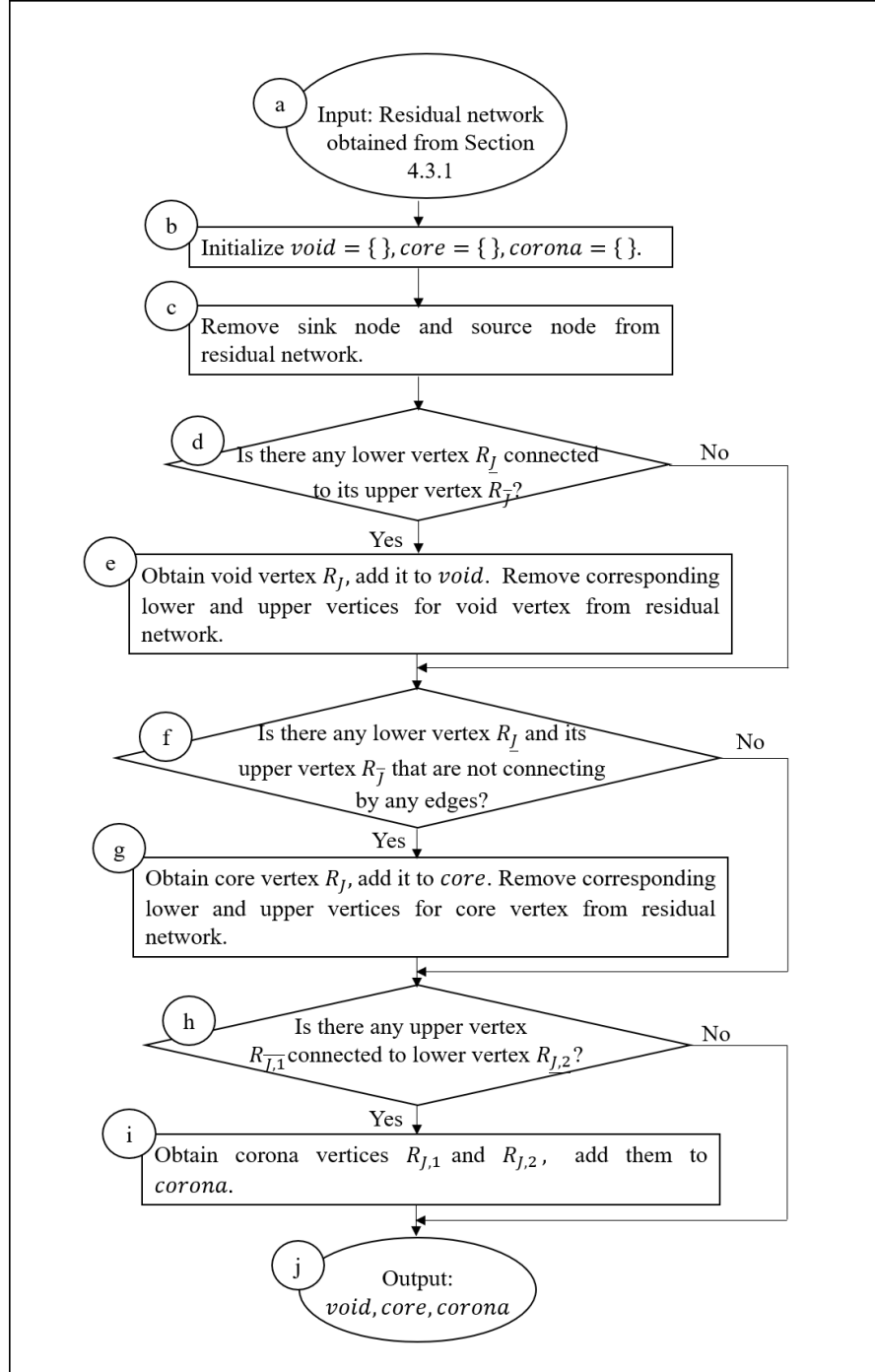


Figure 5.2: Procedure to obtain *void* fuzzy rule, *core* fuzzy rules and *corona* fuzzy rules.

The details of the procedure are outlined, as follows.

- (a) Input: A residual network obtained from Section 4.3.1. Figure 5.3 depicts an example of a residual network for $G_{NM}''(\mathbf{R3})$, obtained in Section 4.3.1.

- (b) Initialize *void*, *core*, and *corona* as an empty set, i.e., $void = \{\}$, $core = \{\}$, and $corona = \{\}$.
- (c) Remove the sink and source nodes from the residual network. Figure 5.4 depicts the residual network after removing the source and sink nodes as well as the edges connected to them.
- (d) If there exists a lower vertex $R_{\underline{j}}$ connected to its upper vertex $R_{\overline{j}}$ in the residual network, go to Step (e). Otherwise, go to Step (f).
- (e) The corresponding vertex R_j is the *void fuzzy rule*. Add the *void fuzzy rule* into set *void*.
As an example, in Figure 5.4, the *void fuzzy rules* for **R3** are R_4 and R_{19} , i.e., $void = \{R_4, R_{19}\}$. After identifying the *void fuzzy rules*, removes the lower vertex $R_{\underline{j}}$ and upper vertex $R_{\overline{j}}$ of the *void fuzzy rules* R_j , as well as the edges connected to them from the residual network. Figure 5.5 depicts the residual network, with the removal of the lower vertex $R_{\underline{j}}$ and upper vertex $R_{\overline{j}}$ of the *void fuzzy rules* R_j .
- (f) If there exists a lower vertex $R_{\underline{j}}$ and its upper vertex $R_{\overline{j}}$ that are not connected by any edges, go to Step (g). Otherwise, go to Step (h).
- (g) The corresponding vertex R_j is the *core fuzzy rule*. Add the *core fuzzy rule* into set *core*. In Figure 5.5, $core = \{R_5, R_9, R_{10}, R_{14}, R_{18}\}$. After identifying the *core fuzzy rules*, removes the lower vertex $R_{\underline{j}}$ and upper vertex $R_{\overline{j}}$ of the *core fuzzy rules* R_j , as well as the edges connected to them from the residual network. Figure 5.6 depicts the residual network, with the removal of the lower vertex $R_{\underline{j}}$ and upper vertex $R_{\overline{j}}$ of the *core fuzzy rules* R_j .
- (h) If there exists an upper vertex $R_{\overline{j,1}}$ connected to a lower vertex $R_{\underline{j,2}}$ in the residual network, go to Step (i). Otherwise, go to Step (j).

- (i) The corresponding vertices $R_{j,1}$ and $R_{j,2}$ are the *corona fuzzy rule*. Add the *corona fuzzy rule* into set *corona*. Figure 5.6 shows that $R_{\underline{7}}$ are connected to $R_{\overline{2}}$, therefore, fuzzy rules R_2 and R_7 constitutes the *corona fuzzy rules*, i.e., $corona = \{R_2, R_7\}$.
- (j) Output a set of *void fuzzy rules*, *core fuzzy rules*, and *corona fuzzy rules*, respectively. As an example, for **R3**, $void = \{R_4, R_{19}\}$, $core = \{R_5, R_9, R_{10}, R_{14}, R_{18}\}$ and $corona = \{R_2, R_7\}$.

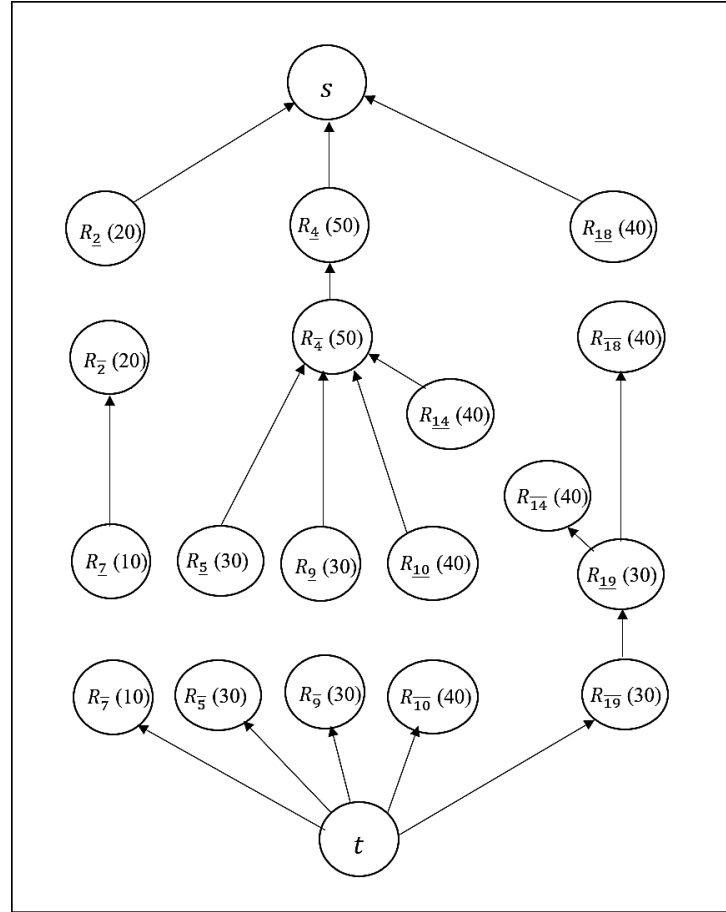


Figure 5.3: Residual network for $G_{NM}''(\mathbf{R3})$

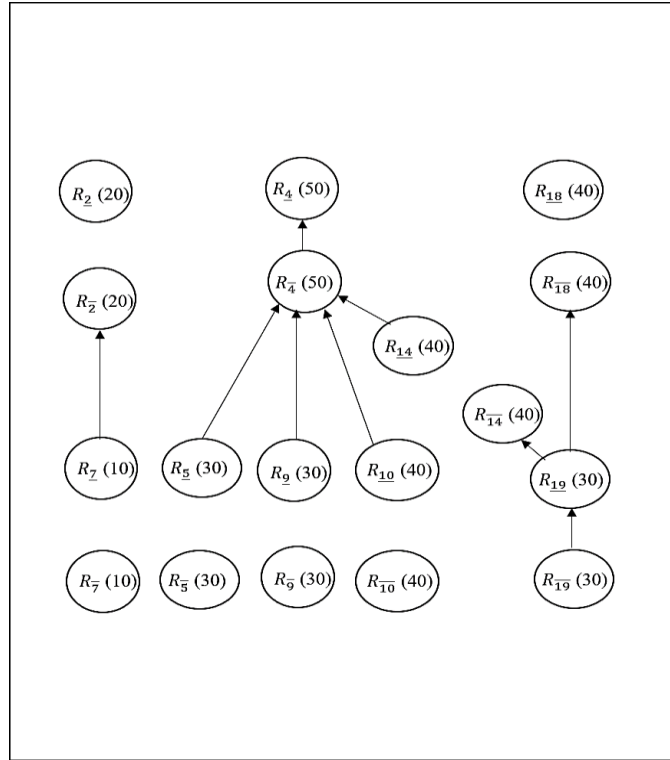


Figure 5.4: Removal of the source node and sink node

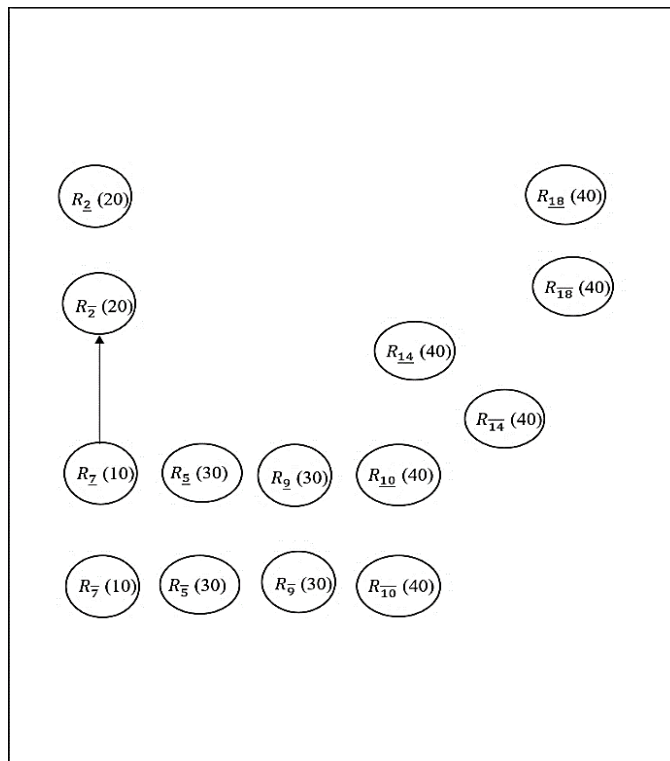


Figure 5.5: Removal of the void vertices

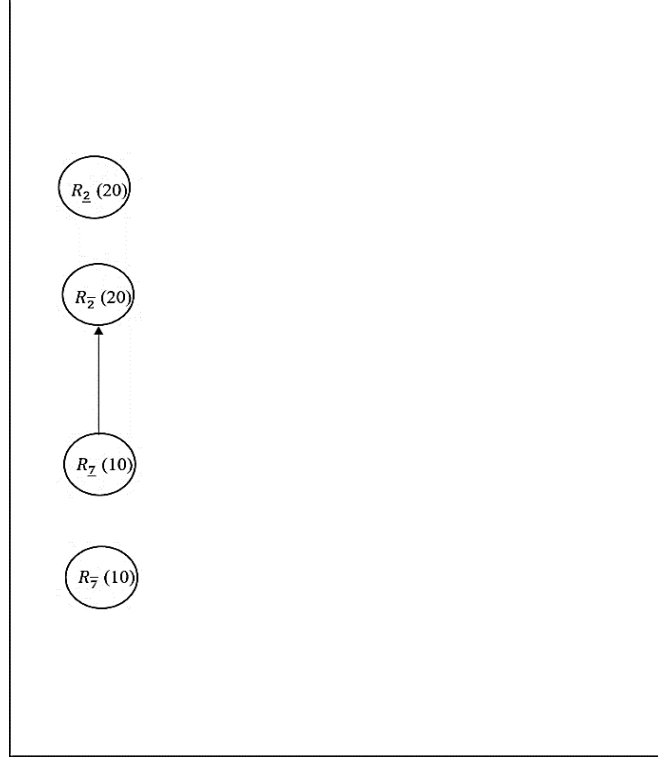


Figure 5.6: Removal of the core vertices

Since the *core fuzzy rules* do not need to be relabeled, we can combine the *core fuzzy rules* with $D(\mathbf{R})$. The *void fuzzy rules* are the fuzzy rules that always need to be relabeled, therefore they always are included in K_s . With the example, K_s is always initialized with $K_s = \{R_4, R_{19}\}$. The *corona fuzzy rules* are the fuzzy rules that cause different combinations of K_s . The goal of MFRR-H is to enumerate different combinations of *corona fuzzy rules* with *void fuzzy rules*, to form K_s . Since the value of k is known, we can determine how many *corona fuzzy rules* need to be combined with *void fuzzy rules*, by using Eq. (5.2).

$$n_c = k - |\text{void}| \quad (5.2)$$

where n_c are the number of *corona fuzzy rules* to be combined, k is the minimum number of fuzzy rules to be relabeled, and $|\text{void}|$ is the number of *void fuzzy rules*. In this example, $k = 3$. The number of *void fuzzy rules* is 2, then $n_c = 3 - 2 = 1$. The enumerated

combination of 1 *corona fuzzy rule* are $\{R_2\}$ and $\{R_7\}$. Combining the combination of 1 *corona fuzzy rule* with *void fuzzy rules* gives $K_s = \{R_2, R_4, R_{19}\}$ and $\{R_4, R_7, R_{19}\}$, which is the same as K_s obtained using MFRR-BFBT. In general, the MFRR-H need to enumerate for $C(|corona|, n_c)$ combination for searching the optimal solution, where $C(|corona|, n_c)$ is the number of n_c -combination given the size of *corona fuzzy rule* (i.e., $|corona|$).

Remark 5.3. Stage-1 optimality and Stage-1 completeness is guaranteed by using MFRR-H.

5.3.4 Experiments for Stage-1

Table 5.1 summarizes the k -noisy fuzzy rules obtained by MFRR-BFBT, MFRR-NF and MFRR-H, using **R3**. A minimum of 3 non-monotone fuzzy rules need to be relabeled, in order to form a monotone fuzzy rule base. MFRR-BFBT and MFRR-H can search for all feasible combinations of k -noisy fuzzy rules (i.e., 2 feasible combinations are found), while MFRR-NF can only search for one feasible combination of k -noisy fuzzy rules. Stage-1 optimality is fulfilled by MFRR-BFBT, MFRR-NF and MFRR-H, as these three methods are able to search for the minimum number of fuzzy rules to be relabeled. Stage-1 completeness is fulfilled by MFRR-BFBT and MFRR-H, but not for MFRR-NF. In addition, the computational time for MFRR-BFBT, MFRR-NF and MFRR-H are 0.185 seconds, 0.019 seconds, and 0.031 seconds, respectively. MFRR-BFBT has the highest computational cost, which is followed by MFRR-H and MFRR-NF. This results are in line with those obtained in Chapter 3.

Table 5.1: k -noisy fuzzy rules enumerated by MFRR-NF, MFRR-BFBT and MFRR-H in Stage-1, for **R3**

Information	MFRR-BFBT	MFRR-NF	MFRR-H
k	3	3	3
Number of K_s	2	1	2
K_s	$\{R_2, R_4, R_{19}\},$ $\{R_4, R_7, R_{19}\}$	$\{R_2, R_4, R_{19}\}$	$\{R_2, R_4, R_{19}\},$ $\{R_4, R_7, R_{19}\}$
Computational time (s)	0.185	0.019	0.031

5.4 Stage-2: Relabeling of k -Noisy Fuzzy Rules

Stage-2 aims to relabel the k -noisy fuzzy rules obtained in Stage-1 into a monotone fuzzy rule base. Let $U(B)$ be a set of unique fuzzy singletons in $\mathbf{R}$, i.e., $U(B) = \{b_{u,1}, b_{u,2}, \dots, b_{u,r_u}\}$. For an obtained K_s , MFRR attempts to relabel the consequent part of fuzzy rules in K_s with the values in $U(B)$. As mentioned in Section 3.7, the relabeling process can be considered as a process of traversing a k -depth tree in a depth-first order. The k -depth tree represents the solutions with k variables. The relabeling process is initialized with an empty vector as the partial solution (i.e., $R_{partial}$). The partial solution is defined as follows:

Definition 5.4. A partial solution (i.e., $R_{partial}$) of K_s is a vector with variables fewer than k , obtained by MFRR, in such a way that the union of $R_{partial}$ and $R_{remaining}$ satisfies **Condition 2**.

Only the values that fall within the lower and upper limits of a particular fuzzy rule are used to relabel the consequent part of fuzzy rules. For a fuzzy rule $R_{(i_1, i_2, \dots, i_n)}$, its lower and upper limits of $b_{(i_1, i_2, \dots, i_n)}$, are obtained using Eq. (3.4) and Eq. (3.5), as explained in Section 3.7. The candidate for relabeling $b_{(i_1, i_2, \dots, i_n)}$ (i.e., b_{rel}) is obtained using Eq. (3.6).

During the relabeling process, a loss function is used to measure the cost incurred for relabeling the fuzzy rules with a new value. The requirements that a function needs to satisfy in order to be naturally considered as a loss function are given in Definition 5.5.

Definition 5.5. An arbitrary loss function L that satisfies four rational properties (i.e., P1, P2, P3 and P4), for $b_i, b_j, b_k \in U(B)$, is considered. An additional property (P5) is outlined.

$$(P1). L(\lambda b_i + (1 - \lambda)b_j) \leq \lambda L(b_i) + (1 - \lambda)L(b_j) , \text{ for } \lambda \in [0,1]$$

$$(P2). L(b_i, b_j) \geq 0$$

$$(P3) L(b_i, b_j) = 0 \text{ iff } b_i = b_j$$

$$(P4). b_i < b_j < b_k \vee b_i > b_j > b_k \implies L(b_i, b_j) \leq L(b_i, b_k) \wedge L(b_j, b_k) \leq L(b_i, b_k)$$

$$(P5). b_i < b_j < b_k \vee b_i > b_j > b_k \implies L(b_i, b_j) < L(b_i, b_k) \wedge L(b_j, b_k) < L(b_i, b_k)$$

The first property (P1) states that a loss function must be a convex function. The second property (P2) is known as the non-negativity property. The third property (P3) is known as the identity of indiscernible. The fourth property (P4) is a weak monotonicity-like property (Rademaker et al., 2009). (P5) is a stronger monotonicity-like property, a special case of (P4). In this chapter, three commonly used loss functions, i.e., absolute loss (L1 loss), square loss (L2 loss), and zero-one loss (0-1 loss) are considered. They are defined as follows:

$$\text{Absolute loss: } L(b_j, b_j^{rel}) = |b_j - b_j^{rel}| \quad (5.3)$$

$$\text{Square loss: } L(b_j, b_j^{rel}) = (b_j - b_j^{rel})^2 \quad (5.4)$$

$$\text{Zero - One loss: } L(b_j, b_j^{rel}) = \min(1, |b_j - b_j^{rel}|) \quad (5.5)$$

where b_j is the original consequent value for the relabeled fuzzy rule R_j and $b_j^{rel} \in b_{rel}$. The total loss, i.e., $Loss$ is computed by summing the loss incurred for each relabeled fuzzy rule, as follows:

$$Loss = \sum_{j_n=1}^{j_n=m_n} \dots \sum_{j_2=1}^{j_2=m_2} \sum_{j_1=1}^{j_1=m_1} L(b_{j_1,j_2,\dots,j_n}, b_{j_1,j_2,\dots,j_n}^{rel}) \quad (5.6)$$

Stage-2 optimality is fulfilled if the relabeled fuzzy rule base (i.e., $b_{j_1,j_2,\dots,j_n}^{rel}$) has the minimum loss. Stage-2 optimality and completeness are always fulfilled as Stage-2 itself is a brute force search. MFRR optimality and completeness are expected to be satisfied using MFRR-BFBT and MFRR-H. However, MFRR-NF produces only Stage-2 optimality for a combination of k -noisy fuzzy rules from Stage-1, in which MFRR optimality may not always be satisfied.

5.4.1 Experiments for Stage-2

In this section, the results of **R3** from MFRR-BFBT, MFRR-NF and MFRR-H with three different loss functions incorporated in Stage-2 are compared. Table 5.2 to Table 5.5 shows the optimal relabeled fuzzy rule bases obtained by using MFRR-BFBT, MFRR-NF and MFRR-H with different setting of loss functions. The obtained results are further summarized in Table 5.6.

Table 5.2: Optimal relabeled fuzzy rule base obtained by MFRR-BFBT, MFRR-NF and MFRR-H using L1 loss function, L2 loss function and 0-1 loss function (with minimum loss value of 40, 600 and 3 respectively)

	x_2				
	μ_2^1	μ_2^2	μ_2^3	μ_2^4	μ_2^5
μ_1^1	$b_{1(1,1)} = 10$	$b_{6(1,2)} = 10$	$b_{11(1,3)} = 20$	$b_{16(1,4)} = 20$	$b_{21(1,5)} = 30$
μ_1^2	$b_{2(2,1)}^{rel} = 10$	$b_{7(2,2)} = 10$	$b_{12(2,3)} = 20$	$b_{17(2,4)} = 30$	$b_{22(2,5)} = 40$
μ_1^3	$b_{3(3,1)} = 20$	$b_{8(3,2)} = 30$	$b_{13(3,3)} = 30$	$b_{18(3,4)} = 40$	$b_{23(3,5)} = 40$
μ_1^4	$b_{4(4,1)}^{rel} = 30$	$b_{9(4,2)} = 30$	$b_{14(4,3)} = 40$	$b_{19(4,4)}^{rel} = 40$	$b_{24(4,5)} = 50$
μ_1^5	$b_{5(5,1)} = 30$	$b_{10(5,2)} = 40$	$b_{15(5,3)} = 40$	$b_{20(5,4)} = 40$	$b_{25(5,5)} = 50$

Table 5.3: Optimal relabeled fuzzy rule base obtained by MFRR-BFBT and MFRR-H using L1 loss function, L2 loss function and 0-1 loss function (with minimum loss value of 40, 600 and 3 respectively)

	x_2				
	μ_2^1	μ_2^2	μ_2^3	μ_2^4	μ_2^5
x_1 μ_1^1	$b_{1(1,1)} = 10$	$b_{6(1,2)} = 10$	$b_{11(1,3)} = 20$	$b_{16(1,4)} = 20$	$b_{21(1,5)} = 30$
μ_1^2	$b_{2(2,1)} = 20$	$b_{7(2,2)}^{rel} = 20$	$b_{12(2,3)} = 20$	$b_{17(2,4)} = 30$	$b_{22(2,5)} = 40$
μ_1^3	$b_{3(3,1)} = 20$	$b_{8(3,2)} = 30$	$b_{13(3,3)} = 30$	$b_{18(3,4)} = 40$	$b_{23(3,5)} = 40$
μ_1^4	$b_{4(4,1)}^{rel} = 30$	$b_{9(4,2)} = 30$	$b_{14(4,3)} = 40$	$b_{19(4,4)}^{rel} = 40$	$b_{24(4,5)} = 50$
μ_1^5	$b_{5(5,1)} = 30$	$b_{10(5,2)} = 40$	$b_{15(5,3)} = 40$	$b_{20(5,4)} = 40$	$b_{25(5,5)} = 50$

Table 5.4: Optimal relabeled fuzzy rule base obtained by MFRR-BFBT, MFRR-NF and MFRR-H using 0-1 loss function (with minimum loss value of 3)

	x_2				
	μ_2^1	μ_2^2	μ_2^3	μ_2^4	μ_2^5
x_1 μ_1^1	$b_{1(1,1)} = 10$	$b_{6(1,2)} = 10$	$b_{11(1,3)} = 20$	$b_{16(1,4)} = 20$	$b_{21(1,5)} = 30$
μ_1^2	$b_{2(2,1)}^{rel} = 10$	$b_{7(2,2)} = 10$	$b_{12(2,3)} = 20$	$b_{17(2,4)} = 30$	$b_{22(2,5)} = 40$
μ_1^3	$b_{3(3,1)} = 20$	$b_{8(3,2)} = 30$	$b_{13(3,3)} = 30$	$b_{18(3,4)} = 40$	$b_{23(3,5)} = 40$
μ_1^4	$b_{4(4,1)}^{rel} = 20$	$b_{9(4,2)} = 30$	$b_{14(4,3)} = 40$	$b_{19(4,4)}^{rel} = 40$	$b_{24(4,5)} = 50$
μ_1^5	$b_{5(5,1)} = 30$	$b_{10(5,2)} = 40$	$b_{15(5,3)} = 40$	$b_{20(5,4)} = 40$	$b_{25(5,5)} = 50$

Table 5.5: Optimal relabeled fuzzy rule base obtained by MFRR-BFBT and MFRR-H using 0-1 loss function (with minimum loss value of 3)

	x_2				
	μ_2^1	μ_2^2	μ_2^3	μ_2^4	μ_2^5
x_1 μ_1^1	$b_{1(1,1)} = 10$	$b_{6(1,2)} = 10$	$b_{11(1,3)} = 20$	$b_{16(1,4)} = 20$	$b_{21(1,5)} = 30$
μ_1^2	$b_{2(2,1)} = 20$	$b_{7(2,2)}^{rel} = 20$	$b_{12(2,3)} = 20$	$b_{17(2,4)} = 30$	$b_{22(2,5)} = 40$
μ_1^3	$b_{3(3,1)} = 20$	$b_{8(3,2)} = 30$	$b_{13(3,3)} = 30$	$b_{18(3,4)} = 40$	$b_{23(3,5)} = 40$
μ_1^4	$b_{4(4,1)}^{rel} = 20$	$b_{9(4,2)} = 30$	$b_{14(4,3)} = 40$	$b_{19(4,4)}^{rel} = 40$	$b_{24(4,5)} = 50$
μ_1^5	$b_{5(5,1)} = 30$	$b_{10(5,2)} = 40$	$b_{15(5,3)} = 40$	$b_{20(5,4)} = 40$	$b_{25(5,5)} = 50$

Table 5.6: Minimum loss obtained with different loss functions in Stage-2 for **R3**.

Information		MFRR-BFBT			MFRR-NF			MFRR-H		
		L1	L2	0-1	L1	L2	0-1	L1	L2	0-1
Number of	optimal solutions	2	2	4	1	1	2	2	2	4
Minimum loss value		40	600	3	40	600	3	40	600	3

As shown in Table 5.6, MFRR-BFBT, MFRR-NF and MFRR-H produce a loss of 40, 600 and 3, for the L1 loss function, L2 loss function, and 0-1 loss function, respectively. MFRR-BFBT and MFRR-H produce two optimal solutions for the L1 loss function and L2 loss function, and four optimal solutions for the 0-1 loss function. All optimal solutions are obtained using the MFRR-BFBT and MFRR-H procedures in Stage-2. More optimal solutions can be obtained from the 0-1 loss function, as it does not satisfy (P5).

Using MFRR-NF, only one optimal solution is obtained for the L1 loss function and L2 loss function, and two optimal solutions are obtained for the 0-1 loss function. In this example, while MFRR optimality is satisfied, MFRR completeness cannot be achieved by using MFRR-NF, mainly due to the failure of fulfilling Stage-1 completeness. Although MFRR-NF does not guarantee an MFRR optimal solution, it is still useful for solving large-size problems, since its computational time is shorter than those of MFRR-BFBT and MFRR-H.

5.5 Stage-3: Selection of the Relabeled Fuzzy Rule Base

While Stage-2 may produce multiple optimal solutions, it is not clear how to select a suitable solution among the optimal solutions. In Stage-3, the optimal solutions are examined subjectively or objectively. Subjectively, the optimal solutions generated by MFRR can be examined and selected by the domain experts, based on their knowledge and experience. MFRR is useful for assisting domain experts in identifying noisy fuzzy rules and offering a number of optimal solutions for selection. Objectively, the optimal solutions can be analyzed by adopting the following criteria:

- i. Histogram analysis

The relabeled values for the noisy fuzzy rules are analyzed. A more uniform distribution for the relabeled values is preferred.

ii. Smooth surface curve

The optimal solutions can be chosen based on their surface curve. A smooth surface curve, by visual inspection, is preferred.

As an example, the four relabeled fuzzy rule bases in Tables 5.2, 5.3, 5.4 and 5.5 are considered. The histogram distribution for each relabeled fuzzy rule base are shown in Figures 5.7, 5.8, 5.9 and 5.10, respectively. It can be observed that the relabeled fuzzy rule bases as shown in Tables 5.2, 5.3 and 5.4 are associated with uniform distributions. However, this is not the case for the relabeled fuzzy rule base in Table 5.5. In short, the relabeled fuzzy rule bases in Tables 5.2, 5.3 and 5.4 are the preferred.

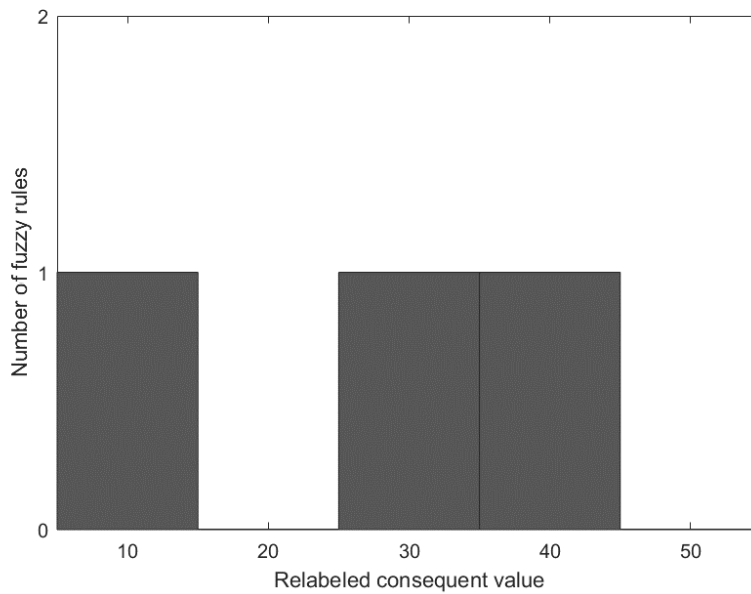


Figure 5.7: Histogram distribution for relabeled fuzzy rule base in Table 5.2

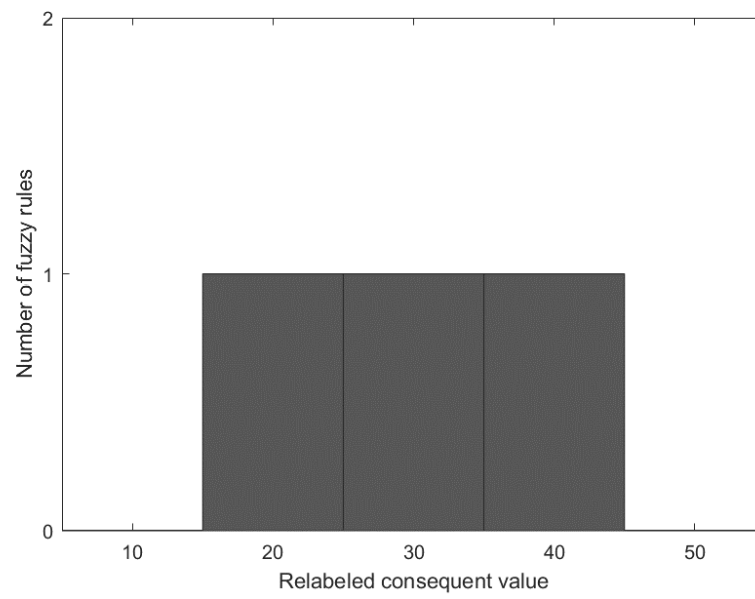


Figure 5.8: Histogram distribution for relabeled fuzzy rule base in Table 5.3

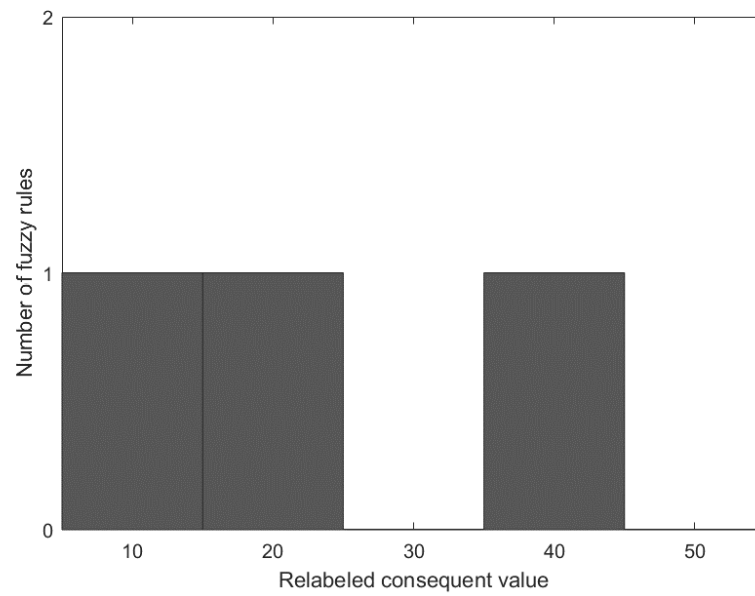


Figure 5.9: Histogram distribution for relabeled fuzzy rule base in Table 5.4

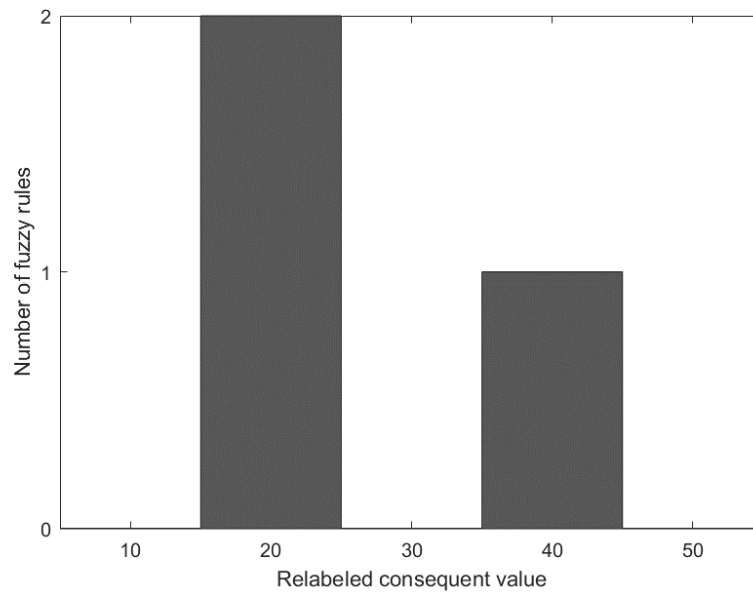


Figure 5.10: Histogram distribution for relabeled fuzzy rule base in Table 5.5

The three potential optimal solutions could also be further evaluated using the second selection criteria. Figures 5.11, 5.12 and 5.13 show the surface plots for the relabeled fuzzy rule base in Tables 5.2, 5.3 and 5.4, respectively. Relabeled fuzzy rule base in Table 5.3 has the smoothest surface plot (as shown in Figure 5.12), and it maybe preferred in many applications.

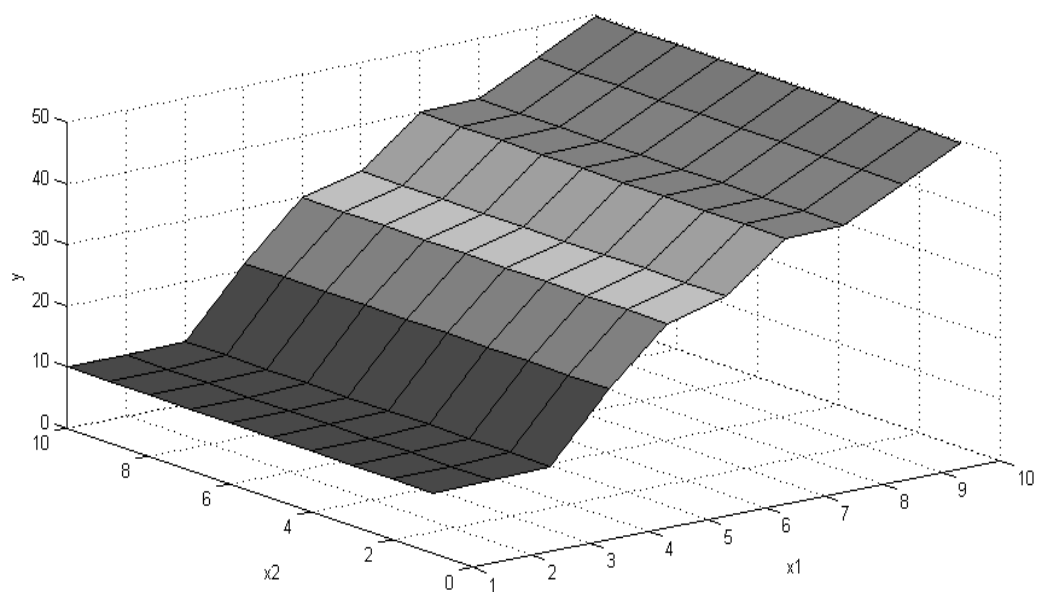


Figure 5.11: Surface plot of the two input FIS-model using relabeled fuzzy rule base in Table 5.2

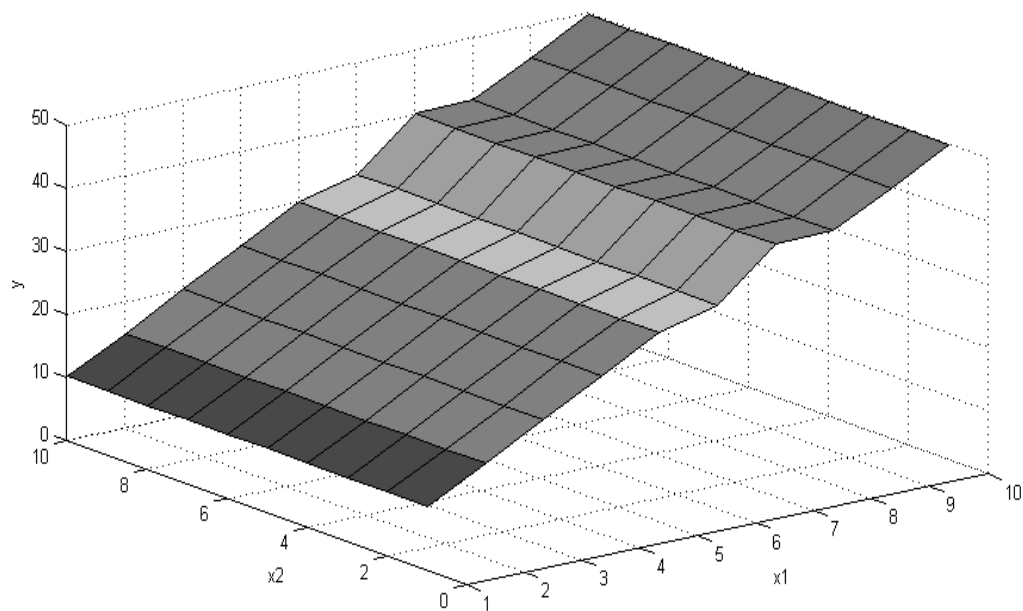


Figure 5.12: Surface plot of the two input FIS-model using relabeled fuzzy rule base in Table 5.3

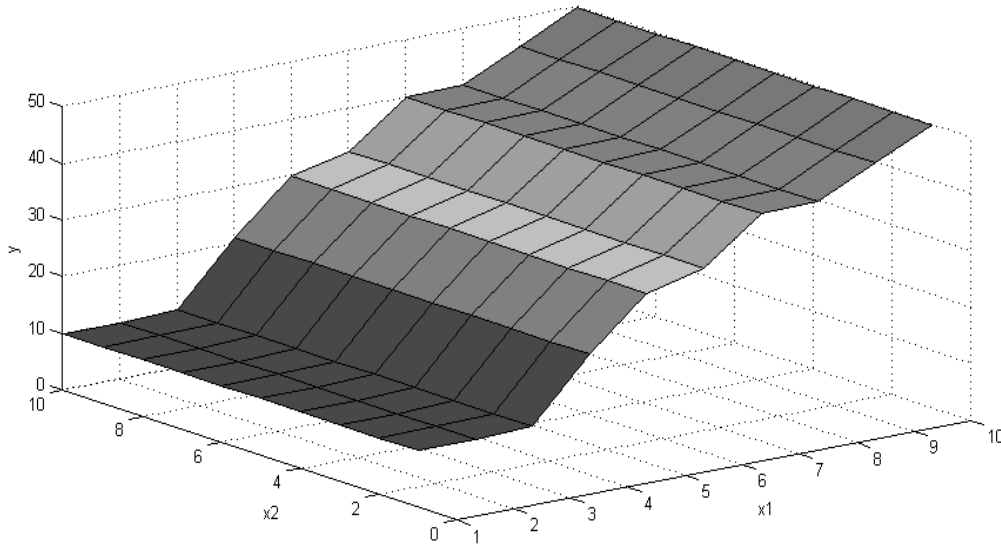


Figure 5.13: Surface plot of the two input FIS-model using relabeled fuzzy rule base in Table 5.4

5.6 Summary

In this chapter, MFRR is formulated as a three-stage framework for implementation. Stage-1 is identified as a combinatorial problem, which is time consuming when the problem size is large. Three monotone fuzzy rule relabeling techniques, i.e., MFRR-BFBT, MFRR-NF and MFRR-H, have been proposed and evaluated. MFRR-NF outperforms MFRR-BFBT and MFRR-H in terms of computational time, but it does not guarantee global optimal solutions. While MFRR-BFBT and MFRR-H provide global optimal solutions, they require a long computational time to yield the solutions. Comparatively, MFRR-H is faster than MFRR-BFBT, but its computational cost is still high. In general, the computation time for the proposed MFRR approaches are arranged in order, i.e., $\text{MFRR-NF} < \text{MFRR-H} < \text{MFRR-BFBT}$. In the next chapter, application of MFRR to Failure Mode and Effect Analysis (FMEA) will be presented.

CHAPTER 6

APPLICATIONS TO FAILURE MODE AND EFFECT ANALYSIS

6.1 Introduction

In the previous chapter, a practical MFRR framework for zero-order TSK fuzzy inference system was devised. In this chapter, the importance of using the developed monotone fuzzy rules relabeling (MFRR) method in conjunction with FIS models that utilise fuzzy rules provided by humans for tackling assessment/evaluation and decision-making problems is shown. Specifically, the MFRR framework is applied to undertake Failure Mode and Effect Analysis (FMEA). Two publicly available information pertaining to FMEA of a sewage treatment plant (Hsieh, 2006) and a public hospital (Kumru & Kumru, 2013) are used to demonstrate the usefulness of the proposed MFRR framework, and promising results are obtained.

This chapter is organized as follows. In Section 6.2, the background of Failure Mode and Effect Analysis is introduced and explained. In Section 6.3, two benchmark information are presented, and their experimental results are further discussed. The concluding remarks for Chapter 6 is given in Section 6.4.

6.2 Background

FMEA is a useful reliability analysis tool for evaluating the risks associated with potential failure modes of a complex system or process (Bowles & Pelaez, 1995; Jee et al., 2015; Kerk et al., 2017). FMEA is first developed as a formal design methodology by the

aerospace industry in 1960s (Bowles & Pelaez, 1995), since then, it has found successful application in a wide range of industries such as automotive (Xu, Tang, Xie, Ho, & Zhu, 2002), nuclear (Guimarães & Lapa, 2004; Tashjian, 1975), semiconductor (Tay & Lim, 2006), agriculture (Jong, Tay, & Lim, 2013), etc. The main purpose of FMEA is to help recognizing and evaluating the potential failures of a product, systems, process or services and its effect, as well as identifies corrective actions that can eliminate or reduce the chances of potential failures/risks from occurring/recurring (Stamatis, 2003).

In conventional FMEA, the risk of a failure mode is determined by a Risk Priority Number (RPN) model (Stamatis, 2003). The RPN score is obtained by multiplication of three input factors, i.e., Severity (S), Occurrence (O) and Detection (D), as shown in Eq. (6.1), whereby S is an assessment of the seriousness or effects of the failure, O is the frequency or likelihood a failure occurring, and D is the ability of detecting the failure.

$$RPN = S \times O \times D \quad (6.1)$$

Even though the conventional FMEA methodology has been a well-accepted safety analysis method (Pillay & Wang, 2003), however, it suffers from several limitations. Information of FMEA are often qualitatively expressed and described in natural language, such as “likely”, “very high” etc., and thus it is difficult for conventional FMEA to precisely determine or evaluate these linguistic variables (Xu et al., 2002). Besides that, the evaluations of conventional FMEA approach are quantitative, ignoring the fact that relationship between the output RPN score and inputs S, O and D are depend on qualitative assessment (Bowles & Pelaez, 1995). In addition, different product combinations of S, O and D input factors may produce identical value of RPN score, even though the hidden risk implications may be completely different (Pillay & Wang, 2003; Xu et al., 2002).

As such, various techniques have been proposed as solutions to tackle the limitations imposed by the conventional FMEA methodology (Liu, Liu, & Liu, 2013). These solutions can be classified into five categories, i.e., artificial intelligence based method, mathematical programming methods, multi-criteria decision-making methods, integrated methods and other methods (Liu et al., 2013). In this thesis, the focus is on artificial intelligence based method, specifically, using fuzzy inference system (FIS) to replace the conventional RPN model (Bowles & Pelaez, 1995; Jee et al., 2015; Kerk et al., 2017). Fuzzy inference system based RPN model is preferred to the conventional RPN model due to the following reasons:

- i. It allows customization of the RPN model based on expert's knowledge, whereby such knowledge is represented in the form of fuzzy "IF-THEN" rules (Jong et al., 2013).
- ii. It allows non-linear relationship between the RPN score and the input risk factors to be formulated (Bowles & Pelaez, 1995).
- iii. It captures the three inputs risk factors qualitatively, instead of quantitatively (Bowles & Pelaez, 1995).
- iv. It is robust against uncertainty and vagueness due to fuzzy information processing (Pillay & Wang, 2003).

6.2.1 Fuzzy Inference System (FIS)-Based RPN Model

In this section, a zero-order TSK FIS-based RPN model (hereafter denoted as FIS-RPN model) is explained. Figure 6.1 depicts the procedure for the FMEA with FIS-RPN model, with detailed steps explained as follows.

Step 1: Define the scale table of S, O and D.

- Step 2:** Design the membership function for Severity, Occurrence and Detection.
- Step 3:** Gather expert's knowledge in the format of Fuzzy IF-THEN rule.
- Step 4:** Study the purpose, objective and goal of a product/process. Identify interaction among components/process flow diagram followed by analyze each task.
- Step 5:** Identify potential failures of product/process, which include problems, concerns, and opportunity of improvement.
- Step 6:** Identify consequence of failures to other components/next processes, operation, customers, and government regulations.
- Step 7:** Identify the potential root cause of potential failures.
- Step 8:** Conduct procedure to detect/prevent failure of product/process.
- Step 9:** Rank the seriousness/hazard level of the effect of the potential failures.
- Step 10:** Estimate the frequency for a potential cause of failure.
- Step 11:** Determine the likelihood of the process control to detect a specific root cause of failure.
- Step 12:** Compute the fuzzy RPN score, using Eq. (6.2) (Jee et al., 2015).

$$RPN(S, O, D) = \frac{\sum_{n_S=1}^{m_S} \sum_{n_O=1}^{m_O} \sum_{n_D=1}^{m_D} (\mu_S^{n_S}(S) \times \mu_O^{n_O}(O) \times \mu_D^{n_D}(D) \times b^{n_S, n_O, n_D})}{\sum_{n_S=1}^{m_S} \sum_{n_O=1}^{m_O} \sum_{n_D=1}^{m_D} (\mu_S^{n_S}(S) \times \mu_O^{n_O}(O) \times \mu_D^{n_D}(D))} \quad (6.2)$$

where m_S is the number of membership function for S, m_O is the number of membership function for O, m_D is the number of membership function for Detection, $\mu_S^{n_S}(S)$ is the firing strength for S, $\mu_O^{n_O}(O)$ is the firing strength for O, $\mu_D^{n_D}(D)$ is the firing strength for D, and b^{n_S, n_O, n_D} is the fuzzy singleton value for the corresponding fuzzy rules.

Step 13: Determine action of correction. If correction is needed, go back to Step 4.

Step 14: End.

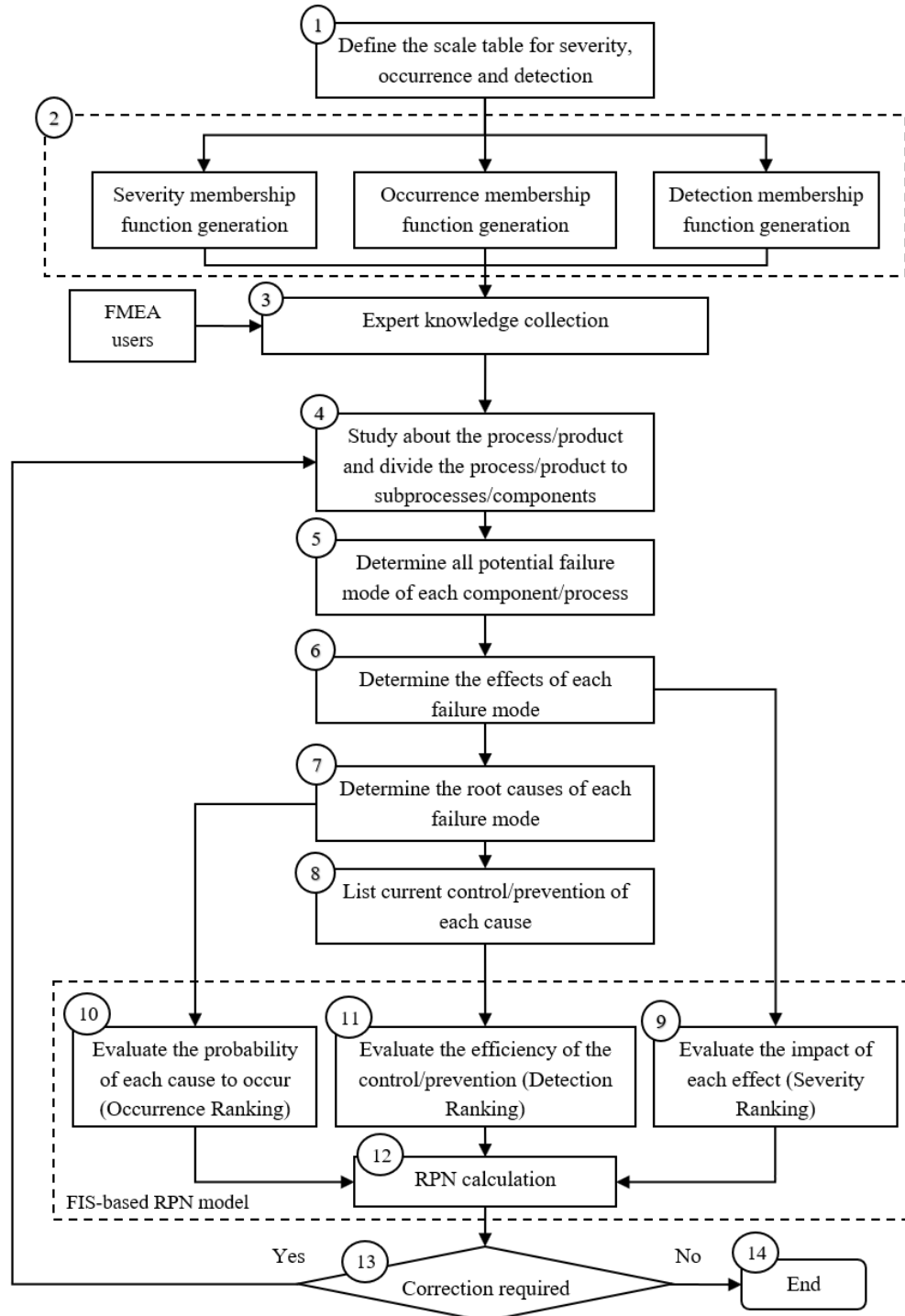


Figure 6.1: FMEA procedure with FIS-RPN model (Tay & Lim, 2006)

6.2.2 Fuzzy Inference System (FIS)-Based RPN Model with Monotone Fuzzy Rule Relabeling

It is important to maintain the monotonicity property of the FIS-RPN model, i.e., the RPN score should not decrease if any of the three risk factors increases (Jee et al., 2015; Kerk et al., 2017). A monotone fuzzy rule base is one of the essential parts for building a monotone FIS-RPN model. Therefore, it is important for the collected fuzzy rules to fulfil the monotonicity property, in order to ensure validity of the produced fuzzy RPN scores. However, in practice, while fuzzy rules are gathered from humans (even from domain experts), it is possible for the resulting fuzzy rule base to violate the monotonicity property due to judgment errors.

To tackle the problems of non-monotone fuzzy rule base, an enhancement to the FIS-RPN model using MFRR is proposed. Figure 6.2 shows the procedure for the FIS-RPN model with MFRR. An additional step (i.e., perform monotone fuzzy rule relabeling) was included in the procedure to relabel the non-monotone fuzzy rules collected from experts. The MFRR framework proposed in Chapter 5 was adopted to ensure the monotonicity of the fuzzy rule base is preserved/restored in an optimal way. The relabeled fuzzy rule base is then can be used to produce a monotone FIS-RPN model.

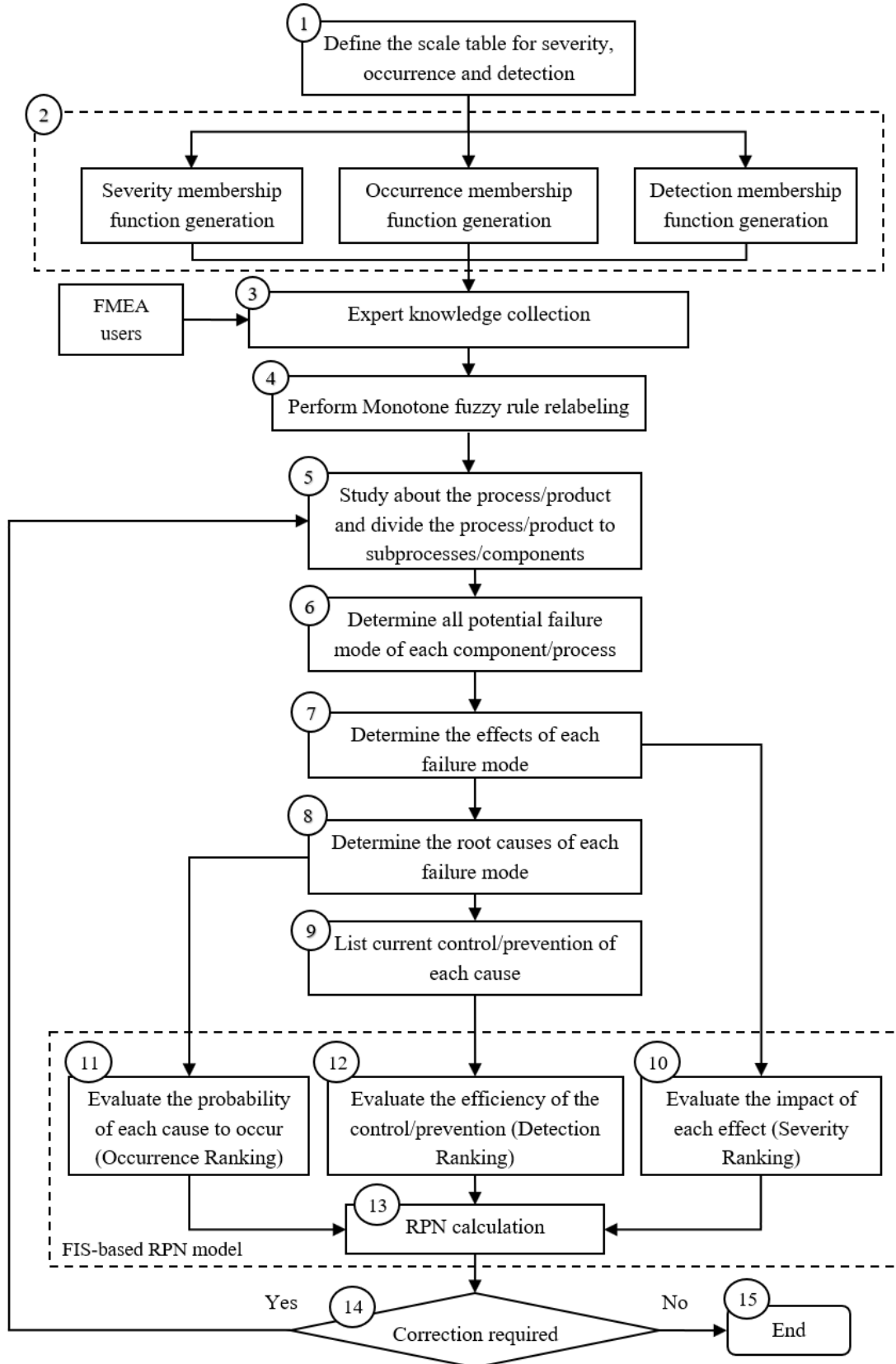


Figure 6.2: FIS-RPN model with enhancement of MFRR

6.3 Benchmark Information

In this section, the importance of the proposed MFRR procedure is demonstrated using two publicly available FMEA fuzzy rule base, i.e. sewage treatment plant (Hsieh, 2006) and the purchasing process of a public hospital(Kumru & Kumru, 2013).

6.3.1 Sewage Treatment Plant (Hsieh, 2006)

6.3.1.1 Background

In this case study, the application of FIS-RPN model to a sewage treatment system in Taiwan is presented. A scale of 1 to 10 is used for quantifying the three risk factors, i.e., $x_{Severity}, x_{Occurrence}, x_{Deteticion} \in [1,10]$. Each input is assigned with five linguistic terms, i.e., *Remote*, *Low*, *Moderate*, *High* and *Very High*, respectively, and their descriptions are explained in Tables 6.1, 6.2 and 6.3 respectively (Hsieh, 2006).

Table 6.1: The representation of linguistic terms for severity (Hsieh, 2006)

Linguistic term	Severity
Remote	The failure has no effect on the performance of system, and customers or operators may not notice the failure.
Low	The failure causes a slight notice, but doesn't deteriorate the system. Few customers will complain the failure.
Moderate	The failure causes a slight deterioration in the system and a high degree of customers and operator dissatisfaction.
High	The failure causes significant deterioration in system performance or minor injuries.
Very High	The failure would seriously affect the ability to complete the function or lead to serious injuries or death.

Table 6.2: The representation of linguistic terms for Occurrence (Hsieh, 2006)

Linguistic term	Occurrence
Remote	The failure is unlikely.
Low	Relatively few failures.
Moderate	Likely to occur more than once.
High	Repeated failures.
Very High	Failure is almost inevitable.

Table 6.3: The representation of linguistic terms for Detection (Hsieh, 2006)

Linguistic term	Detection
Remote	Failure remains undetected until the function of system is ceased.
Low	Failure remains undetected until the system performance is reduced.
Moderate	Failure remains undetected until the system performance is affected.
High	Failure may be detected during inspection.
Very High	Failure would almost be detected during inspection.

The rating scale for the three parameters, i.e., severity, occurrence and detection are given in Table 6.4, 6.5 and 6.6 respectively (Hsieh, 2006). Severity score reflects the seriousness of the effect of a failure mode. Failure mode has a higher rating of severity means it has a greater effect on the subsystem or system. Occurrence is the likelihood or probability of the failure mode occurring. Failure mode has a higher rating of occurrence means it has a higher chance of occurrence. Detection is the inability to detect the failure mode. Failure mode has a higher rating of detection means the failure is harder to be detected if it occurs (Hsieh, 2006).

Table 6.4: Severity rating scale for sewage treatment system (Hsieh, 2006)

Rating	Definition
10	Failure would injure operators or break government regulation without warning.
9	Failure would injure operators or break government regulation with warning.
8	System is inoperable. (loss of primary function)
7	System is operable at a reduced level of performance.
6	System is operable but subsystem is operable at a reduced level of performance.
5	System is operable but has a high level of operator dissatisfaction.
4	Failure causes a slight notice, but there is minor performance loss.
3	Failure causes a slight notice, but without performance loss.
2	Failure has no effect on the performance, and operators may ignore the failure.
1	No significant effect.

Table 6.5: Occurrence rating scale for sewage treatment system (Hsieh, 2006)

Rating	Definition
10	More than one occurrence per day.
9	One occurrence every three to four days.
8	One occurrence per week.
7	One occurrence every month.
6	One occurrence every three months.
5	One occurrence every six months.
4	One occurrence per year.
3	One occurrence every one to three years.
2	One occurrence every three to five years.
1	One occurrence in greater than five years.

Table 6.6: Detection rating scale for sewage treatment system (Hsieh, 2006)

Rating	Definition
10	Design control won't or can't detect this failure.
9	Very remote chance the design control will detect this failure.
8	Remote chance the design control will detect this failure.
7	Very low chance the design control will detect this failure.
6	Low chance the design control will detect this failure.
5	Moderate chance the design control will detect this failure.
4	Moderately high chance the design control will detect this failure.
3	High chance the design control will detect this failure.
2	Very high chance the design control will detect this failure.
1	Design control will almost certainly detect this failure.

The linguistic terms are represented by Gaussian MFs, as shown in Figures 6.3, 6.4 and 6.5. Figures 6.6, 6.7 and 6.8 illustrate the projections of the Gaussian MFs for Severity, Occurrence, and Detection, respectively, using the equation in **Corollary 3.1**. Noted that the Gaussian MFs have been designed to satisfy **Condition 1** of **Theorem 3.1**. As an example, the Gaussian MFs for Severity in Figure 6.3 can be projected as linear lines, as illustrated in Figure 6.6. The linear line with label $E_{Severity}^{Very\ High}$ is always larger in magnitude than that of $E_{Severity}^{High}$. Therefore, $E_{Severity}^{Very\ High} \geq E_{Severity}^{High} \geq E_{Severity}^{Moderate} \geq E_{Severity}^{Low} \geq E_{Severity}^{Remote}$ is always satisfied for $x_{Severity} \in [1,10]$. The same observation applies to Figure 6.7 and Figure 6.8 for Occurrence and Detection.

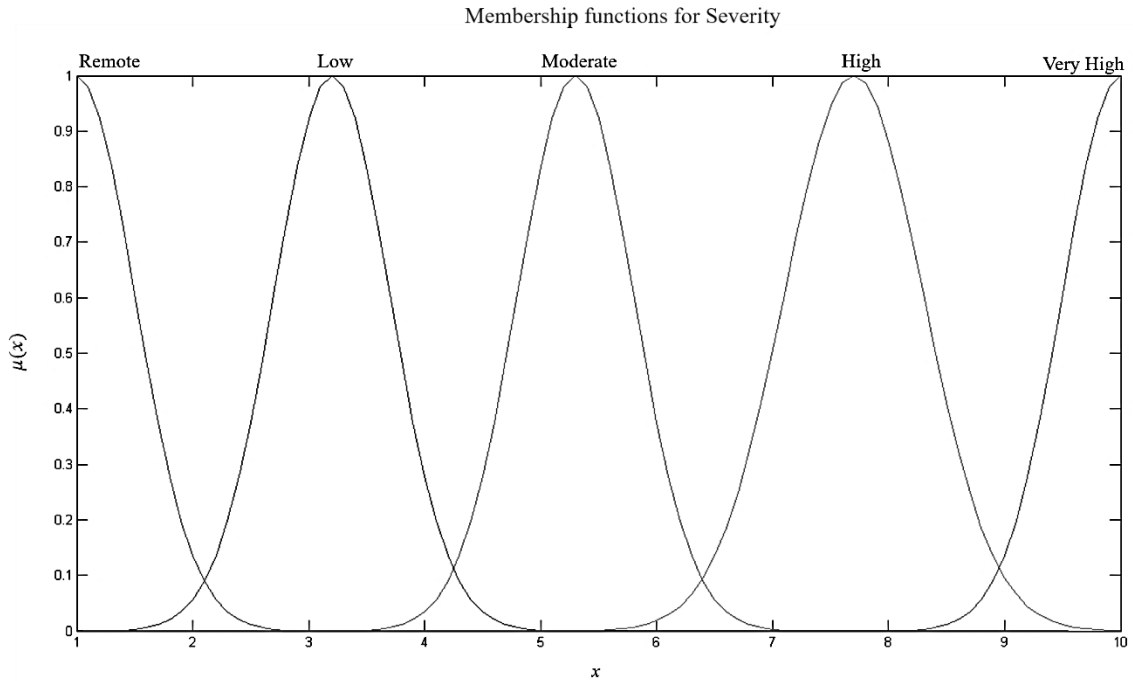


Figure 6.3: The Gaussian membership function for Severity

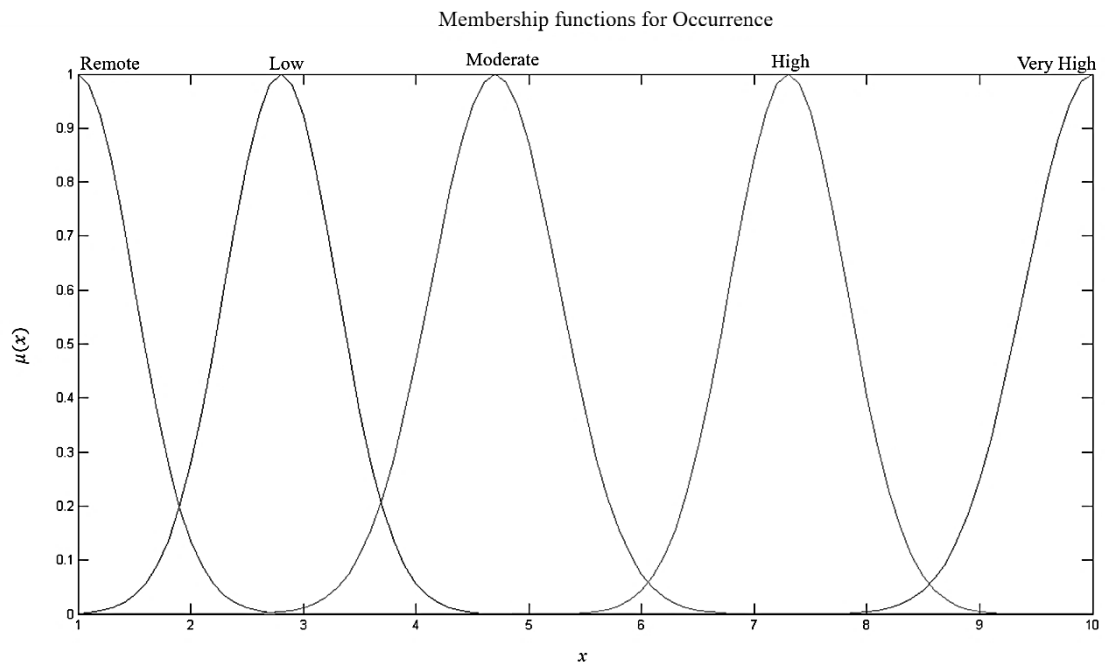


Figure 6.4: The Gaussian membership function for Occurrence

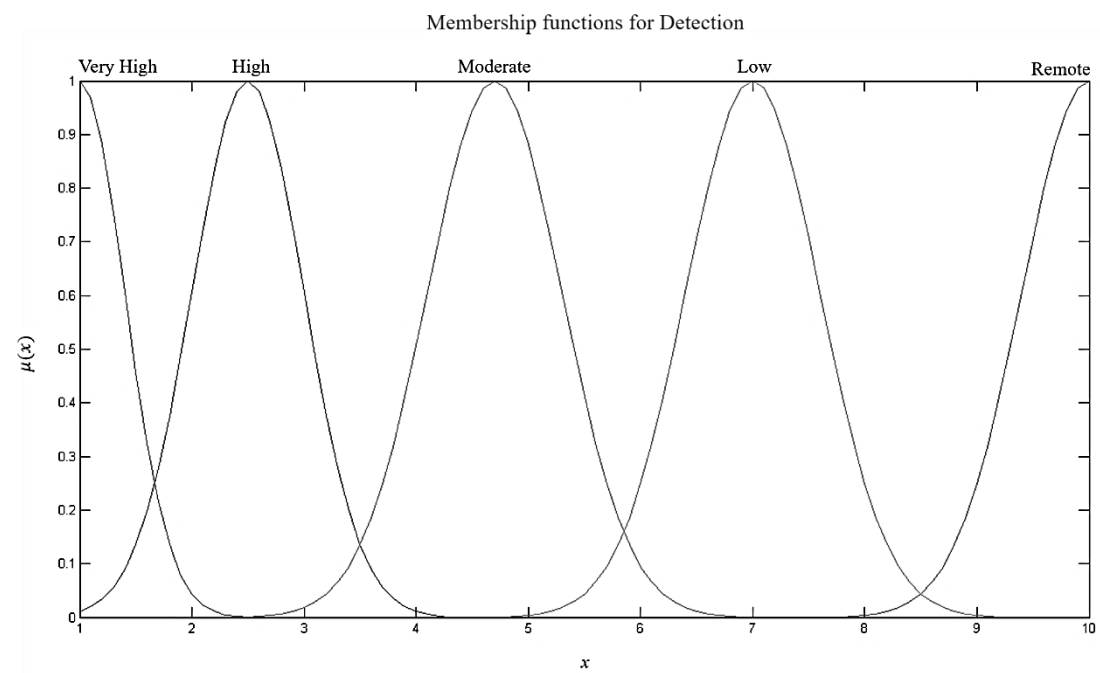


Figure 6.5: The Gaussian membership function for Detection

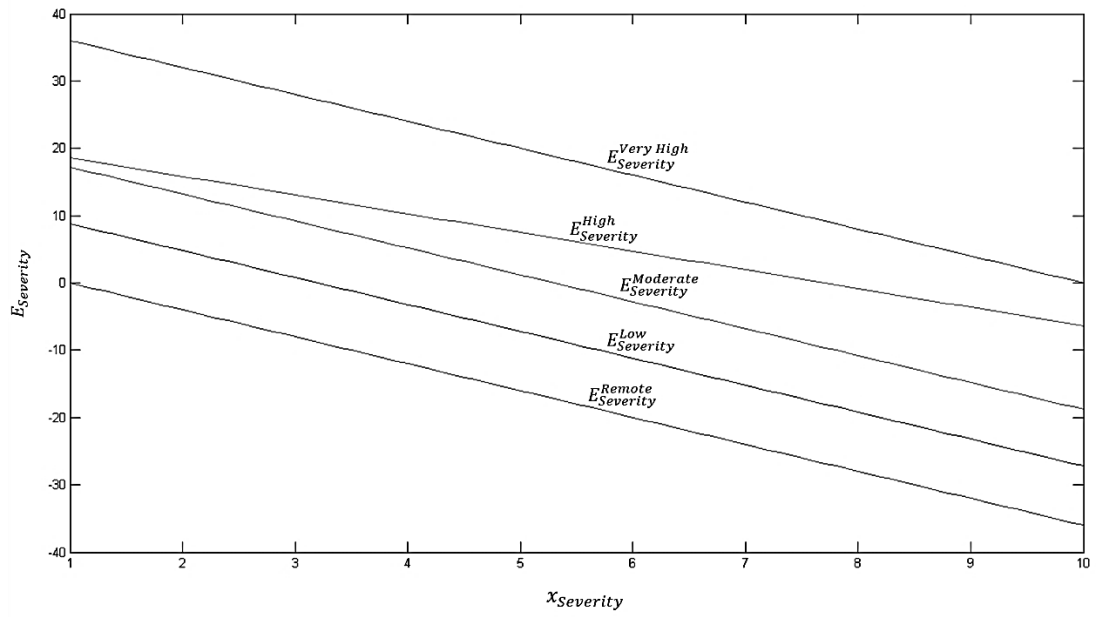


Figure 6.6: Projection of the Gaussian membership functions for Severity

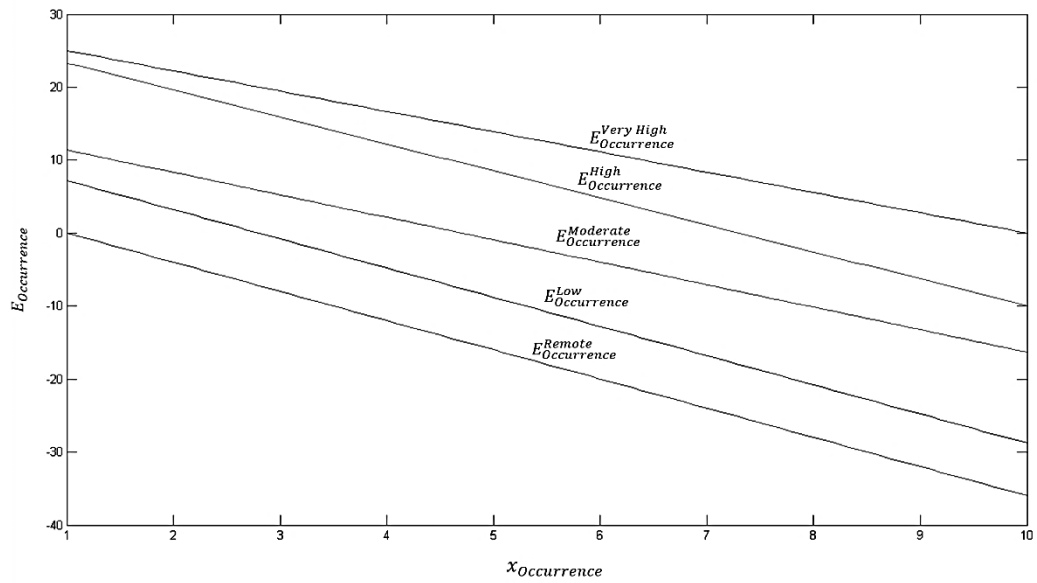


Figure 6.7: Projection of the Gaussian membership functions for Occurrence

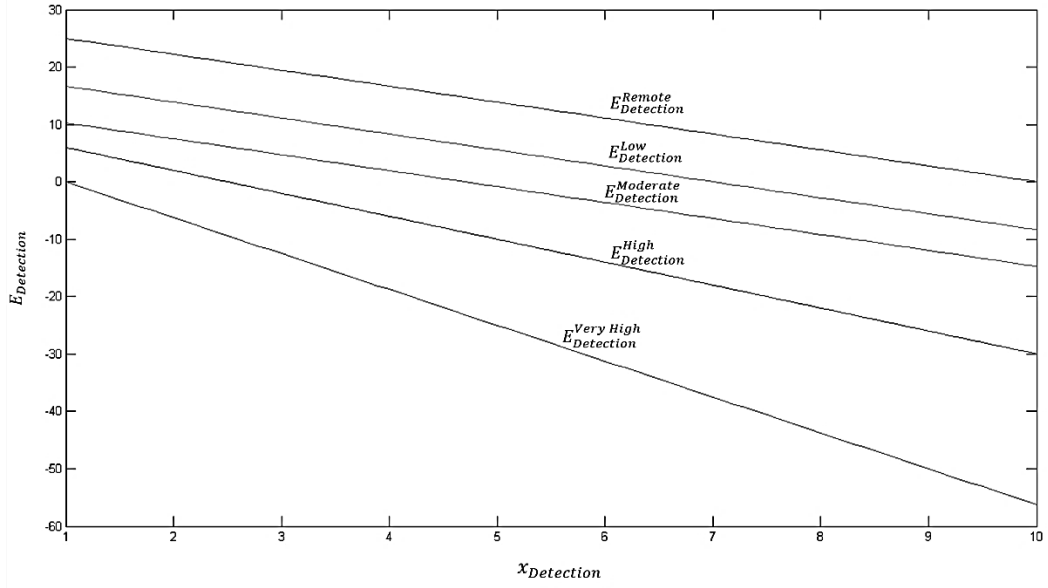


Figure 6.8: Projection of the Gaussian membership functions for Detection

The fuzzy rule consequents are represented with five linguistic terms, i.e., *Low*, *Fairly Low*, *Moderate*, *Fairly High* and *High*. Three experts were asked to provide a value of b to characterize the linguistic terms. Each expert were assigned with a degree of competent, w_i , where $i = 1, 2, 3$, according to their experience and knowledge about this domain. The sum of the degree of competence must equal to one. The value of b for each linguistic term are obtained by using the Eq. (6.3).

$$b = \sum_{i=1}^3 w_i b_i \quad (6.3)$$

The five linguistic terms are characterized by five singleton values, i.e., $b_{Low} = 1$, $b_{Fairly Low} = 2.5$, $b_{Moderate} = 3.7$, $b_{Fairly High} = 6.1$, $b_{High} = 10$ respectively. The relationship between the three inputs and the output are represented by a set of fuzzy IF-THEN rules (Hsieh, 2006).

The collected fuzzy rule base for the sewage treatment plant is denoted as **R4**. A total of 125 fuzzy rules were provided, and parts of the fuzzy rules are shown in Figure 6.9. To ease the explanation, **R4** is reproduced in a grid partition form, as shown in Figure 6.10. As observed, **R4** was non-monotone, i.e., violating **Condition 2** of **Theorem 3.1**. The fuzzy rules that form non-monotone pair with each other were shaded. As an example, R_{18} and R_{19} (see Figure 6.9) contained the same fuzzy linguistic terms for Severity and Occurrence, i.e., *Remote* and *High*, respectively. The fuzzy linguistic terms for Detection in R_{18} and R_{19} were *Moderate* and *Low*, respectively. Therefore, the singleton for fuzzy consequent of R_{19} was expected to be equal or higher than that of R_{18} (i.e., $b_{18} \leq b_{19}$), but this was not the case. Such judgement errors were treated as noise in this experiment. Therefore, MFRR is adopted to relabel **R4** and the results are discussed in the next section.

R_1	: If Severity is Remote and Occurrence is Remote and Detection is Very High Then Risk is Low
R_2	: If Severity is Remote and Occurrence is Remote and Detection is High Then Risk is Low
R_3	: If Severity is Remote and Occurrence is Remote and Detection is Moderate Then Risk is Low
R_4	: If Severity is Remote and Occurrence is Remote and Detection is Low Then Risk is Fairly Low
R_5	: If Severity is Remote and Occurrence is Remote and Detection is Remote Then Risk is Fairly Low
R_6	: If Severity is Remote and Occurrence is Low and Detection is Very High Then Risk is Low
R_7	: If Severity is Remote and Occurrence is Low and Detection is High Then Risk is Low
R_8	: If Severity is Remote and Occurrence is Low and Detection is Moderate Then Risk is Fairly Low
R_9	: If Severity is Remote and Occurrence is Low and Detection is Low Then Risk is Fairly Low
R_{10}	: If Severity is Remote and Occurrence is Low and Detection is Remote Then Risk is Fairly Low
R_{11}	: If Severity is Remote and Occurrence is Moderate and Detection is Very High Then Risk is Low
R_{12}	: If Severity is Remote and Occurrence is Moderate and Detection is High Then Risk is Low
R_{13}	: If Severity is Remote and Occurrence is Moderate and Detection is Moderate Then Risk is Fairly Low
R_{14}	: If Severity is Remote and Occurrence is Moderate and Detection is Low Then Risk is Fairly Low
R_{15}	: If Severity is Remote and Occurrence is Moderate and Detection is Remote Then Risk is Moderate
R_{16}	: If Severity is Remote and Occurrence is High and Detection is Very High Then Risk is Low
R_{17}	: If Severity is Remote and Occurrence is High and Detection is High Then Risk is Fairly High
R_{18}	: If Severity is Remote and Occurrence is High and Detection is Moderate Then Risk is Fairly High
R_{19}	: If Severity is Remote and Occurrence is High and Detection is Low Then Risk is Moderate
R_{20}	: If Severity is Remote and Occurrence is High and Detection is Remote Then Risk is Moderate
R_{21}	: If Severity is Remote and Occurrence is Very High and Detection is Very High Then Risk is Fairly Low
R_{22}	: If Severity is Remote and Occurrence is Very High and Detection is High Then Risk is Fairly Low
R_{23}	: If Severity is Remote and Occurrence is Very High and Detection is Moderate Then Risk is Moderate
R_{24}	: If Severity is Remote and Occurrence is Very High and Detection is Low Then Risk is Moderate
R_{25}	: If Severity is Remote and Occurrence is Very High and Detection is Remote Then Risk is Fairly High
$\vdots$	
R_{86}	: If Severity is High and Occurrence is Medium and Detection is Very High Then Risk is Fairly High
$\vdots$	
R_{111}	: If Severity is Very High and Occurrence is Medium and Detection is Very High Then Risk is Moderate
$\vdots$	
R_{125}	: If Severity is Very High and Occurrence is Very High and Detection is Remote Then Risk is High

Figure 6.9: Part of the fuzzy rules of **R4** (Hsieh, 2006)

		$x_{occurrence}$				
		$\mu_2^1=Remote$	$\mu_2^2=Low$	$\mu_2^3=Moderate$	$\mu_2^4=High$	$\mu_2^5=Very High$
$x_{Severity}$ $\mu_3^1 = Remote$	$x_{Detection}$ $\mu_1^1=Very High$	$b_{1(1,1,1)} = 1$	$b_{6(1,2,1)} = 1$	$b_{11(1,3,1)} = 1$	$b_{16(1,4,1)} = 1$	$b_{21(1,5,1)} = 2.5$
	$\mu_1^2=High$	$b_{2(2,1,1)} = 1$	$b_{7(2,2,1)} = 1$	$b_{12(2,3,1)} = 1$	$b_{17(2,4,1)} = 6.1$	$b_{22(2,5,1)} = 2.5$
	$\mu_1^3=Moderate$	$b_{3(3,1,1)} = 1$	$b_{8(3,2,1)} = 2.5$	$b_{13(3,3,1)} = 2.5$	$b_{18(3,4,1)} = 6.1$	$b_{23(3,5,1)} = 3.7$
	$\mu_1^4=Low$	$b_{4(4,1,1)} = 2.5$	$b_{9(4,2,1)} = 2.5$	$b_{14(4,3,1)} = 2.5$	$b_{19(4,4,1)} = 3.7$	$b_{24(4,5,1)} = 3.7$
	$\mu_1^5=Remote$	$b_{5(5,1,1)} = 2.5$	$b_{10(5,2,1)} = 2.5$	$b_{15(5,3,1)} = 3.7$	$b_{20(5,4,1)} = 3.7$	$b_{25(5,5,1)} = 6.1$
$x_{Severity}$ $\mu_3^2 = Low$	$x_{Detection}$ $\mu_1^1=Very High$	$b_{26(1,1,2)} = 1$	$b_{31(1,2,2)} = 1$	$b_{36(1,3,2)} = 1$	$b_{41(1,4,2)} = 2.5$	$b_{46(1,5,2)} = 2.5$
	$\mu_1^2=High$	$b_{27(2,1,2)} = 1$	$b_{32(2,2,2)} = 2.5$	$b_{37(2,3,2)} = 2.5$	$b_{42(2,4,2)} = 2.5$	$b_{47(2,5,2)} = 3.7$
	$\mu_1^3=Moderate$	$b_{28(3,1,2)} = 2.5$	$b_{33(3,2,2)} = 2.5$	$b_{38(3,3,2)} = 2.5$	$b_{43(3,4,2)} = 3.7$	$b_{48(3,5,2)} = 3.7$
	$\mu_1^4=Low$	$b_{29(4,1,2)} = 2.5$	$b_{34(4,2,2)} = 2.5$	$b_{39(4,3,2)} = 3.7$	$b_{44(4,4,2)} = 3.7$	$b_{49(4,5,2)} = 6.1$
	$\mu_1^5=Remote$	$b_{30(5,1,2)} = 2.5$	$b_{35(5,2,2)} = 3.7$	$b_{40(5,3,2)} = 3.7$	$b_{45(5,4,2)} = 6.1$	$b_{50(5,5,2)} = 6.1$
$x_{Severity}$ $\mu_3^3 = Moderate$	$x_{Detection}$ $\mu_1^1=Very High$	$b_{51(1,1,3)} = 1$	$b_{56(1,2,3)} = 1$	$b_{61(1,3,3)} = 2.5$	$b_{66(1,4,3)} = 2.5$	$b_{71(1,5,3)} = 3.7$
	$\mu_1^2=High$	$b_{52(2,1,3)} = 2.5$	$b_{57(2,2,3)} = 2.5$	$b_{62(2,3,3)} = 2.5$	$b_{67(2,4,3)} = 3.7$	$b_{72(2,5,3)} = 3.7$
	$\mu_1^3=Moderate$	$b_{53(3,1,3)} = 2.5$	$b_{58(3,2,3)} = 2.5$	$b_{63(3,3,3)} = 3.7$	$b_{68(3,4,3)} = 3.7$	$b_{73(3,5,3)} = 6.1$
	$\mu_1^4=Low$	$b_{54(4,1,3)} = 2.5$	$b_{59(4,2,3)} = 3.7$	$b_{64(4,3,3)} = 6.1$	$b_{69(4,4,3)} = 6.1$	$b_{74(4,5,3)} = 6.1$
	$\mu_1^5=Remote$	$b_{55(5,1,3)} = 3.7$	$b_{60(5,2,3)} = 3.7$	$b_{65(5,3,3)} = 6.1$	$b_{70(5,4,3)} = 6.1$	$b_{75(5,5,3)} = 10$
$x_{Severity}$ $\mu_3^4 = High$	$x_{Detection}$ $\mu_1^1=Very High$	$b_{76(1,1,4)} = 2.5$	$b_{81(1,2,4)} = 3.7$	$b_{86(1,3,4)} = 6.1$	$b_{91(1,4,4)} = 6.1$	$b_{96(1,5,4)} = 6.1$
	$\mu_1^2=High$	$b_{77(2,1,4)} = 3.7$	$b_{82(2,2,4)} = 3.7$	$b_{87(2,3,4)} = 6.1$	$b_{92(2,4,4)} = 6.1$	$b_{97(2,5,4)} = 10$
	$\mu_1^3=Moderate$	$b_{78(3,1,4)} = 3.7$	$b_{83(3,2,4)} = 6.1$	$b_{88(3,3,4)} = 6.1$	$b_{93(3,4,4)} = 10$	$b_{98(3,5,4)} = 10$
	$\mu_1^4=Low$	$b_{79(4,1,4)} = 6.1$	$b_{84(4,2,4)} = 6.1$	$b_{89(4,3,4)} = 10$	$b_{94(4,4,4)} = 10$	$b_{99(4,5,4)} = 10$
	$\mu_1^5=Remote$	$b_{80(5,1,4)} = 6.1$	$b_{85(5,2,4)} = 6.1$	$b_{90(5,3,4)} = 10$	$b_{95(5,4,4)} = 10$	$b_{100(5,5,4)} = 10$
$x_{Severity}$ $\mu_3^5 = Very High$	$x_{Detection}$ $\mu_1^1=Very High$	$b_{101(1,1,5)} = 3.7$	$b_{106(1,2,5)} = 3.7$	$b_{111(1,3,5)} = 3.7$	$b_{116(1,4,5)} = 6.1$	$b_{121(1,5,5)} = 6.1$
	$\mu_1^2=High$	$b_{102(2,1,5)} = 3.7$	$b_{107(2,2,5)} = 3.7$	$b_{112(2,3,5)} = 6.1$	$b_{117(2,4,5)} = 6.1$	$b_{122(2,5,5)} = 10$
	$\mu_1^3=Moderate$	$b_{103(3,1,5)} = 3.7$	$b_{108(3,2,5)} = 6.1$	$b_{113(3,3,5)} = 6.1$	$b_{118(3,4,5)} = 10$	$b_{123(3,5,5)} = 10$
	$\mu_1^4=Low$	$b_{104(4,1,5)} = 6.1$	$b_{109(4,2,5)} = 10$	$b_{114(4,3,5)} = 10$	$b_{119(4,4,5)} = 10$	$b_{124(4,5,5)} = 10$
	$\mu_1^5=Remote$	$b_{105(5,1,5)} = 6.1$	$b_{110(5,2,5)} = 10$	$b_{115(5,3,5)} = 10$	$b_{120(5,4,5)} = 10$	$b_{125(5,5,5)} = 10$

Figure 6.10: R4 in grid partition form (Hsieh, 2006)

6.3.1.2 Results and Discussion

A computer with Intel (R) Core (TM) i5-3210M CPU @2.50GHz, 6.00 GB of RAM and Microsoft Window 10 is used for the experiment. With Matlab R2012a, the results for MFRR-BFBT, MFRR-NF and MFRR-H are summarized in Table 6.7.

Table 6.7: Result obtained for MFRR-BFBT, MFRR-NF and MFRR-H

Information	MFRR-BFBT	MFRR-NF	MFRR-H
k	3	3	3
Number of K_s	2	1	2
Computational time (s)	60.15	0.0482	0.0816
Number of optimal solutions	2	1	2
Minimum loss value (L2 loss function)	24.48	24.48	24.48

All 125 fuzzy rules from **R4** were relabeled using MFRR-BFBT, MFRR-NF and MFRR-H. The computational time for MFRR-BFBT, MFRR-NF and MFRR-H are 60.15 seconds, 0.0482 seconds, and 0.0816 seconds, respectively. MFRR-BFBT has the highest computational cost, which is followed by MFRR-H and MFRR-NF. Using MFRR-BFBT and MFRR-H, two optimal solutions were found for relabeling minimum of three fuzzy rules, with the minimum loss measure of 24.48 (using L2 loss function). MFRR-NF can only search for one optimal solution. The first solution suggested that b_{17} , b_{18} and b_{86} should be relabeled to $b_{\text{Fairly Low}}$, b_{Moderate} , and b_{Moderate} , respectively. The second solution suggested that b_{17} , b_{18} and b_{111} should be relabeled to $b_{\text{Fairly Low}}$, b_{Moderate} , and $b_{\text{Fairly High}}$, respectively. Figures 6.11 and 6.12 shows the two obtained optimal relabeled fuzzy rule bases.

		$x_{occurrence}$				
		$\mu_1^1=Remote$	$\mu_2^2=Low$	$\mu_3^3=Moderate$	$\mu_4^4=High$	$\mu_5^5=Very\ High$
$x_{Severity}$ $\mu_3^1 = Remote$	$x_{Detection}$ $\mu_1^1=Very\ High$	$b_{1(1,1,1)} = 1$	$b_{6(1,2,1)} = 1$	$b_{11(1,3,1)} = 1$	$b_{16(1,4,1)} = 1$	$b_{21(1,5,1)} = 2.5$
	$\mu_1^2=High$	$b_{2(2,1,1)} = 1$	$b_{7(2,2,1)} = 1$	$b_{12(2,3,1)} = 1$	$b_{17(2,4,1)}^{rel} = 2.5$	$b_{22(2,5,1)} = 2.5$
	$\mu_1^3=Moderate$	$b_{3(3,1,1)} = 1$	$b_{8(3,2,1)} = 2.5$	$b_{13(3,3,1)} = 2.5$	$b_{18(3,4,1)}^{rel} = 3.7$	$b_{23(3,5,1)} = 3.7$
	$\mu_1^4=Low$	$b_{4(4,1,1)} = 2.5$	$b_{9(4,2,1)} = 2.5$	$b_{14(4,3,1)} = 2.5$	$b_{19(4,4,1)} = 3.7$	$b_{24(4,5,1)} = 3.7$
	$\mu_1^5=Remote$	$b_{5(5,1,1)} = 2.5$	$b_{10(5,2,1)} = 2.5$	$b_{15(5,3,1)} = 3.7$	$b_{20(5,4,1)} = 3.7$	$b_{25(5,5,1)} = 6.1$
$x_{Severity}$ $\mu_3^2 = Low$	$x_{Detection}$ $\mu_1^1=Very\ High$	$b_{26(1,1,2)} = 1$	$b_{31(1,2,2)} = 1$	$b_{36(1,3,2)} = 1$	$b_{41(1,4,2)} = 2.5$	$b_{46(1,5,2)} = 2.5$
	$\mu_1^2=High$	$b_{27(2,1,2)} = 1$	$b_{32(2,2,2)} = 2.5$	$b_{37(2,3,2)} = 2.5$	$b_{42(2,4,2)} = 2.5$	$b_{47(2,5,2)} = 3.7$
	$\mu_1^3=Moderate$	$b_{28(3,1,2)} = 2.5$	$b_{33(3,2,2)} = 2.5$	$b_{38(3,3,2)} = 2.5$	$b_{43(3,4,2)} = 3.7$	$b_{48(3,5,2)} = 3.7$
	$\mu_1^4=Low$	$b_{29(4,1,2)} = 2.5$	$b_{34(4,2,2)} = 2.5$	$b_{39(4,3,2)} = 3.7$	$b_{44(4,4,2)} = 3.7$	$b_{49(4,5,2)} = 6.1$
	$\mu_1^5=Remote$	$b_{30(5,1,2)} = 2.5$	$b_{35(5,2,2)} = 3.7$	$b_{40(5,3,2)} = 3.7$	$b_{45(5,4,2)} = 6.1$	$b_{50(5,5,2)} = 6.1$
$x_{Severity}$ $\mu_3^3 = Moderate$	$x_{Detection}$ $\mu_1^1=Very\ High$	$b_{51(1,1,3)} = 1$	$b_{56(1,2,3)} = 1$	$b_{61(1,3,3)} = 2.5$	$b_{66(1,4,3)} = 2.5$	$b_{71(1,5,3)} = 3.7$
	$\mu_1^2=High$	$b_{52(2,1,3)} = 2.5$	$b_{57(2,2,3)} = 2.5$	$b_{62(2,3,3)} = 2.5$	$b_{67(2,4,3)} = 3.7$	$b_{72(2,5,3)} = 3.7$
	$\mu_1^3=Moderate$	$b_{53(3,1,3)} = 2.5$	$b_{58(3,2,3)} = 2.5$	$b_{63(3,3,3)} = 3.7$	$b_{68(3,4,3)} = 3.7$	$b_{73(3,5,3)} = 6.1$
	$\mu_1^4=Low$	$b_{54(4,1,3)} = 2.5$	$b_{59(4,2,3)} = 3.7$	$b_{64(4,3,3)} = 6.1$	$b_{69(4,4,3)} = 6.1$	$b_{74(4,5,3)} = 6.1$
	$\mu_1^5=Remote$	$b_{55(5,1,3)} = 3.7$	$b_{60(5,2,3)} = 3.7$	$b_{65(5,3,3)} = 6.1$	$b_{70(5,4,3)} = 6.1$	$b_{75(5,5,3)} = 10$
$x_{Severity}$ $\mu_3^4 = High$	$x_{Detection}$ $\mu_1^1=Very\ High$	$b_{76(1,1,4)} = 2.5$	$b_{81(1,2,4)} = 3.7$	$b_{86(1,3,4)}^{rel} = 3.7$	$b_{91(1,4,4)} = 6.1$	$b_{96(1,5,4)} = 6.1$
	$\mu_1^2=High$	$b_{77(2,1,4)} = 3.7$	$b_{82(2,2,4)} = 3.7$	$b_{87(2,3,4)} = 6.1$	$b_{92(2,4,4)} = 6.1$	$b_{97(2,5,4)} = 10$
	$\mu_1^3=Moderate$	$b_{78(3,1,4)} = 3.7$	$b_{83(3,2,4)} = 6.1$	$b_{88(3,3,4)} = 6.1$	$b_{93(3,4,4)} = 10$	$b_{98(3,5,4)} = 10$
	$\mu_1^4=Low$	$b_{79(4,1,4)} = 6.1$	$b_{84(4,2,4)} = 6.1$	$b_{89(4,3,4)} = 10$	$b_{94(4,4,4)} = 10$	$b_{99(4,5,4)} = 10$
	$\mu_1^5=Remote$	$b_{80(5,1,4)} = 6.1$	$b_{85(5,2,4)} = 6.1$	$b_{90(5,3,4)} = 10$	$b_{95(5,4,4)} = 10$	$b_{100(5,5,4)} = 10$
$x_{Severity}$ $\mu_3^5 = Very\ High$	$x_{Detection}$ $\mu_1^1=Very\ High$	$b_{101(1,1,5)} = 3.7$	$b_{106(1,2,5)} = 3.7$	$b_{111(1,3,5)} = 3.7$	$b_{116(1,4,5)} = 6.1$	$b_{121(1,5,5)} = 6.1$
	$\mu_1^2=High$	$b_{102(2,1,5)} = 3.7$	$b_{107(2,2,5)} = 3.7$	$b_{112(2,3,5)} = 6.1$	$b_{117(2,4,5)} = 6.1$	$b_{122(2,5,5)} = 10$
	$\mu_1^3=Moderate$	$b_{103(3,1,5)} = 3.7$	$b_{108(3,2,5)} = 6.1$	$b_{113(3,3,5)} = 6.1$	$b_{118(3,4,5)} = 10$	$b_{123(3,5,5)} = 10$
	$\mu_1^4=Low$	$b_{104(4,1,5)} = 6.1$	$b_{109(4,2,5)} = 10$	$b_{114(4,3,5)} = 10$	$b_{119(4,4,5)} = 10$	$b_{124(4,5,5)} = 10$
	$\mu_1^5=Remote$	$b_{105(5,1,5)} = 6.1$	$b_{110(5,2,5)} = 10$	$b_{115(5,3,5)} = 10$	$b_{120(5,4,5)} = 10$	$b_{125(5,5,5)} = 10$

Figure 6.11: First relabeled fuzzy rule base for **R4** (obtained by MFRR-BFBT, MFRR-NF and MFRR-H)

		$x_{occurrence}$				
		$\mu_2^1=Remote$	$\mu_2^2=Low$	$\mu_2^3=Moderate$	$\mu_2^4=High$	$\mu_2^5=Very High$
$x_{Severity}$ $\mu_3^1 = Remote$	$x_{Detection}$ $\mu_1^1=Very High$	$b_{1(1,1,1)} = 1$	$b_{6(1,2,1)} = 1$	$b_{11(1,3,1)} = 1$	$b_{16(1,4,1)} = 1$	$b_{21(1,5,1)} = 2.5$
	$\mu_1^2=High$	$b_{2(2,1,1)} = 1$	$b_{7(2,2,1)} = 1$	$b_{12(2,3,1)} = 1$	$b_{17(2,4,1)}^{rel} = 2.5$	$b_{22(2,5,1)} = 2.5$
	$\mu_1^3=Moderate$	$b_{3(3,1,1)} = 1$	$b_{8(3,2,1)} = 2.5$	$b_{13(3,3,1)} = 2.5$	$b_{18(3,4,1)}^{rel} = 3.7$	$b_{23(3,5,1)} = 3.7$
	$\mu_1^4=Low$	$b_{4(4,1,1)} = 2.5$	$b_{9(4,2,1)} = 2.5$	$b_{14(4,3,1)} = 2.5$	$b_{19(4,4,1)} = 3.7$	$b_{24(4,5,1)} = 3.7$
	$\mu_1^5=Remote$	$b_{5(5,1,1)} = 2.5$	$b_{10(5,2,1)} = 2.5$	$b_{15(5,3,1)} = 3.7$	$b_{20(5,4,1)} = 3.7$	$b_{25(5,5,1)} = 6.1$
$x_{Severity}$ $\mu_3^2 = Low$	$x_{Detection}$ $\mu_1^1=Very High$	$b_{26(1,1,2)} = 1$	$b_{31(1,2,2)} = 1$	$b_{36(1,3,2)} = 1$	$b_{41(1,4,2)} = 2.5$	$b_{46(1,5,2)} = 2.5$
	$\mu_1^2=High$	$b_{27(2,1,2)} = 1$	$b_{32(2,2,2)} = 2.5$	$b_{37(2,3,2)} = 2.5$	$b_{42(2,4,2)} = 2.5$	$b_{47(2,5,2)} = 3.7$
	$\mu_1^3=Moderate$	$b_{28(3,1,2)} = 2.5$	$b_{33(3,2,2)} = 2.5$	$b_{38(3,3,2)} = 2.5$	$b_{43(3,4,2)} = 3.7$	$b_{48(3,5,2)} = 3.7$
	$\mu_1^4=Low$	$b_{29(4,1,2)} = 2.5$	$b_{34(4,2,2)} = 2.5$	$b_{39(4,3,2)} = 3.7$	$b_{44(4,4,2)} = 3.7$	$b_{49(4,5,2)} = 6.1$
	$\mu_1^5=Remote$	$b_{30(5,1,2)} = 2.5$	$b_{35(5,2,2)} = 3.7$	$b_{40(5,3,2)} = 3.7$	$b_{45(5,4,2)} = 6.1$	$b_{50(5,5,2)} = 6.1$
$x_{Severity}$ $\mu_3^3 = Moderate$	$x_{Detection}$ $\mu_1^1=Very High$	$b_{51(1,1,3)} = 1$	$b_{56(1,2,3)} = 1$	$b_{61(1,3,3)} = 2.5$	$b_{66(1,4,3)} = 2.5$	$b_{71(1,5,3)} = 3.7$
	$\mu_1^2=High$	$b_{52(2,1,3)} = 2.5$	$b_{57(2,2,3)} = 2.5$	$b_{62(2,3,3)} = 2.5$	$b_{67(2,4,3)} = 3.7$	$b_{72(2,5,3)} = 3.7$
	$\mu_1^3=Moderate$	$b_{53(3,1,3)} = 2.5$	$b_{58(3,2,3)} = 2.5$	$b_{63(3,3,3)} = 3.7$	$b_{68(3,4,3)} = 3.7$	$b_{73(3,5,3)} = 6.1$
	$\mu_1^4=Low$	$b_{54(4,1,3)} = 2.5$	$b_{59(4,2,3)} = 3.7$	$b_{64(4,3,3)} = 6.1$	$b_{69(4,4,3)} = 6.1$	$b_{74(4,5,3)} = 6.1$
	$\mu_1^5=Remote$	$b_{55(5,1,3)} = 3.7$	$b_{60(5,2,3)} = 3.7$	$b_{65(5,3,3)} = 6.1$	$b_{70(5,4,3)} = 6.1$	$b_{75(5,5,3)} = 10$
$x_{Severity}$ $\mu_3^4 = High$	$x_{Detection}$ $\mu_1^1=Very High$	$b_{76(1,1,4)} = 2.5$	$b_{81(1,2,4)} = 3.7$	$b_{86(1,3,4)} = 6.1$	$b_{91(1,4,4)} = 6.1$	$b_{96(1,5,4)} = 6.1$
	$\mu_1^2=High$	$b_{77(2,1,4)} = 3.7$	$b_{82(2,2,4)} = 3.7$	$b_{87(2,3,4)} = 6.1$	$b_{92(2,4,4)} = 6.1$	$b_{97(2,5,4)} = 10$
	$\mu_1^3=Moderate$	$b_{78(3,1,4)} = 3.7$	$b_{83(3,2,4)} = 6.1$	$b_{88(3,3,4)} = 6.1$	$b_{93(3,4,4)} = 10$	$b_{98(3,5,4)} = 10$
	$\mu_1^4=Low$	$b_{79(4,1,4)} = 6.1$	$b_{84(4,2,4)} = 6.1$	$b_{89(4,3,4)} = 10$	$b_{94(4,4,4)} = 10$	$b_{99(4,5,4)} = 10$
	$\mu_1^5=Remote$	$b_{80(5,1,4)} = 6.1$	$b_{85(5,2,4)} = 6.1$	$b_{90(5,3,4)} = 10$	$b_{95(5,4,4)} = 10$	$b_{100(5,5,4)} = 10$
$x_{Severity}$ $\mu_3^5 = Very High$	$x_{Detection}$ $\mu_1^1=Very High$	$b_{101(1,1,5)} = 3.7$	$b_{106(1,2,5)} = 3.7$	$b_{111(1,3,5)}^{rel} = 6.1$	$b_{116(1,4,5)} = 6.1$	$b_{121(1,5,5)} = 6.1$
	$\mu_1^2=High$	$b_{102(2,1,5)} = 3.7$	$b_{107(2,2,5)} = 3.7$	$b_{112(2,3,5)} = 6.1$	$b_{117(2,4,5)} = 6.1$	$b_{122(2,5,5)} = 10$
	$\mu_1^3=Moderate$	$b_{103(3,1,5)} = 3.7$	$b_{108(3,2,5)} = 6.1$	$b_{113(3,3,5)} = 6.1$	$b_{118(3,4,5)} = 10$	$b_{123(3,5,5)} = 10$
	$\mu_1^4=Low$	$b_{104(4,1,5)} = 6.1$	$b_{109(4,2,5)} = 10$	$b_{114(4,3,5)} = 10$	$b_{119(4,4,5)} = 10$	$b_{124(4,5,5)} = 10$
	$\mu_1^5=Remote$	$b_{105(5,1,5)} = 6.1$	$b_{110(5,2,5)} = 10$	$b_{115(5,3,5)} = 10$	$b_{120(5,4,5)} = 10$	$b_{125(5,5,5)} = 10$

Figure 6.12: Second relabeled fuzzy rule base for **R4** (obtained by MFRR-BFBT and MFRR-H)

Figure 6.13 shows the surface plot of the FIS-based RPN model, using the original non-monotone fuzzy rule base (i.e., without relabeling), and a non-monotone surface curve can be observed (i.e., as indicated in Area 1). Figures 6.14 and 6.15 show the surface plots of the FIS-based RPN models, using the two suggested optimal relabeled fuzzy rules, respectively, and two monotone surface curves can be observed (as indicated in Area 2 and

Area 3). As such, both the new monotone FIS-based RPN models with relabeled fuzzy rules could serve as optimal alternatives for FMEA users to handle the problem in sewage treatment systems.

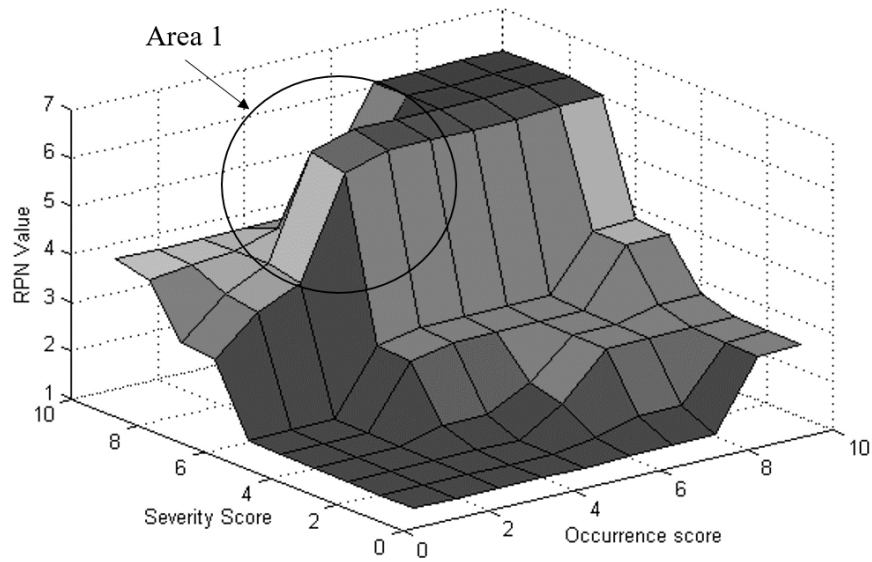


Figure 6.13: Surface plot of the FIS-based RPN model (before MFRR) with Detection=1.

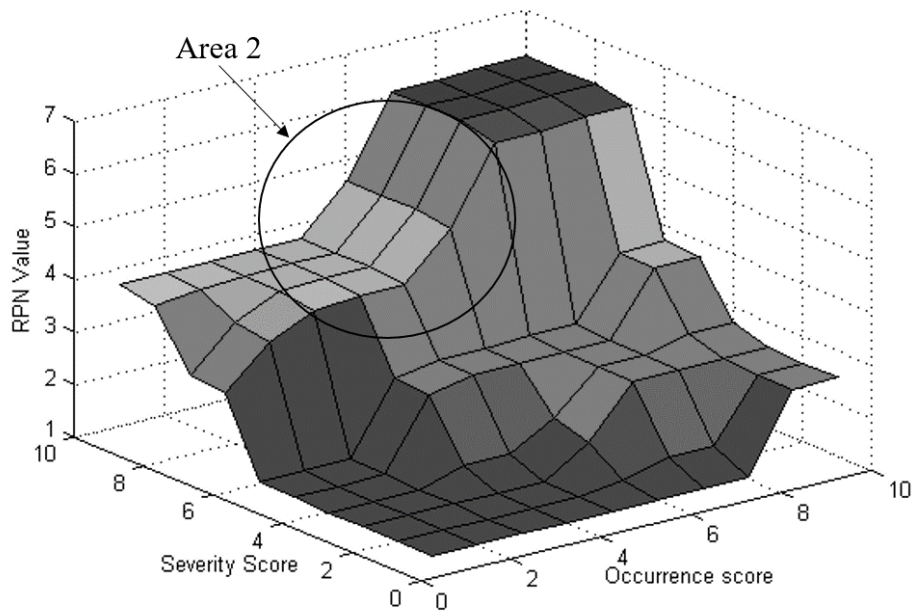
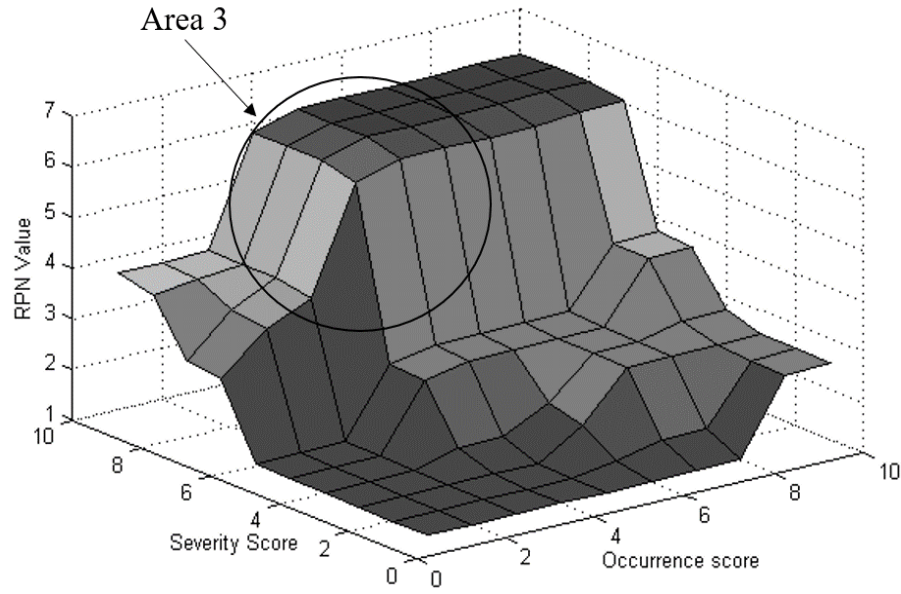


Figure 6.14: Surface plot of the FIS-based RPN model based on the first optimal solution with Detection=1



(c)

Figure 6.15: Surface plot of the FIS-based FMEA model based on the second optimal solution with Detection=1.

In the absence of MFRR, it is possible for a set of noisy fuzzy rules (collected from humans) to produce a non-monotone FIS model, as shown in the FMEA example. With the use of MFRR, meaningful and useful FIS models can be generated. Based on the FMEA example in sewage treatment systems, we can observe that noisy fuzzy rules lead to a non-monotone FIS-based RPN model. By using MFRR, the potential errors due to noise can be identified by generating two monotone and useful FIS-based RPN models.

Note that MFRR may produce more than one optimal solutions (or extrema). The number of optimal solutions (or extrema) generated by MFRR depends on the combination of the noisy fuzzy rules, or more specifically, the fuzzy rules in the k -subset. We found that MFRR tends to generate multiple optimal solutions when the cardinality of k -subset is more than one, and there exist combinations of noisy fuzzy rules that differ between each other. When multiple optimal solutions are generated, seeking human judgement serves as a good

way to choose a suitable FIS-based model. FMEA users can evaluate the two potential FIS-based RPN models based on the surface plots, e.g. Figure 6.14 is smoother than that of Figure 6.15. FMEA users can also pin-point the noisy fuzzy rules sets before implementation.

6.3.2 Hospital Purchasing Process (Kumru & Kumru, 2013)

6.3.2.1 Background

In this section, an application of FIS-RPN model to improve purchasing process in a public hospital is presented (Kumru & Kumru, 2013). The FIS-based RPN model considers three risk factors as the inputs, i.e., Severity (S), Occurrence (O) and Not Detection (ND), and produces an RPN score as the output. A scale of 1 to 10 is used for quantifying the three risk factors, i.e., $x_{Severity}, x_{Occurrence}, x_{Not\ Detection} \in [1,10]$. Each input is assigned with five linguistic terms, i.e., *almost none*, *low*, *medium*, *high* and *very high*, respectively.

The linguistic terms are represented by Gaussian MFs. The Gaussian MFs used in this experiment are satisfying **Condition 1** of **Theorem 3.1**. The relationship between the three inputs and the output are represented by a set of fuzzy IF-THEN rules. There are total number of 125 fuzzy rules were given (i.e., $r = 125$). The fuzzy rule consequents are represented with ten linguistic terms, i.e., *none*, *very low*, *low*, *high low*, *low medium*, *medium*, *high medium*, *low high*, *high* and *very high*. They are characterized by ten singleton values, i.e., $b_{None} = 1$, $b_{Very\ low} = 2$, $b_{Low} = 3$, $b_{High\ low} = 4$, $b_{Low\ medium} = 5$, $b_{Medium} = 6$, $b_{High\ medium} = 7$, $b_{Low\ High} = 8$, $b_{High} = 9$, $b_{Very\ High} = 10$, respectively.

Figure 6.16 shows the fuzzy rule base given in (Kumru & Kumru, 2013), denoted as **R5**. It can be noticed that the collected fuzzy rule base **R5** does not satisfy **Condition 2** of **Theorem 3.1**, i.e. the fuzzy rule base is not monotone. The fuzzy rules that form non-

monotone pair with each other were shaded. As an example, R_{22} and R_{23} are not monotone to each other (which can be observed from the consequent value of R_{22} and R_{23} , i.e., b_{22} and b_{23} in Figure 6.16). R_{22} and R_{23} contain the same fuzzy linguistic terms for S and ND , i.e., almost none and very high, respectively. The fuzzy linguistic terms for O in R_{22} and R_{23} are low and medium, respectively. Therefore, the singleton for consequent of R_{23} is expected to be equal or higher than that of R_{22} (i.e., $b_{23} \geq b_{22}$), but the fuzzy rule base shows that b_{22} is larger than b_{23} .

		$x_{Not\ Detection}$				
		$\mu_2^1 = \text{Almost none}$	$\mu_2^2 = \text{Low}$	$\mu_2^3 = \text{Medium}$	$\mu_2^4 = \text{High}$	$\mu_2^5 = \text{Very high}$
$x_{Occurrence}$ $\mu_3^1 = \text{Almost none}$	$x_{Severity}$ $\mu_1^1 = \text{Almost none}$	$b_{1(1,1,1)} = 1$	$b_{6(1,2,1)} = 1$	$b_{11(1,3,1)} = 2$	$b_{16(1,4,1)} = 3$	$b_{21(1,5,1)} = 3$
	$\mu_1^2 = \text{Low}$	$b_{2(2,1,1)} = 2$	$b_{7(2,2,1)} = 3$	$b_{12(2,3,1)} = 3$	$b_{17(2,4,1)} = 4$	$b_{22(2,5,1)} = 5$
	$\mu_1^3 = \text{medium}$	$b_{3(3,1,1)} = 2$	$b_{8(3,2,1)} = 3$	$b_{13(3,3,1)} = 3$	$b_{18(3,4,1)} = 4$	$b_{23(3,5,1)} = 4$
	$\mu_1^4 = \text{high}$	$b_{4(4,1,1)} = 3$	$b_{9(4,2,1)} = 4$	$b_{14(4,3,1)} = 5$	$b_{19(4,4,1)} = 6$	$b_{24(4,5,1)} = 7$
	$\mu_1^5 = \text{Very high}$	$b_{5(5,1,1)} = 4$	$b_{10(5,2,1)} = 5$	$b_{15(5,3,1)} = 6$	$b_{20(5,4,1)} = 7$	$b_{25(5,5,1)} = 9$
$x_{Occurrence}$ $\mu_3^2 = \text{Low}$	$x_{Severity}$ $\mu_1^1 = \text{Almost none}$	$b_{26(1,1,2)} = 1$	$b_{31(1,2,2)} = 1$	$b_{36(1,3,2)} = 2$	$b_{41(1,4,2)} = 3$	$b_{46(1,5,2)} = 3$
	$\mu_1^2 = \text{Low}$	$b_{27(2,1,2)} = 2$	$b_{32(2,2,2)} = 3$	$b_{37(2,3,2)} = 4$	$b_{42(2,4,2)} = 5$	$b_{47(2,5,2)} = 6$
	$\mu_1^3 = \text{medium}$	$b_{28(3,1,2)} = 4$	$b_{33(3,2,2)} = 5$	$b_{38(3,3,2)} = 6$	$b_{43(3,4,2)} = 7$	$b_{48(3,5,2)} = 8$
	$\mu_1^4 = \text{high}$	$b_{29(4,1,2)} = 5$	$b_{34(4,2,2)} = 6$	$b_{39(4,3,2)} = 7$	$b_{44(4,4,2)} = 8$	$b_{49(4,5,2)} = 9$
	$\mu_1^5 = \text{Very high}$	$b_{30(5,1,2)} = 5$	$b_{35(5,2,2)} = 5$	$b_{40(5,3,2)} = 6$	$b_{45(5,4,2)} = 9$	$b_{50(5,5,2)} = 9$
$x_{Occurrence}$ $\mu_3^3 = \text{Medium}$	$x_{Severity}$ $\mu_1^1 = \text{Almost none}$	$b_{51(1,1,3)} = 2$	$b_{56(1,2,3)} = 2$	$b_{61(1,3,3)} = 3$	$b_{66(1,4,3)} = 4$	$b_{71(1,5,3)} = 5$
	$\mu_1^2 = \text{Low}$	$b_{52(2,1,3)} = 3$	$b_{57(2,2,3)} = 4$	$b_{62(2,3,3)} = 5$	$b_{67(2,4,3)} = 6$	$b_{72(2,5,3)} = 7$
	$\mu_1^3 = \text{medium}$	$b_{53(3,1,3)} = 5$	$b_{58(3,2,3)} = 6$	$b_{63(3,3,3)} = 7$	$b_{68(3,4,3)} = 8$	$b_{73(3,5,3)} = 9$
	$\mu_1^4 = \text{high}$	$b_{54(4,1,3)} = 3$	$b_{59(4,2,3)} = 4$	$b_{64(4,3,3)} = 5$	$b_{69(4,4,3)} = 7$	$b_{74(4,5,3)} = 8$
	$\mu_1^5 = \text{Very high}$	$b_{55(5,1,3)} = 8$	$b_{60(5,2,3)} = 6$	$b_{65(5,3,3)} = 7$	$b_{70(5,4,3)} = 8$	$b_{75(5,5,3)} = 9$
$x_{Occurrence}$ $\mu_3^4 = \text{High}$	$x_{Severity}$ $\mu_1^1 = \text{Almost none}$	$b_{76(1,1,4)} = 1$	$b_{81(1,2,4)} = 2$	$b_{86(1,3,4)} = 3$	$b_{91(1,4,4)} = 3$	$b_{96(1,5,4)} = 4$
	$\mu_1^2 = \text{Low}$	$b_{77(2,1,4)} = 2$	$b_{82(2,2,4)} = 3$	$b_{87(2,3,4)} = 4$	$b_{92(2,4,4)} = 5$	$b_{97(2,5,4)} = 6$
	$\mu_1^3 = \text{medium}$	$b_{78(3,1,4)} = 3$	$b_{83(3,2,4)} = 4$	$b_{88(3,3,4)} = 5$	$b_{93(3,4,4)} = 6$	$b_{98(3,5,4)} = 7$
	$\mu_1^4 = \text{high}$	$b_{79(4,1,4)} = 5$	$b_{84(4,2,4)} = 6$	$b_{89(4,3,4)} = 7$	$b_{94(4,4,4)} = 8$	$b_{99(4,5,4)} = 9$
	$\mu_1^5 = \text{Very high}$	$b_{80(5,1,4)} = 6$	$b_{85(5,2,4)} = 7$	$b_{90(5,3,4)} = 8$	$b_{95(5,4,4)} = 9$	$b_{100(5,5,4)} = 10$
$x_{Occurrence}$ $\mu_3^5 = \text{Very High}$	$x_{Severity}$ $\mu_1^1 = \text{Almost none}$	$b_{101(1,1,5)} = 2$	$b_{106(1,2,5)} = 2$	$b_{111(1,3,5)} = 3$	$b_{116(1,4,5)} = 3$	$b_{121(1,5,5)} = 4$
	$\mu_1^2 = \text{Low}$	$b_{102(2,1,5)} = 3$	$b_{107(2,2,5)} = 4$	$b_{112(2,3,5)} = 4$	$b_{117(2,4,5)} = 5$	$b_{122(2,5,5)} = 6$
	$\mu_1^3 = \text{medium}$	$b_{103(3,1,5)} = 4$	$b_{108(3,2,5)} = 4$	$b_{113(3,3,5)} = 5$	$b_{118(3,4,5)} = 6$	$b_{123(3,5,5)} = 7$
	$\mu_1^4 = \text{high}$	$b_{104(4,1,5)} = 5$	$b_{109(4,2,5)} = 6$	$b_{114(4,3,5)} = 7$	$b_{119(4,4,5)} = 8$	$b_{124(4,5,5)} = 9$
	$\mu_1^5 = \text{Very high}$	$b_{105(5,1,5)} = 7$	$b_{110(5,2,5)} = 8$	$b_{115(5,3,5)} = 9$	$b_{120(5,4,5)} = 10$	$b_{125(5,5,5)} = 10$

Figure 6.16: R_5 in grid partition form (Kumru & Kumru, 2013)

Using the network flow approach in Section 5.3, the minimum number of fuzzy rules to be relabeled is 26 (i.e., $k=26$). Note that in Stage-1, MFRR-BFBT needs to search for $C(125,26) = 5 \times 10^{26}$ possible number of combinations of k -subset of fuzzy rules, which is not practical to be conducted using a laptop computer. Therefore, only MFRR-NF and MFRR-H are experimented.

6.3.2.2 Results and Analysis

A computer with Intel (R) Core (TM) i5-3210M CPU @2.50GHz, 6.00 GB of RAM and Microsoft Window 10 is used for the experiment. With Matlab R2012a, the results for MFRR-NF and MFRR-H are summarized in Table 6.8.

Table 6.8: Result obtained for MFRR-NF and MFRR-H

Information	MFRR-NF	MFRR-H
K	26	26
Number of K_s	1	224
Computational time (s)	2.38	389654.39
Number of optimal solutions	1	128
Minimum loss value (L2 loss function)	53	53

Both MFRR-NF and MFRR-H produce a relabeled fuzzy rule base with a minimum loss of 53. The L2 loss function is used in this experiment to quantify the total loss incurred during the relabeling process. MFRR-NF is able to generate an optimal relabeled fuzzy rule base with a faster computational time, i.e., 2.38 seconds, as it searches only one K_s . On the other hand, MFRR-H is able to generate all the optimal solutions with 224 K_s . A total of 128 optimal relabeled fuzzy rules are produced, with computational time of about 4.5 days. For MFRR-H, the number of *corona fuzzy rules* obtained are 28, and the number of *void fuzzy rules* obtained are 12. Another additional 14 fuzzy rules need to be combined with *void fuzzy rules* to form K_s . These 14 additional fuzzy rules are selected from 28 *corona fuzzy rules*. MFRR-H attempts to search for $C(28,14) = 40116600$ combinations of the fuzzy rules, in

order to generate K_s . There are 224 K_s obtained by MFRR-H. in which 128 optimal solutions are obtained by relabeling the fuzzy rules in all 224 K_s . In this simulation, MFRR optimality is achieved using both MFRR-NF and MFRR-H.

Figure 6.17 shows the optimal relabeled fuzzy rule base obtained using MFRR-NF. The 26-noisy fuzzy rules identified (highlighted in grey) in MFRR Stage-1 are relabeled, which leads to the optimal solution obtained using MFRR-H. Figure 6.18 shows one of the optimal solutions for relabeled fuzzy rule base obtained by using MFRR-H, based on histogram selection method. A more uniform distribution for the relabeled values is preferred. Its histogram distribution (the full list of histogram distribution is attached in Appendix A) is shown in Figure 6.19. The optimal solution in Figure 6.18 has more uniform distribution for the relabeled value.

		$x_{Not\ Detection}$				
		$\mu_2^1=Almost\ none$	$\mu_2^2=Low$	$\mu_2^3=Medium$	$\mu_2^4=High$	$\mu_2^5=Very\ high$
$x_{Severity}$ $\mu_3^1 = Almost\ none$	$x_{occurrence}$ $\mu_1^1=Almost\ none$	$b_{1(1,1,1)} = 1$	$b_{6(1,2,1)} = 1$	$b_{11(1,3,1)} = 2$	$b_{16(1,4,1)} = 3$	$b_{21(1,5,1)} = 3$
	$\mu_1^2=Low$	$b_{2(2,1,1)} = 2$	$b_{7(2,2,1)} = 3$	$b_{12(2,3,1)} = 3$	$b_{17(2,4,1)} = 4$	$b_{22(2,5,1)} = 4$
	$\mu_1^3=medium$	$b_{3(3,1,1)} = 2$	$b_{8(3,2,1)} = 3$	$b_{13(3,3,1)} = 3$	$b_{18(3,4,1)} = 4$	$b_{23(3,5,1)} = 4$
	$\mu_1^4=high$	$b_{4(4,1,1)} = 3$	$b_{9(4,2,1)} = 4$	$b_{14(4,3,1)} = 5$	$b_{19(4,4,1)} = 6$	$b_{24(4,5,1)} = 7$
	$\mu_1^5=Very\ high$	$b_{5(5,1,1)} = 4$	$b_{10(5,2,1)} = 5$	$b_{15(5,3,1)} = 6$	$b_{20(5,4,1)} = 7$	$b_{25(5,5,1)} = 9$
$x_{Severity}$ $\mu_3^2 = Low$	$x_{occurrence}$ $\mu_1^1=Almost\ none$	$b_{26(1,1,2)} = 1$	$b_{31(1,2,2)} = 1$	$b_{36(1,3,2)} = 2$	$b_{41(1,4,2)} = 3$	$b_{46(1,5,2)} = 3$
	$\mu_1^2=Low$	$b_{27(2,1,2)} = 2$	$b_{32(2,2,2)} = 3$	$b_{37(2,3,2)} = 4$	$b_{42(2,4,2)} = 5$	$b_{47(2,5,2)} = 6$
	$\mu_1^3=medium$	$b_{28(3,1,2)} = 3$	$b_{33(3,2,2)} = 4$	$b_{38(3,3,2)} = 5$	$b_{43(3,4,2)} = 6$	$b_{48(3,5,2)} = 7$
	$\mu_1^4=high$	$b_{29(4,1,2)} = 3$	$b_{34(4,2,2)} = 4$	$b_{39(4,3,2)} = 5$	$b_{44(4,4,2)} = 7$	$b_{49(4,5,2)} = 8$
	$\mu_1^5=Very\ high$	$b_{30(5,1,2)} = 5$	$b_{35(5,2,2)} = 5$	$b_{40(5,3,2)} = 6$	$b_{45(5,4,2)} = 8$	$b_{50(5,5,2)} = 9$
$x_{Severity}$ $\mu_3^3 = Medium$	$x_{occurrence}$ $\mu_1^1=Almost\ none$	$b_{51(1,1,3)} = 1$	$b_{56(1,2,3)} = 2$	$b_{61(1,3,3)} = 3$	$b_{66(1,4,3)} = 3$	$b_{71(1,5,3)} = 4$
	$\mu_1^2=Low$	$b_{52(2,1,3)} = 2$	$b_{57(2,2,3)} = 3$	$b_{62(2,3,3)} = 4$	$b_{67(2,4,3)} = 5$	$b_{72(2,5,3)} = 6$
	$\mu_1^3=medium$	$b_{53(3,1,3)} = 3$	$b_{58(3,2,3)} = 4$	$b_{63(3,3,3)} = 5$	$b_{68(3,4,3)} = 6$	$b_{73(3,5,3)} = 7$
	$\mu_1^4=high$	$b_{54(4,1,3)} = 3$	$b_{59(4,2,3)} = 4$	$b_{64(4,3,3)} = 5$	$b_{69(4,4,3)} = 7$	$b_{74(4,5,3)} = 8$
	$\mu_1^5=Very\ high$	$b_{55(5,1,3)} = 6$	$b_{60(5,2,3)} = 6$	$b_{65(5,3,3)} = 7$	$b_{70(5,4,3)} = 8$	$b_{75(5,5,3)} = 9$
$x_{Severity}$ $\mu_3^4 = High$	$x_{occurrence}$ $\mu_1^1=Almost\ none$	$b_{76(1,1,4)} = 1$	$b_{81(1,2,4)} = 2$	$b_{86(1,3,4)} = 3$	$b_{91(1,4,4)} = 3$	$b_{96(1,5,4)} = 4$
	$\mu_1^2=Low$	$b_{77(2,1,4)} = 2$	$b_{82(2,2,4)} = 3$	$b_{87(2,3,4)} = 4$	$b_{92(2,4,4)} = 5$	$b_{97(2,5,4)} = 6$
	$\mu_1^3=medium$	$b_{78(3,1,4)} = 3$	$b_{83(3,2,4)} = 4$	$b_{88(3,3,4)} = 5$	$b_{93(3,4,4)} = 6$	$b_{98(3,5,4)} = 7$
	$\mu_1^4=high$	$b_{79(4,1,4)} = 5$	$b_{84(4,2,4)} = 6$	$b_{89(4,3,4)} = 7$	$b_{94(4,4,4)} = 8$	$b_{99(4,5,4)} = 9$
	$\mu_1^5=Very\ high$	$b_{80(5,1,4)} = 6$	$b_{85(5,2,4)} = 7$	$b_{90(5,3,4)} = 8$	$b_{95(5,4,4)} = 9$	$b_{100(5,5,4)} = 10$
$x_{Severity}$ $\mu_3^5 = Very\ High$	$x_{occurrence}$ $\mu_1^1=Almost\ none$	$b_{101(1,1,5)} = 2$	$b_{106(1,2,5)} = 2$	$b_{111(1,3,5)} = 3$	$b_{116(1,4,5)} = 3$	$b_{121(1,5,5)} = 4$
	$\mu_1^2=Low$	$b_{102(2,1,5)} = 3$	$b_{107(2,2,5)} = 4$	$b_{112(2,3,5)} = 4$	$b_{117(2,4,5)} = 5$	$b_{122(2,5,5)} = 6$
	$\mu_1^3=medium$	$b_{103(3,1,5)} = 4$	$b_{108(3,2,5)} = 4$	$b_{113(3,3,5)} = 5$	$b_{118(3,4,5)} = 6$	$b_{123(3,5,5)} = 7$
	$\mu_1^4=high$	$b_{104(4,1,5)} = 5$	$b_{109(4,2,5)} = 6$	$b_{114(4,3,5)} = 7$	$b_{119(4,4,5)} = 8$	$b_{124(4,5,5)} = 9$
	$\mu_1^5=Very\ high$	$b_{105(5,1,5)} = 7$	$b_{110(5,2,5)} = 8$	$b_{115(5,3,5)} = 9$	$b_{120(5,4,5)} = 10$	$b_{125(5,5,5)} = 10$

Figure 6.17: The relabeled fuzzy rule base with MFRR-NF

		$x_{Not\ Detection}$				
		$\mu_2^1=Almost\ none$	$\mu_2^2=Low$	$\mu_2^3=Medium$	$\mu_2^4=High$	$\mu_2^5=Very\ high$
$x_{Severity}$ $\mu_3^1 = Almost\ none$	$x_{Occurrence}$ $\mu_1^1=Almost\ none$	$b_{1(1,1,1)} = 1$	$b_{6(1,2,1)} = 1$	$b_{11(1,3,1)} = 2$	$b_{16(1,4,1)} = 3$	$b_{21(1,5,1)} = 3$
	$\mu_1^2=Low$	$b_{2(2,1,1)} = 2$	$b_{7(2,2,1)} = 3$	$b_{12(2,3,1)} = 3$	$b_{17(2,4,1)} = 4$	$b_{22(2,5,1)} = 5$
	$\mu_1^3=medium$	$b_{3(3,1,1)} = 2$	$b_{8(3,2,1)} = 3$	$b_{13(3,3,1)} = 3$	$b_{18(3,4,1)} = 4$	$b_{23(3,5,1)} = 5$
	$\mu_1^4=high$	$b_{4(4,1,1)} = 3$	$b_{9(4,2,1)} = 4$	$b_{14(4,3,1)} = 5$	$b_{19(4,4,1)} = 6$	$b_{24(4,5,1)} = 7$
	$\mu_1^5=Very\ high$	$b_{5(5,1,1)} = 4$	$b_{10(5,2,1)} = 5$	$b_{15(5,3,1)} = 6$	$b_{20(5,4,1)} = 7$	$b_{25(5,5,1)} = 9$
$x_{Severity}$ $\mu_3^2 = Low$	$x_{Occurrence}$ $\mu_1^1=Almost\ none$	$b_{26(1,1,2)} = 1$	$b_{31(1,2,2)} = 1$	$b_{36(1,3,2)} = 2$	$b_{41(1,4,2)} = 3$	$b_{46(1,5,2)} = 3$
	$\mu_1^2=Low$	$b_{27(2,1,2)} = 2$	$b_{32(2,2,2)} = 3$	$b_{37(2,3,2)} = 4$	$b_{42(2,4,2)} = 5$	$b_{47(2,5,2)} = 6$
	$\mu_1^3=medium$	$b_{28(3,1,2)} = 3$	$b_{33(3,2,2)} = 4$	$b_{38(3,3,2)} = 5$	$b_{43(3,4,2)} = 6$	$b_{48(3,5,2)} = 7$
	$\mu_1^4=high$	$b_{29(4,1,2)} = 3$	$b_{34(4,2,2)} = 4$	$b_{39(4,3,2)} = 5$	$b_{44(4,4,2)} = 7$	$b_{49(4,5,2)} = 9$
	$\mu_1^5=Very\ high$	$b_{30(5,1,2)} = 5$	$b_{35(5,2,2)} = 5$	$b_{40(5,3,2)} = 6$	$b_{45(5,4,2)} = 8$	$b_{50(5,5,2)} = 9$
$x_{Severity}$ $\mu_3^3 = Medium$	$x_{Occurrence}$ $\mu_1^1=Almost\ none$	$b_{51(1,1,3)} = 1$	$b_{56(1,2,3)} = 2$	$b_{61(1,3,3)} = 3$	$b_{66(1,4,3)} = 3$	$b_{71(1,5,3)} = 4$
	$\mu_1^2=Low$	$b_{52(2,1,3)} = 2$	$b_{57(2,2,3)} = 3$	$b_{62(2,3,3)} = 4$	$b_{67(2,4,3)} = 5$	$b_{72(2,5,3)} = 6$
	$\mu_1^3=medium$	$b_{53(3,1,3)} = 3$	$b_{58(3,2,3)} = 4$	$b_{63(3,3,3)} = 5$	$b_{68(3,4,3)} = 6$	$b_{73(3,5,3)} = 7$
	$\mu_1^4=high$	$b_{54(4,1,3)} = 3$	$b_{59(4,2,3)} = 4$	$b_{64(4,3,3)} = 5$	$b_{69(4,4,3)} = 7$	$b_{74(4,5,3)} = 9$
	$\mu_1^5=Very\ high$	$b_{55(5,1,3)} = 6$	$b_{60(5,2,3)} = 6$	$b_{65(5,3,3)} = 7$	$b_{70(5,4,3)} = 8$	$b_{75(5,5,3)} = 9$
$x_{Severity}$ $\mu_3^4 = High$	$x_{Occurrence}$ $\mu_1^1=Almost\ none$	$b_{76(1,1,4)} = 1$	$b_{81(1,2,4)} = 2$	$b_{86(1,3,4)} = 3$	$b_{91(1,4,4)} = 3$	$b_{96(1,5,4)} = 4$
	$\mu_1^2=Low$	$b_{77(2,1,4)} = 2$	$b_{82(2,2,4)} = 3$	$b_{87(2,3,4)} = 4$	$b_{92(2,4,4)} = 5$	$b_{97(2,5,4)} = 6$
	$\mu_1^3=medium$	$b_{78(3,1,4)} = 3$	$b_{83(3,2,4)} = 4$	$b_{88(3,3,4)} = 5$	$b_{93(3,4,4)} = 6$	$b_{98(3,5,4)} = 7$
	$\mu_1^4=high$	$b_{79(4,1,4)} = 5$	$b_{84(4,2,4)} = 6$	$b_{89(4,3,4)} = 7$	$b_{94(4,4,4)} = 8$	$b_{99(4,5,4)} = 9$
	$\mu_1^5=Very\ high$	$b_{80(5,1,4)} = 6$	$b_{85(5,2,4)} = 7$	$b_{90(5,3,4)} = 8$	$b_{95(5,4,4)} = 9$	$b_{100(5,5,4)} = 10$
$x_{Severity}$ $\mu_3^5 = Very\ High$	$x_{Occurrence}$ $\mu_1^1=Almost\ none$	$b_{101(1,1,5)} = 2$	$b_{106(1,2,5)} = 2$	$b_{111(1,3,5)} = 3$	$b_{116(1,4,5)} = 3$	$b_{121(1,5,5)} = 4$
	$\mu_1^2=Low$	$b_{102(2,1,5)} = 3$	$b_{107(2,2,5)} = 4$	$b_{112(2,3,5)} = 4$	$b_{117(2,4,5)} = 5$	$b_{122(2,5,5)} = 6$
	$\mu_1^3=medium$	$b_{103(3,1,5)} = 4$	$b_{108(3,2,5)} = 4$	$b_{113(3,3,5)} = 5$	$b_{118(3,4,5)} = 6$	$b_{123(3,5,5)} = 7$
	$\mu_1^4=high$	$b_{104(4,1,5)} = 5$	$b_{109(4,2,5)} = 6$	$b_{114(4,3,5)} = 7$	$b_{119(4,4,5)} = 8$	$b_{124(4,5,5)} = 9$
	$\mu_1^5=Very\ high$	$b_{105(5,1,5)} = 7$	$b_{110(5,2,5)} = 8$	$b_{115(5,3,5)} = 9$	$b_{120(5,4,5)} = 10$	$b_{125(5,5,5)} = 10$

Figure 6.18: A relabeled fuzzy rule base with MFRR-H, selected from 128 solutions using histogram analysis

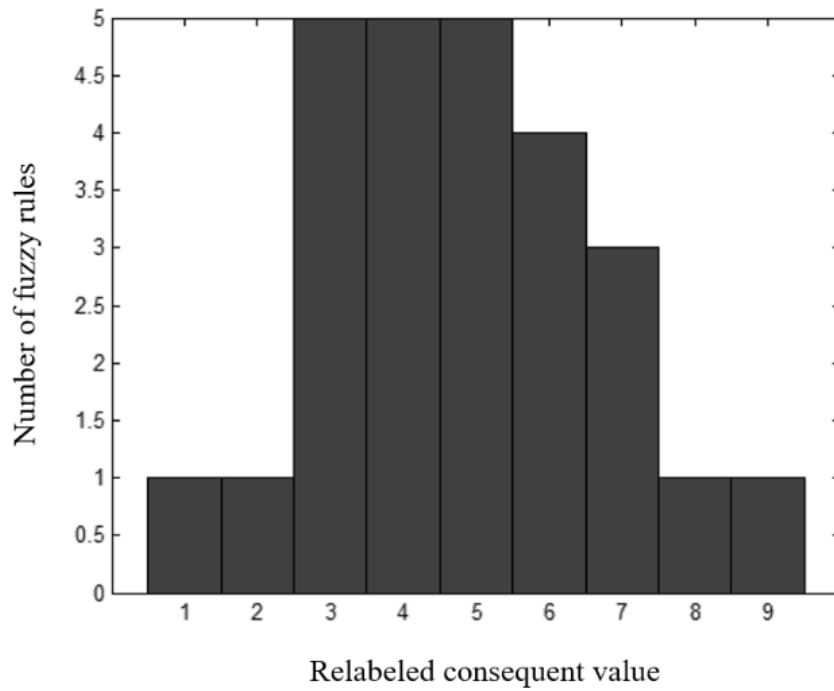


Figure 6.19: Histogram distribution for relabeled fuzzy rule base in Figure 6.18

Figure 6.20 shows the surface plot of the FIS-based RPN model using the original non-monotone fuzzy rule base **R5**. A non-monotone surface curve is observed. Figure 6.21 and Figure 6.22 show the surface plots of the FIS-based RPN model using the relabeled fuzzy rule bases obtained by MFRR-NF (Figure 6.17) and MFRR-H (Figure 6.18), respectively, and monotone surface curves are observed.

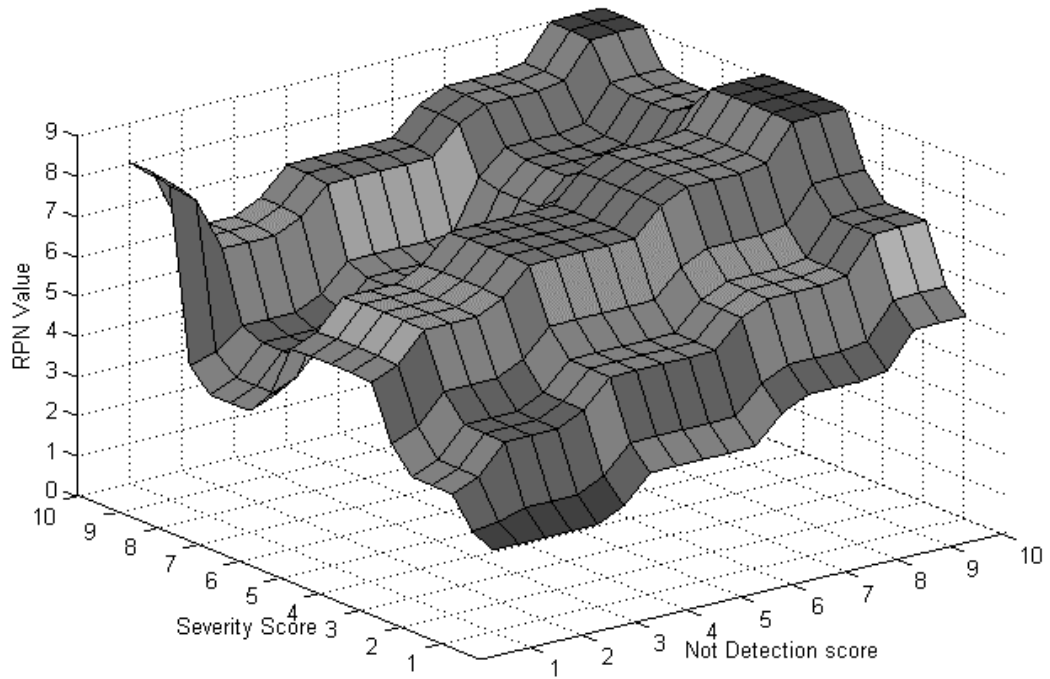


Figure 6.20: Surface plot of the FIS-based RPN model (before MFRR) with Occurrence=5

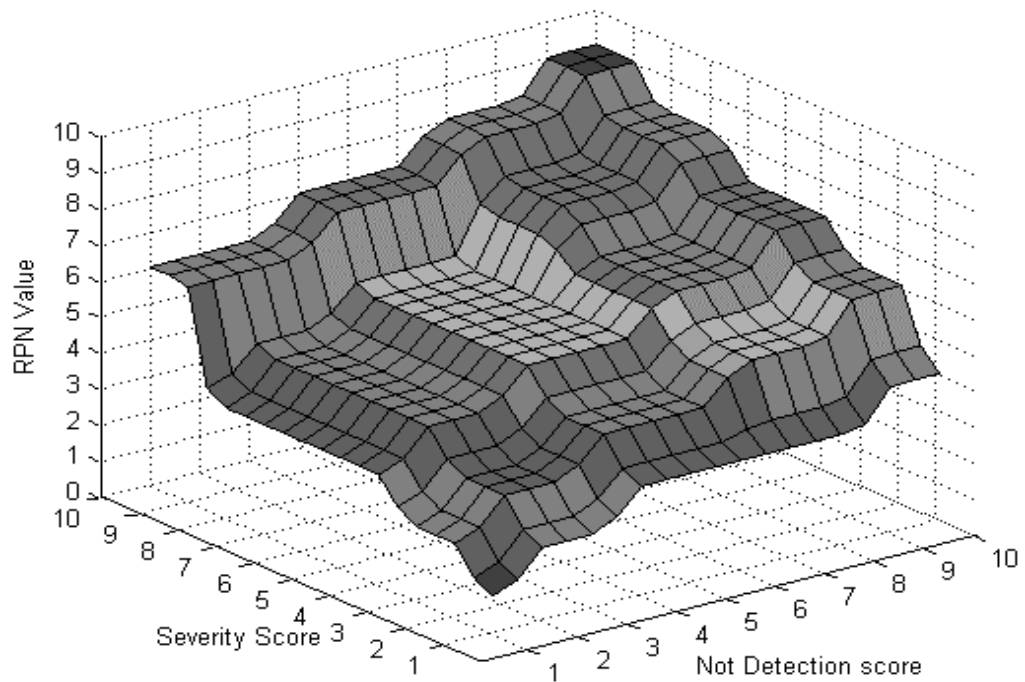


Figure 6.21: Surface plot of the FIS-based RPN model (after MFRR-NF) with Occurrence=5

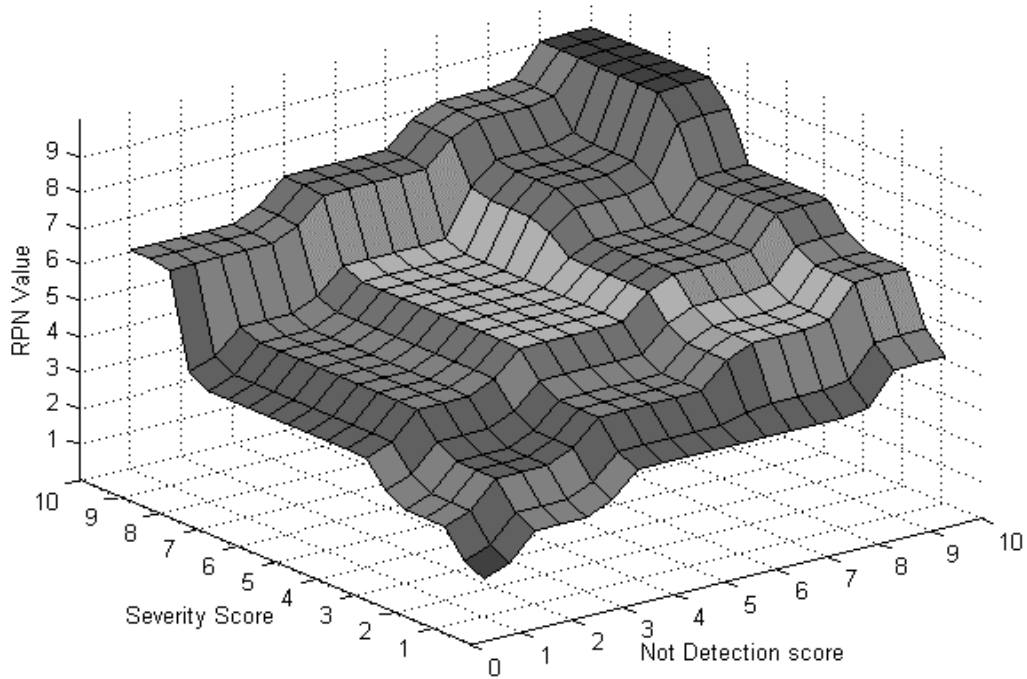


Figure 6.22: Surface plot of the FIS-based RPN model (after MFRR-H) with Occurrence=5

Both MFRR-NF and MFRR-H are useful to restore the monotonicity property of the FIS-based RPN model in an optimal way. A trade-off exists between the computational cost and completeness of solutions. MFRR-NF is able to produce the solutions in a faster way, but it can only search for one Stage-2 optimal solution, in which MFRR optimality is not guaranteed. However, it is useful as it can solve the problem in a practical timeframe.

6.4 Summary

In this chapter, the usefulness of MFRR is demonstrated using two set of publicly available information pertaining to FMEA of a sewage treatment plant and a public hospital purchasing process. MFRR has shown to be useful to restore the monotonicity of a non-monotone fuzzy rule base in an optimal way, to generate a monotone FIS-RPN model. In

the next chapter, application of MFRR framework to an education assessment problem will be presented.

CHAPTER 7

APPLICATION TO FUZZY INFERENCE SYSTEM -BASED CRITERION-REFERENCED ASSESSMENT

7.1 Introduction

In the previous chapter, the usefulness of MFRR is demonstrated using two publicly available information pertaining to FMEA problems. In this chapter, the use of MFRR as a solution to handle a non-monotone fuzzy rule base for a FIS-based Criterion Referenced Assessment (CRA) is presented.

The organization of this chapter is as follows. The background of educational assessment as well as its challenges are introduced in Section 7.2. In Section 7.3, the background of FIS-based CRA, the monotonicity property and its sufficient conditions are reviewed. In Section 7.4, the proposed FIS-based CRA model with the monotone fuzzy rule relabeling technique is presented. In Section 7.5, the case study and the results obtained are analyzed and discussed. Finally, concluding remarks are presented in Section 7.6.

7.2 Background

Educational assessment is concerned with judging the quality and extent of achievements or performances of a student, or a group of students, in a teaching and learning process (Groulund & Linn, 1990; Sadler, 2005). The focus of this chapter is on the use of Criterion-Referenced Assessment (CRA) in tertiary education. CRA is different from Norm-Referenced Assessment (NRA)(Burton, 2007). In NRA, a student's performance is

compared with those from other students in the same cohort (e.g. the top 5% of the class receives “A”), and this method is used in Scholastic Assessment Test and *Graduate Record Examination* (Lee, 2005). Instead of judging a student’s performance against those from his/her peers, CRA evaluates the student’s performance individually. In CRA, the student’s performance on a given test item (or task) is evaluated using a set of clearly defined criteria as the learning outcomes and/or standards (Dunn, Parry, & Morgan, 2002; Sadler, 2005). Usually, a score is given to each test item using a designated scoring rubric, and an aggregated score is produced by combining all scores from different test items (Sadler, 2005). CRA is becoming more popular in educational assessment practice owing to the following reasons: (i) the assessment aims to provide a measure of performance that is interpretable in terms of a clearly defined and delimited domain of learning tasks (Dunn et al., 2002; Sadler, 2005); (ii) grades are awarded to students based on their own competence (Neil, Wadley, & Phinn, 1999); and (iii) CRA is relatively transparent, accountable, and consistent (Dunn et al., 2002; Neil et al., 1999). As a result, CRA has been adopted widely in higher educational institutions in recent years (Dunn et al., 2002).

Despite its popularity, there are two key challenges for practical implementation of CRA: (i) the choice of the most meaningful measuring index (e.g., percentile score, raw score, or standard score); (ii) the method to obtain a meaningful measuring index (Popham & Husek, 1969). These problems are related to qualitative criteria, which are often imprecise and vague (Saliu, 2005). Normally, the qualitative criteria used in CRA are distinguishable attributes or rules that are useful for making judgments (Sadler, 2005). They are usually elaborated in terms of a marking grid, e.g., a simple verbal scale with qualitative linguistic terms, such as poor, acceptable, or excellent (Sadler, 2005).

To solve these problems, comprehensive mathematical approaches (instead of simple or weighted addition) that are able to handle uncertain and imprecise information in qualitative assessment are needed. In this aspect, fuzzy set-based approaches (Kwok, Zhou, Zhang, & Ma, 2007; Saliu, 2005; Tay & Lim, 2011a) offer an effective solution. Fuzzy set is useful for tackling ill-defined boundaries of activity or observation (Zadeh, 1965). In many practical cases, human preferences are qualitative and subjective. Therefore, it is not easy to assign exact numerical values to precisely describe the preferences (Jang et al., 1997; Tseng, 2010). Fuzzy set-based approaches can mathematically handle both numerical data and subjective preferences, which are often vague and imprecise (Jang et al., 1997; Mendel, 1995; Tseng, 2010). Besides that, fuzzy set related techniques are able to provide a meaningful measuring index (Boongoen, Shen, & Price, 2011; Kwok et al., 2007; Saliu, 2005; Tay & Lim, 2011a; Wang & Chen, 2008), hence solving the problems associated with practical implementation of CRA.

In this chapter, the focus is on the application of a popular fuzzy set-related technique, i.e., the Fuzzy Inference System (FIS), to CRA, resulting in an FIS-based CRA model. FIS models have been adopted in many educational assessment problems (Saliu, 2005; Tay & Lim, 2011a). The benefits of using FIS-based CRA are: (i) the rubric criteria of CRA can be qualitative rather than quantitative (Saliu, 2005), and FIS is a useful approach to keep qualitative assessment accountable; (ii) FIS can be used as an alternative to model the relationship between the score of each test item and the aggregated score (Tay & Lim, 2011a); (iii) FIS is able to incorporate expert knowledge in the form of linguistic or imprecise statements in its reasoning process (Saliu, 2005); and (iv) FIS can be used as a structured framework to increase the validity and reliability of educational assessment (Saliu, 2005).

Besides that, it is worth noting that other fuzzy set related techniques have previously been used in other education practice too (Hossain & Brooks, 2008; Tseng, 2010).

FIS-based CRA considers the score given to each test item as the input, and produces an aggregated score that reflects the student's performance as the output (i.e., the measuring index). A challenge for practical implementation of FIS-based CRA is the fulfillment of the monotonicity property of the measuring index (Saliu, 2005; Tay & Lim, 2011a). In essence, the monotonicity property suggests that a student with a better score for a test item should not be awarded a lower aggregated score. The monotonicity property is important to ensure the validity of the assessment, as highlighted in (Saliu, 2005). To preserve the monotonicity property, the use of sufficient conditions has been introduced as a set of governing equations to construct a monotonic FIS-based CRA model (Tay & Lim, 2008a, 2008b, 2011b). The sufficient conditions suggest that a complete and monotonically-ordered fuzzy rule base is required. However, in practice, it is difficult to establish a complete fuzzy rule base as it may contain a large number of fuzzy rules (Tay & Lim, 2008a, 2008b, 2011b). Previous research findings assume that the constructed fuzzy rule base follows a monotonic order, which can be either complete (Tay & Lim, 2011a) or incomplete (Jee, Tay, & Ng, 2013). However, it is always difficult to gather a set of monotonically-ordered fuzzy rules in practice. Therefore, further research is necessary to solve this problem, and this is the main motivation of this work.

The aim of this chapter is to investigate on the use of MFRR as a solution to handle a non-monotonic fuzzy rule base constructed based on the appraiser's evaluation. While the importance of fuzzy rule relabeling has been discussed, its application to real-world problems is still new. More importantly, incorporating the fuzzy rule relabeling technique into the FIS-

based CRA model is a novel approach to tackle the non-monotonic rule base problem. This is the key contribution of this chapter.

On the other hand, FIS-based CRA has been used in a number of educational assessment areas, e.g., students' writing sample (Nolan, 1998), micro-computer design (Saliu, 2005), English proficiency (Cin & Baba, 2008), and laboratory project assessment (Tay & Lim, 2011a). However, to the best of our knowledge, the use of FIS-based CRA in civil engineering is new. From the civil engineering perspective, CRA is important because the students (i.e., potential future civil engineers) should be able to produce a structural design that satisfies the required professional standards and specifications. In this aspect, it is the responsibility of the students (i.e., who have passed the structural engineering course) to produce structural designs that are strong enough to safely support all anticipated loadings, and must not deflect, tilt, vibrate, or crack in a way that could impair their usefulness (MacGregor, 1997; Nilson, Darwin, & Dolan, 2011). In this chapter, the efficacy of FIS-based CRA for the assessment of students' projects in a structural engineering subject, which is compulsory for civil engineering students in Universiti Malaysia Sarawak (UNIMAS), is reported. As expected, a non-monotonic fuzzy rule base has been provided by the appraiser during the case study (owing to qualitative human assessments). As such, we describe how the MFRR can be adopted to allow the fuzzy rules to be relabeled; hence ensuring the validity of the FIS-based CRA model in the case study. In short, this chapter reports a new application of the proposed fuzzy rule relabeling technique to educational assessment in civil engineering. It provides a solution for solving the common problem in implementing a valid FIS-based CRA model based on non-monotonic fuzzy rules from the appraiser. In addition, the use of a monotonicity test (Tay & Lim, 2011c) to validate the FIS-based CRA outputs is demonstrated.

7.3 FIS-Based CRA Model

A project with n test items (i.e., $Test\#i$, where $i = 1, 2, \dots, n$) is considered. Each test item is associated with a set of criteria, which is summarized as a scoring rubric. The score for $Test\#i$ (i.e., $x_i \in [\underline{x}_i, \overline{x}_i]$) is awarded by the appraiser in accordance with the designated scoring rubric, where $\underline{x}_i$ and $\overline{x}_i$ are the lower and upper bounds for $Test\#i$, respectively. The scores are combined by the FIS-based CRA model to produce an aggregated score, i.e., $aggregated\ score = f_{CRA}(x_1, x_2, \dots, x_n)$.

Each scoring rubric is further divided into $m_{Test\#i}$ partitions. Each partition is assigned a linguistic term, i.e., $A_i^{j_i}$, where $j_i = 1, 2, \dots, m_{Test\#i}$. In this paper, the Gaussian membership functions (i.e., $\mu_i^{j_i}(x_i)$) are used to represent these partitions. A fuzzy rule base is a collection of knowledge from the appraiser in the IF-THEN format. A specific fuzzy rule, i.e., $R_{j_1, j_2, \dots, j_n}: \bigwedge_{i=1}^n \mu_i^{j_i}(x_i) \rightarrow b_{j_1, j_2, \dots, j_n}$, of the proposed FIS-based CRA model is written as:

$$R_{j_1, j_2, \dots, j_n}: IF\ x_1\ is\ A_1^{j_1}\ and\ x_2\ is\ A_2^{j_2}\ and\ \dots\ and\ x_n\ is\ A_n^{j_n}\ THEN\ aggregated\ score\ is\ B_{j_1, j_2, \dots, j_n},$$

where $B_{j_1, j_2, \dots, j_n}$ is the membership function of the aggregated score and $b_{j_1, j_2, \dots, j_n}$ is the fuzzy singleton for $B_{j_1, j_2, \dots, j_n}$. To simplify the notation, $R_{j_1, j_2, \dots, j_n}$ is also denoted as $R_j: A_j \rightarrow b_j$, where $j(j_1, j_2, \dots, j_n) = j_1 + \sum_{k=2}^n [(j_k - 1)(\prod_{p=1}^{k-1} m_{Test\#p})]$. Using the zero-order Sugeno FIS model, the FIS-based CRA model is obtained using Eq. (7.1).

$$aggregated\ score = \frac{\sum_{j_1=1}^{m_{Test\#1}} \sum_{j_2=1}^{m_{Test\#2}} \dots \sum_{j_n=1}^{m_{Test\#n}} (\mu_1^{j_1}(x_1) \times \mu_2^{j_2}(x_2) \times \dots \times \mu_n^{j_n}(x_n) \times b_{j_1, j_2, \dots, j_n})}{\sum_{j_1=1}^{m_{Test\#1}} \sum_{j_2=1}^{m_{Test\#2}} \dots \sum_{j_n=1}^{m_{Test\#n}} (\mu_1^{j_1}(x_1) \times \mu_2^{j_2}(x_2) \times \dots \times \mu_n^{j_n}(x_n))} \quad (7.1)$$

7.3.1 The Monotonicity Property and the Sufficient Conditions

For the FIS-based CRA model to satisfy the monotonicity property, its output (i.e., *aggregatedscore*) must increase monotonically when its input (i.e., x_i) increases, as shown in Equation (7.2):

$$f_{CRA}(x_1, x_2, \dots, x_i^1, \dots, x_n) \leq f_{CRA}(x_1, x_2, \dots, x_i^2, \dots, x_n) \text{ where } x_i^1 < x_i^2 \quad (7.2)$$

The *sufficient conditions* in Section 3.3.2 can be used to ensure the monotonicity property (Tay & Lim, 2008a, 2011b). Two conditions are required, as follows:

Condition (1): The membership functions should be tuned in such a way that at the rule antecedent, $\frac{d\mu_i^{j_i=q_1}(x_i)/dx_i}{\mu_i^{j_i=q_1}(x_i)} \leq \frac{d\mu_i^{j_i=q_2}(x_i)/dx_i}{\mu_i^{j_i=q_2}(x_i)}$ where $q_1 < q_2$ and $x_i \in [\underline{x_i}, \overline{x_i}]$. Note that $\frac{d\mu_i^{j_i}(x_i)/dx_i}{\mu_i^{j_i}(x_i)}$ is the ratio of the rate of change of the membership function and the membership function itself (Tay & Lim, 2008a, 2011b).

Condition (2): A monotonic fuzzy rule base should be maintained in such a way that at the rule consequent, $b_{j_1, j_2, \dots, j_i=q_2, \dots, j_n} - b_{j_1, j_2, \dots, j_i=q_1, \dots, j_n} \geq 0$, where $q_1 < q_2$.

7.4 Constructing the FIS-Based CRA Model with the MFRR

7.4.1 Procedure for Constructing the FIS-Based CRA Model

The procedure for constructing the FIS-based CRA model is outlined in Figure 7.1. A structural engineering course project titled “Reinforced Concrete Design” for third year civil engineering students in UNIMAS is considered. Information from the course project is

used to clarify the procedure for constructing the FIS-based CRA model. The details of each step of the procedure are explained, as follows:

- (a) Determine the learning objectives and learning outcomes (see Section 7.4.2).
- (b) Design the test items (see Section 7.4.3).
- (c) Define the assessment criteria for each test item. The scoring rubric is developed based on the defined criteria (see Section 7.4.4).
- (d) Design the fuzzy membership functions for each test item with Condition (1) (see Section 7.4.5).
- (e) Collect a set of fuzzy rules from the appraiser to form a fuzzy rule base (see Section 7.4.6).
- (f) Perform monotone fuzzy rules relabeling (see Section 7.4.7).
- (g) Rate each test item (see Section 7.4.8).

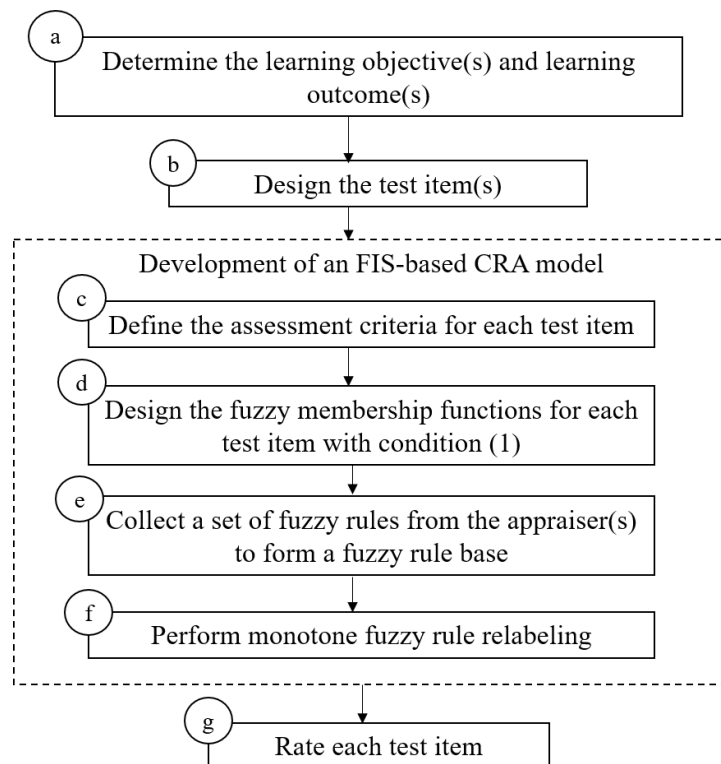


Figure 7.1: The procedure for constructing the FIS-based CRA model with the proposed monotone fuzzy rule relabeling technique

7.4.2 Determine the Learning Objectives and Learning Outcomes

The learning objectives and learning outcomes need to be clearly specified at the beginning of the course. Specifically, for the “Reinforced Concrete Design” course, its learning objectives include equipping students with the fundamental knowledge and skills in reinforced concrete design and the related principles in the design of structural elements in buildings (i.e. beams, slabs, columns, footings, and staircases) by following the professional codes of practice (British Standards Institution, 1997). At the end of the course, the students are expected to be able to design and analyze reinforced concrete beams, slabs, staircases, columns, and footings, as well as to illustrate their designs as a structural key plan.

7.4.3 Design the Test Items

The test items are designed in accordance with the learning objectives and learning outcomes. In this case, the students are expected to perform three test items (i.e., $n = 3$), namely *structural key plans (skp)*, *analysis of structural elements (ase)*, and *detailed sketching (ds)*, i.e., $i = \{skp, ase, ds\}$. The first test item (i.e., *skp*) requires the students to design structural key plans and draw them according to scale. The second test item (i.e., *ase*) requires the students to analyze and design the structural elements with proper steel reinforcement. The third test item (i.e., *ds*) requires the students to sketch the detailed design according to scale, in which the steel reinforcement elements should be clearly labeled, with all dimensions clearly indicated.

7.4.4 Define the Assessment Criteria for Each Test Item

For each test item (i.e., $Test\#i$), a scoring rubric with a predefined scale (i.e., $[\underline{x}_i, \bar{x}_i]$) is developed. The scoring rubrics for skp , ase , and ds are described in Tables 7.1 to 7.3, respectively. A score from 1 to 10 is used for skp and ds (i.e., $x_{skp}, x_{ds} \in [1, 10]$), while a score from 1 to 20 is used for ase (i.e., $x_{ase} \in [1, 20]$). Each scoring rubric is divided into five partitions, i.e., $m_{skp} = m_{ase} = m_{ds} = 5$. Each partition is represented by a linguistic term, i.e. “Unsatisfactory”, “Satisfactory”, “Good”, “Very Good”, and “Excellent”, respectively. A criterion is further linked to each partition. As an example, the third partition for skp is linked to “Good” (i.e. $A_{skp}^3 = \text{Good}$), which indicates (1) plans are drawn to scale; (2) the layout of all the structural elements follows the logic of load transfer; (3) some of the dimensions are not indicated and some structural elements are not marked.

Table 7.1: The scoring rubrics for *structural key plans*

Score	Linguistic Terms	Criteria
10	Excellent	Plans are drawn to scale. The layout of all structural elements follows the logic of load transfer. All the dimensions are clearly indicated. Every structural element is clearly marked.
9-8	Very Good	Plans are drawn to scale. The layout of all the structural elements follows the logic of load transfer. All the dimensions are clearly indicated. Some structural elements are not marked.
7-6	Good	Plans are drawn to scale. The layout of all the structural elements follows the logic of load transfer. Some of the dimensions are not indicated. Some structural elements are not marked.
5-3	Satisfactory	Plans are not drawn to scale. Most of the structural elements follow the logic of load transfer. Some of the dimensions are not indicated. Most of the structural elements are not marked.
2-1	Unsatisfactory	Plans are not drawn to scale. Most of the structural elements do not follow the logic of load transfer. The dimensions are poorly indicated. The structural elements are not marked.

Table 7.2: The scoring rubrics for *analysis of structural elements*

Score	Linguistic Terms	Criteria
20	Excellent	Analysis and design are complete. Able to apply knowledge in reinforced concrete design. Able to calculate loads, analyze the structural elements correctly and provide proper steel reinforcement.
19-16	Very Good	Analysis and design are complete. Able to apply most of the learned knowledge. Able to calculate loads, analyze the structural elements correctly and provide proper steel reinforcement.
15-11	Good	Analysis and design are mostly complete. Able to apply most of the learned knowledge. Minor mistakes in calculating loads, minor mistakes in analyzing the structural elements and provide proper steel reinforcement.
10-6	Satisfactory	Analysis and design are mostly complete. Able to apply some of the learned knowledge. Obvious mistakes in calculating loads, and analyzing the structural elements. Most of the steel reinforcement is provided incorrectly.
5-1	Unsatisfactory	Analysis and design are incomplete. Apply very little of learned knowledge. Incorrect use of the analysis and design methods. Steel reinforcement is provided incorrectly.

Table 7.3: The scoring rubrics for *detailed sketching*

Score	Linguistic Terms	Criteria
10	Excellent	Sketches are drawn approximately to scale. All the steel reinforcement arrangements are correctly drawn with clear labels. All the dimensions are indicated. Ready to be transferred into detail drawings.
9-8	Very Good	Sketches are drawn approximately to scale. All the steel reinforcement arrangements are drawn with some missing labels. All the dimensions are indicated. Ready to be transferred into detail drawings with minor amendment.
7-6	Good	Sketches are drawn approximately to scale. Most of the steel reinforcement arrangements are correctly drawn with some missing. The dimensions are not clearly indicated. Ready to be transferred into detail drawings with some amendment.
5-3	Satisfactory	Sketches are drawn. Some of the steel reinforcement arrangements are drawn with incomplete labels. Most of the dimensions are not indicated. Ready to be transferred into detail drawings with major amendment.
2-1	Unsatisfactory	Sketches are drawn. Most of the steel reinforcement arrangements are not drawn and with incomplete labels. Most of the dimensions are not indicated. Not ready to be transferred into detail drawings.

7.4.5 Design the Fuzzy Membership Functions for Each Test Item with Condition (1)

The Gaussian membership function is adopted to represent each partition of the scoring rubric, as follows.

$$\mu_i^{j_i}(x: c_i^{j_i}, \sigma_i^{j_i}) = e^{-[x - c_i^{j_i}]^2 / 2\sigma_i^{j_i^2}} \quad (7.3)$$

where $c_i^{j_i}$ and $\sigma_i^{j_i}$ are the center and width of the Gaussian membership function, respectively.

The Gaussian membership functions for *skp*, *ase*, and *ds* are designed according to Condition 1, and are labeled as $\mu_{skp}^{j_{skp}}$, $\mu_{ase}^{j_{ase}}$, and $\mu_{ds}^{j_{ds}}$, respectively. Figure 7.2 shows the fuzzy membership functions for *skp*. As an example, $\mu_{skp}^2(x_{skp})$ represents the second partition (i.e., “Satisfactory” with a score between 3 and 5) of the scoring rubric of *skp*. Similarly, the fuzzy membership functions for *ase* and *ds* are depicted as in Figure 7.3 and Figure 7.4, respectively.

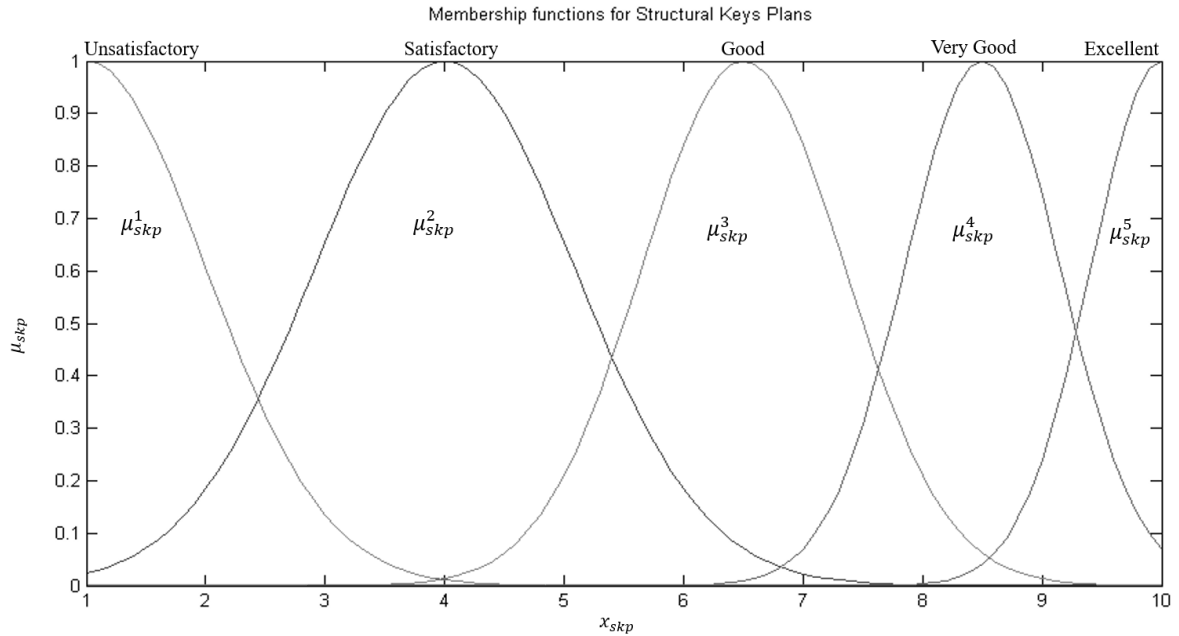


Figure 7.2: Gaussian membership functions for *structural key plans*

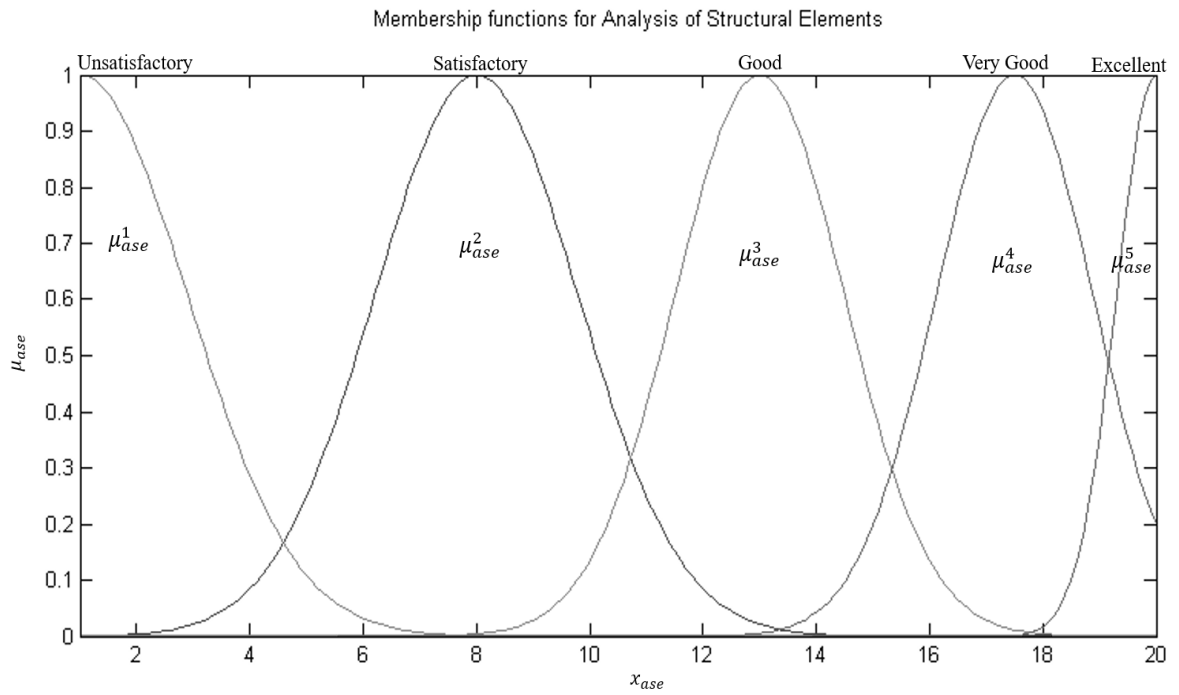


Figure 7.3: Gaussian membership functions for *analysis of structural elements*

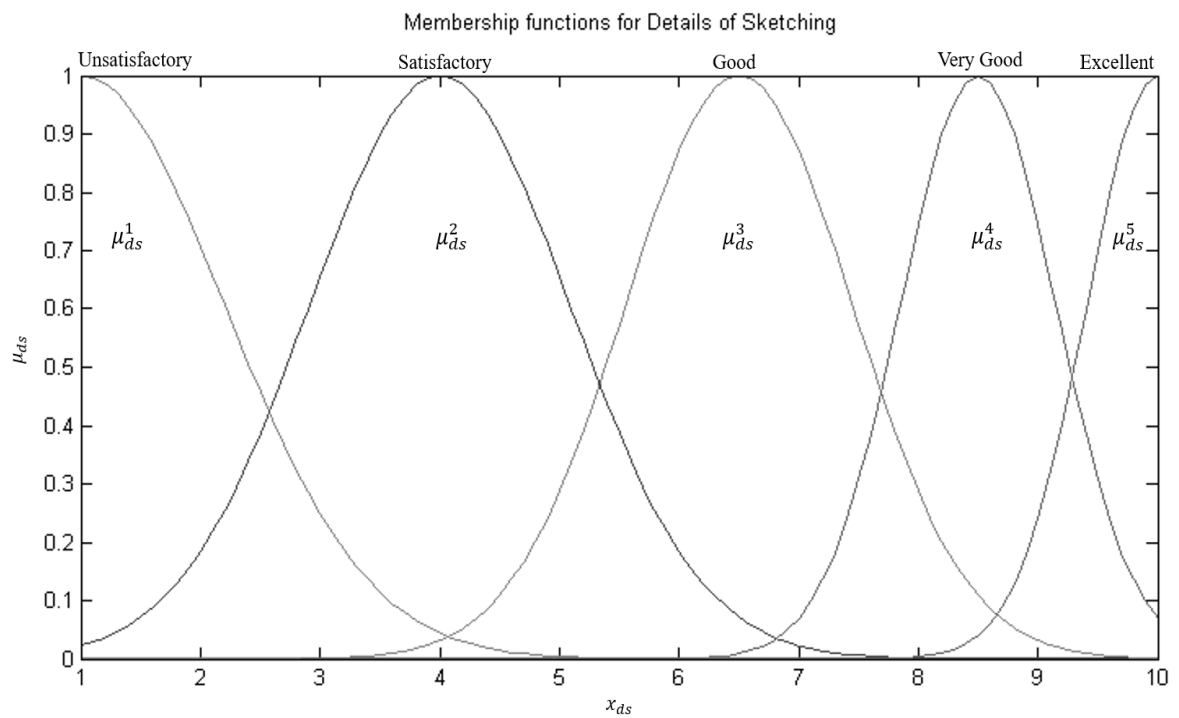


Figure 7.4: Gaussian membership functions for *detailed sketching*

Each Gaussian membership function can be projected and visualized by using the ratio of $\mu_i^{ji'}(x_i)/\mu_i^{ji}(x_i)$, i.e., Eq. (7.4). A linear function should be obtained. Figures 7.5, 7.6, and 7.7 illustrate the projections of the fuzzy membership functions for *skp*, *ase*, and *ds*, respectively. It can be observed that all the designed Gaussian membership functions satisfy Condition (1). As an example, in Figure 7.5, the linear line of “ $E_{skp}^{Excellent}$ ” is higher than that of “ $E_{skp}^{Very Good}$ ”. In other words, $E_{skp}^{Excellent} \geq E_{skp}^{Very Good} \geq E_{skp}^{Good} \geq E_{skp}^{Satisfactory} \geq E_{skp}^{Unsatisfactory}$ is satisfied for $x_{skp} \in [1, 10]$. The same observation applies to Figures 7.6 and 7.7 for $x_{ase} \in [1, 20]$ and $x_{ds} \in [1, 10]$, respectively.

$$E_i^{A_i^{ji}}(x_i) = \mu_i^{ji'}(x_i)/\mu_i^{ji}(x_i) = -(1/\sigma_i^2)x_i + (c_i/\sigma_i^2) \quad (7.4)$$

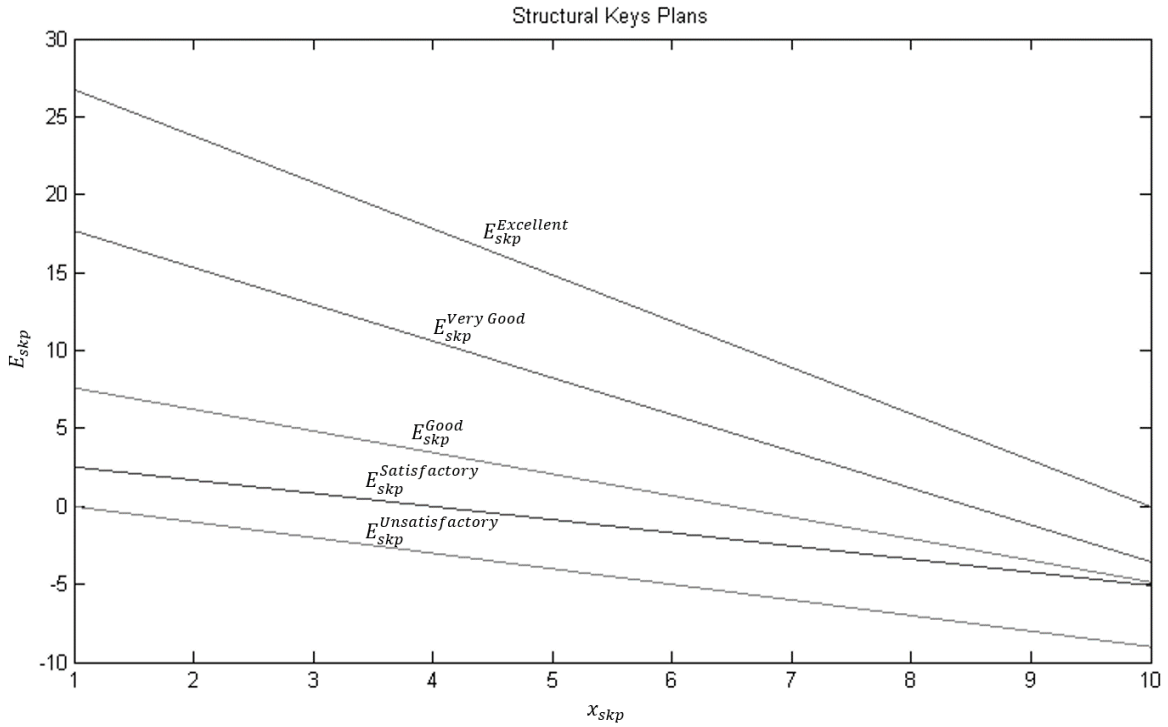


Figure 7.5: Projection of the fuzzy membership functions for *structural key plans*

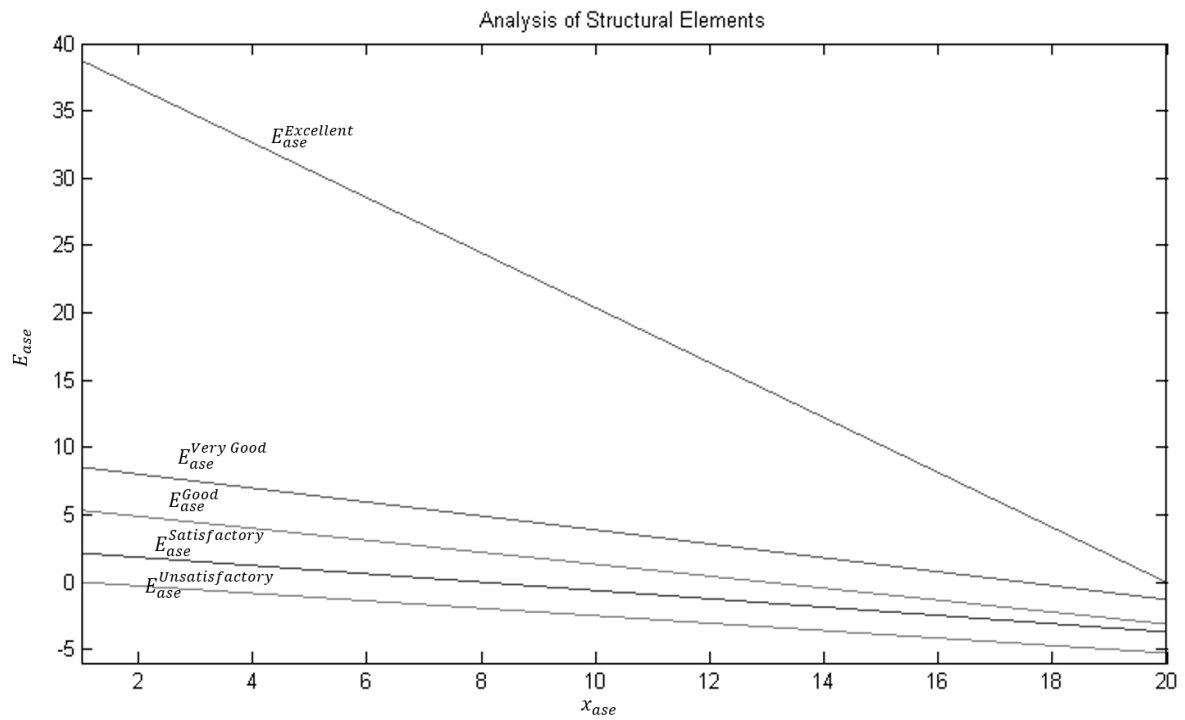


Figure 7.6: Projection of the fuzzy membership functions for *analysis of structural elements*

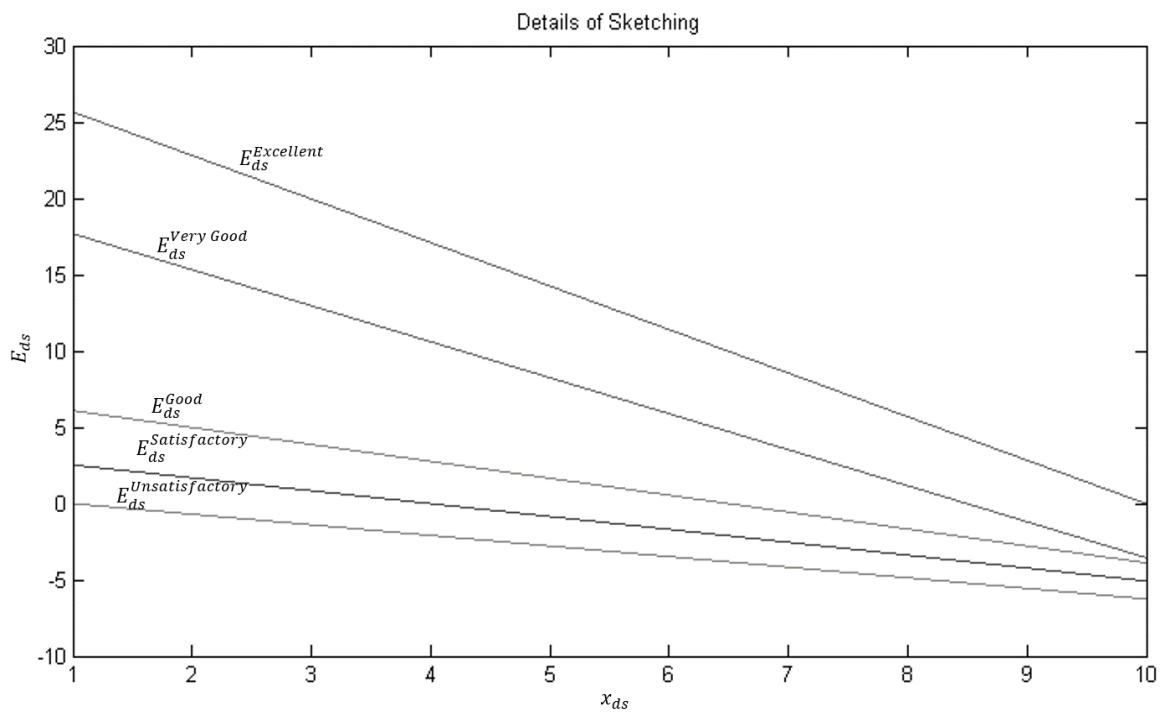


Figure 7.7: Projection of the fuzzy membership functions for *detailed sketching*

7.4.6 Collect a Set of Fuzzy Rules from the Appraiser

In this step, the output membership functions are defined. In this paper, the FIS-based CRA output (i.e., the aggregated score) varies from 1 to 100. It is represented by seven fuzzy membership functions. These fuzzy membership functions represent the linguistic terms of “Unsatisfactory”, “Very Weak”, “Weak”, “Fair”, “Good”, “Very Good”, and “Excellent”, respectively, and they are characterized by seven singleton values, i.e., $b_{unsatisfactory} = 20$, $b_{Very\ Weak} = 40$, $b_{Weak} = 50$, $b_{Fair} = 70$, $b_{Good} = 80$, $b_{Very\ Good} = 90$, $b_{Excellent} = 100$, respectively. These singleton values were assigned by a lecturer who teach structural engineering course project in UNIMAS.

A fuzzy rule base is constructed based on the information from the appraiser. The fuzzy rule base is a collection of domain expert's (i.e., the appraiser in this case study) knowledge in the IF-THEN format. Each rule describes the relationship between the test item and aggregated score. In this case study, a total of 125 ($r = 5 (skp) \times 5 (ase) \times 5 (ds) = 125$) fuzzy rules are required, and they are denoted as “ $R_{j_{skp}, j_{ase}, j_{ds}}: \mu_{skp}^{j_{skp}}(x_{skp}) \wedge \mu_{ase}^{j_{ase}}(x_{ase}) \wedge \mu_{ds}^{j_{ds}}(x_{ds}) \rightarrow b_{j_{skp}, j_{ase}, j_{ds}}$ ”. Each fuzzy rule is interpreted as: “IF *structural key plans* is $\mu_{skp}^{j_{skp}}$ and *analysis of structural elements* is $\mu_{ase}^{j_{ase}}$ and *detailed sketching* is $\mu_{ds}^{j_{ds}}$, THEN the *aggregated score* is $B_{j_{skp}, j_{ase}, j_{ds}}$ ”, where $b_{j_{skp}, j_{ase}, j_{ds}}$ is the fuzzy singleton for $B_{j_{skp}, j_{ase}, j_{ds}}$. Figure 7.8 shows part of the fuzzy rule base collected from the appraiser (the full list of fuzzy rules is attached in Appendix B). As an example, $R_{1,3,3}$ is interpreted as “IF *structural key plans* is Unsatisfactory and *analysis of structural elements* is Good and *detailed sketching* is Good, THEN the *aggregated score* is Very Weak”; hence, $b_{1,3,3}=40$.

It is worth mentioning that the fuzzy rules obtained from the appraiser may not be monotonically-ordered, owing to human subjective assessments. From Figure 7.8, *Condition (2)* suggests that $b_{1,5,3} \geq b_{1,4,3}$ should be satisfied. However, as can be seen in Figure 7.8, $b_{1,5,3} = 40$ and $b_{1,4,3} = 50$, hence, $b_{1,5,3} \geq b_{1,4,3}$ is not satisfied. As a result, it is necessary to relabel the fuzzy rules in order to ensure the validity of the FIS-based CRA output.

$R_{1,3,3}$: IF <i>Structural key plans</i> is Unsatisfactory and <i>analysis of structural elements</i> is Good and <i>detailed sketching</i> is Good THEN <i>aggregatedscore</i> is Very Weak. $R_{1,4,3}$: IF <i>Structural key plans</i> is Unsatisfactory and <i>analysis of structural elements</i> is Very Good and <i>detailed sketching</i> is Good THEN <i>aggregatedscore</i> is <u>Weak</u>. $R_{1,5,3}$: IF <i>Structural key plans</i> is Unsatisfactory and <i>analysis of structural elements</i> is Excellent and <i>detailed sketching</i> is Good THEN <i>aggregatedscore</i> is Very Weak $R_{2,4,4}$: IF <i>Structural key plans</i> is Satisfactory and <i>analysis of structural elements</i> is Very Good and <i>detailed sketching</i> is Very Good THEN <i>aggregatedscore</i> is Good $R_{2,4,5}$: IF <i>Structural key plans</i> is Satisfactory and <i>analysis of structural elements</i> is Very Good and <i>detailed sketching</i> is Excellent THEN <i>aggregatedscore</i> is <u>Fair</u>
--

Figure 7.8: Part of the fuzzy rule base

7.4.7 Perform Monotone Fuzzy Rule Relabeling

In this step, the proposed MFRR-H (see section 5.3.3 in Chapter 5) is used to relabel the non-monotone fuzzy rule base. The relabeling outcome can be observed in Figure 7.9, in which minimum two fuzzy rules are relabeled, with minimum loss of 200. The fuzzy singletons (on the aggregated score domain) for $R_{1,4,3}$ and $R_{2,4,5}$ are relabeled to be “Very Weak” and “Good”, respectively.

$R_{1,3,3}$: IF <i>Structural key plans</i> is Unsatisfactory and <i>Analysis of structural elements</i> is Good and <i>Detailed sketching</i> is Good THEN aggregated score is Very Weak .
$R_{1,4,3}^{rel}$: IF <i>Structural key plans</i> is Unsatisfactory and <i>Analysis of structural elements</i> is Very Good and <i>Detailed sketching</i> is Good THEN aggregated score is <u>Very Weak</u> .
$R_{1,5,3}$: IF <i>Structural key plans</i> is Unsatisfactory and <i>Analysis of structural elements</i> is Excellent and <i>Detailed sketching</i> is Good THEN aggregated score is Very Weak
$R_{2,4,4}$: IF <i>Structural key plans</i> is Satisfactory and <i>Analysis of structural elements</i> is Very Good and <i>Detailed sketching</i> is Very Good THEN aggregated score is Good
$R_{2,4,5}^{rel}$: IF <i>Structural key plans</i> is Satisfactory and <i>Analysis of structural elements</i> is Very Good and <i>Detailed sketching</i> is Excellent THEN aggregated score is <u>Good</u>

Figure 7.9: Examples of some fuzzy rules after fuzzy rule relabeling

A computer with Intel (R) Core (TM) i5-3210M CPU @2.50GHz, 6.00 GB of RAM and Microsoft Window 10 Single Language has been used for the experiment. With Matlab R2012a, the search process consumed 0.046435 seconds.

7.4.8 Rate Each Test Item

A score is awarded to each test item with reference to its associated scoring rubric, as in Section 7.4.4. Eq. (7.1) in Section 7.3 is used to aggregate the score. The relabeled fuzzy rule base from Step (f) is used.

7.5 Experimental Results and Discussion

In this section, the experimental results are analyzed and discussed. A number of surface plots are first depicted and discussed. Then, a monotonicity test to evaluate the fulfillment of the monotonicity property by the FIS-based CRA model is conducted. Discussion of the results ensues.

7.5.1 Surface Plots

Figures 7.10 and 7.11 illustrate the surfaces plot of the aggregated scores versus x_{ase} and x_{ds} (with $x_{skp} = 5$) without and with MFRR, respectively. A non-monotonic area is observed in Figure 7.10, i.e., Area 1. This is because the fuzzy rules are not monotonically-ordered. After MFRR, a monotonic surface plot is observed in Figure 7.11, whereby Area 1 (in Figure 7.10) now satisfies the monotonicity property (i.e., Area 2).

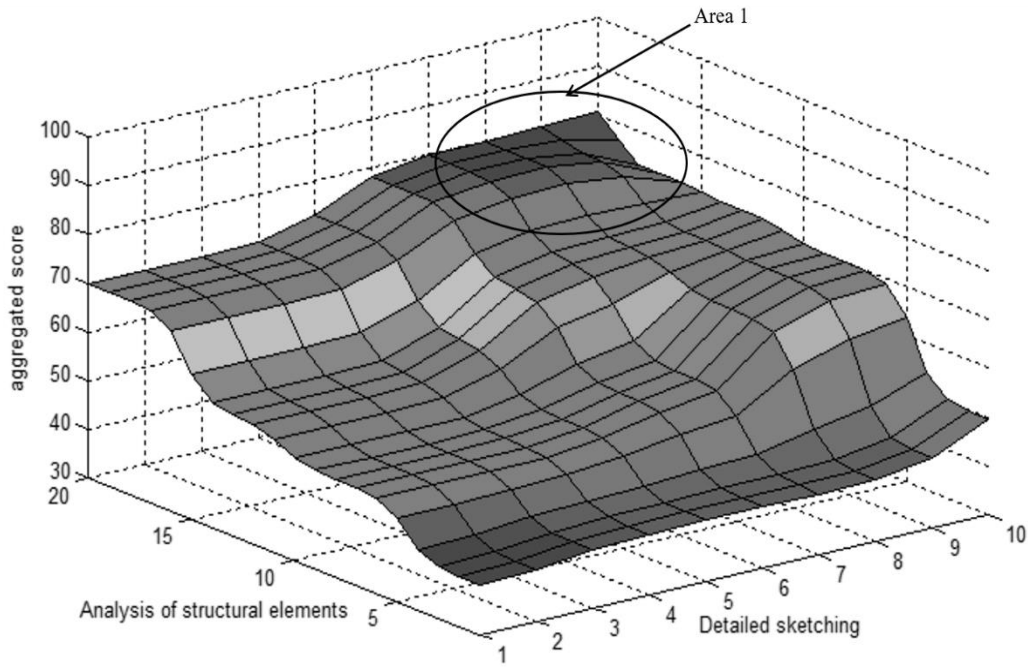


Figure 7.10: The surface plot of the aggregated scores versus ase and ds (with $x_{skp} = 5$) without MFRR

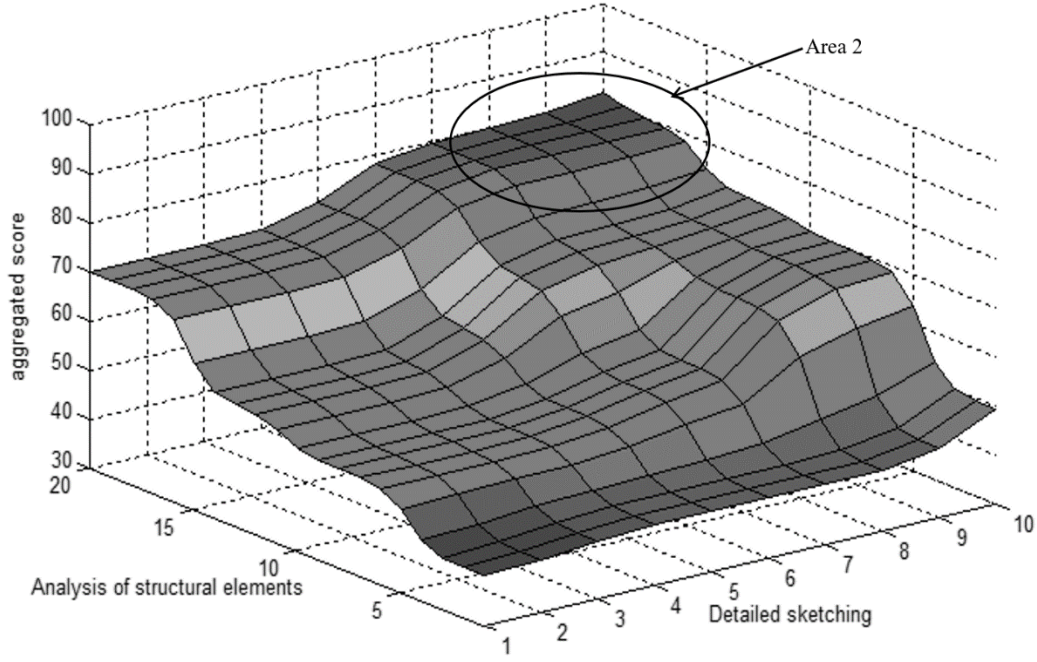


Figure 7.11: The surface plot of the aggregated scores versus ase and ds (with $x_{skp} = 5$) with MFRR.

7.5.2 Monotonicity Test

In this section, the monotonicity test introduced in (Tay & Lim, 2011c) is used to evaluate the degree of fulfillment of the monotonicity property by the FIS-based CRA model. Specifically, the monotonic relationship between the output (i.e., aggregated score) and each input of the FIS-based CRA model (denoted as x_i , where $x_i \in [x_{skp}, x_{ase}, x_{ds}]$), is evaluated.

Let $\bar{s}$ denote a subset of $[x_{skp}, x_{ase}, x_{ds}]$, where x_i is excluded, i.e., $x_1, x_2 \in \bar{s}$, $\bar{s} \subset \bar{x}$; $x_i \notin \bar{s}$. In the monotonicity test, Equation (7.1) is written as $aggregated\ score_{x_{skp}, x_{ase}, x_{ds}} = f_{CRA}(x_{skp}, x_{ase}, x_{ds})$, and Equation (7.5), i.e., $monotone(aggregated\ score_{x_i, \bar{s}})$, is adopted. To evaluate the degree of fulfillment of the monotonicity property, Equation (7.6) is computed. If $Monotonicity\ test(x_i) = \bar{x}_2 \times \bar{x}_1 \times (\bar{x}_i - 1)$, the FIS-based CRA model is said to satisfy the monotonicity property.

$$\begin{aligned}
& \text{monotone}(\text{aggregated score}_{x_i, \bar{s}}) = \\
& \begin{cases} 1, & \text{monotone}(\text{aggregated score}_{x_i, \bar{s}}) \leq \text{monotone}(\text{aggregated score}_{x_{i+1}, \bar{s}}) \\ 0, & \text{else} \end{cases} \quad (7.5)
\end{aligned}$$

$$\text{Monotonicity test}(x_i) = \sum_{x_2=1}^{x_2=\bar{x}_2} \sum_{x_1=1}^{x_1=\bar{x}_1} \sum_{x_i=1}^{x_i=\bar{x}_i-1} \left(\text{monotone}(\text{aggregated score}_{x_i, \bar{s}}) \right) \quad (7.6)$$

Table 7.4 summaries the test results of $x_i \in (x_{skp}, x_{ase}, x_{ds})$, for both with and without MFRR. When $x_i = x_{skp}$, $\bar{x}_2 \times \bar{x}_1 \times (\bar{x}_i - 1) = 10 \times 20 \times (10 - 1) = 1800$; hence $\text{Monotonicity test}(x_{skp}) = 1800$ is obtained for both with and without MFRR. In other words, the monotonicity property is satisfied for skp before and after fuzzy rule relabeling.

When $x_i = x_{ase}$, $\bar{x}_2 \times \bar{x}_1 \times (\bar{x}_i - 1) = 10 \times 10 \times (20 - 1) = 1900$. The monotonicity test, $\text{Monotonicity test}(x_{ase})$, produces 1863 and 1900 for without and with MFRR, respectively. These results indicate that the FIS-based CRA model is not able to satisfy the monotonicity property without fuzzy rule relabeling. The same can be observed for $x_i = x_{ds}$, as shown in Table 7.4.

Table 7.4: Results of the monotonicity test

x_i	$\bar{x}_2 \times \bar{x}_1 \times (\bar{x}_i - 1)$	$\text{Monotonicity test}(x_i)$	
		Without MFRR	With MFRR
x_{skp}	1800	1800	1800
x_{ase}	1900	1863	1900
x_{ds}	1800	1751	1800

7.5.3 Results and Discussion

Table 7.5 summarizes the evaluation results for three CRA models (conventional model as well as FIS-based CRA models with and without MFRR). Column “No.” is the label of students. Columns “ x_{skip} ”, “ x_{ase} ”, and “ x_{ds} ” list the scores for all test items, respectively. Column “CRA score” shows the assessment results with the conventional CRA model, i.e., $aggregated\ score = \left\{ \frac{(x_{skip} + x_{ase} + x_{ds})}{(x_{skip} + x_{ase} + x_{ds})} \times 100\% \right\}$. Columns “FIS-based CRA (without MFRR)” and “FIS-based CRA (with MFRR)” indicate the assessment results (i.e., the aggregated scores and expert’s knowledge) with the FIS-based CRA models, for both without and with MFRR, respectively.

Table 7.5: Assessment results of three CRA models

No.	Scores			CRA score (%)	FIS-based CRA (without MFRR)				FIS-based CRA (with MFRR)			
	x_{skip}	x_{ase}	x_{ds}		<i>aggregated score</i> (%)	Expert’s Knowledge		<i>aggregated score</i> (%)	Expert’s knowledge		<i>b</i>	
						Linguistic term	<i>b</i>		Linguistic term			
1	2	3	3	20.0000	25.1433	Unsatisfactory	20	25.1433	Unsatisfactory		20	
2	3	3	3	22.5000	36.9617	Very Weak	40	36.9617	Very Weak		40	
3	3	5	2	25.0000	42.8388	Very Weak	40	42.8388	Very Weak		40	
4	1	15	10	65.0000	49.9938	Weak	50	50.0567	Weak		50	
5	2	13	6.5	53.7500	46.6527	Very Weak	40	46.6161	Very Weak		40	
6	2	20	6.5	71.2500	50.4176	Very Weak	40	49.2427	Very Weak		40	
7	2	17	6.5	63.7500	56.0532	Weak	50	49.1672	Very Weak		40	
8	4	6	3	32.5000	49.2854	Weak	50	49.2854	Weak		50	
9	5	7	3	37.5000	49.9368	Weak	50	49.9368	Weak		50	
10	5	7	7	47.5000	55.7510	Weak	50	55.7510	Weak		50	
11	5	10	7	55.0000	58.6166	Weak	50	58.6166	Weak		50	
12	4	18	10	80.0000	70.7233	Fair	70	79.6372	Good		80	
13	4	18	8.5	76.2500	79.2973	Good	80	79.6225	Good		80	
14	6	11	7	60.0000	68.5495	Fair	70	68.5495	Fair		70	
15	8	16	9	82.5000	85.9476	Very Good	90	85.9499	Very Good		90	
16	9	17	9	87.5000	90.1409	Very Good	90	90.1410	Very Good		90	
17	10	17	9	90.0000	91.9102	Very Good	90	91.9102	Very Good		90	
18	10	18	9	92.5000	92.2882	Very Good	90	92.2882	Very Good		90	
19	10	19	9	95.0000	94.9462	Very Good	90	94.9462	Very Good		90	
20	10	19	10	97.5000	99.1940	Excellent	100	99.1940	Excellent		100	

As an example, the assessment outcome of student #19 is analyzed, as follows. Figure 7.12 depicts the structural key plans designed by student #19. The student was awarded a score of 10 for *skp* because the plan was drawn to scale with the layout of all structural elements followed the logic of load transfer. In addition, all dimensions in the plan were clearly indicated, and every structural element was clearly marked. The student was given a score of 19 for *ase* because the analysis and design were complete, and the student was able to calculate loads, analyse the structural elements correctly, and provide proper steel reinforcement. In other words, the student demonstrated the ability to apply most of the learned knowledge. However, one mark was deducted due to minor mistakes in calculation. The student was awarded a score of 9 for *ds* because the sketch was drawn approximately to scale. All steel reinforcement arrangements were correctly drawn, but with some missing labels; hence one mark was deducted. Besides that, all dimensions were indicated and ready to be transferred into detailed drawings. The aggregated scores obtained by the student were 95%, 94.9462%, and 94.9462% using the conventional CRA model, FIS-based CRA models without and with fuzzy rule relabeling, respectively.

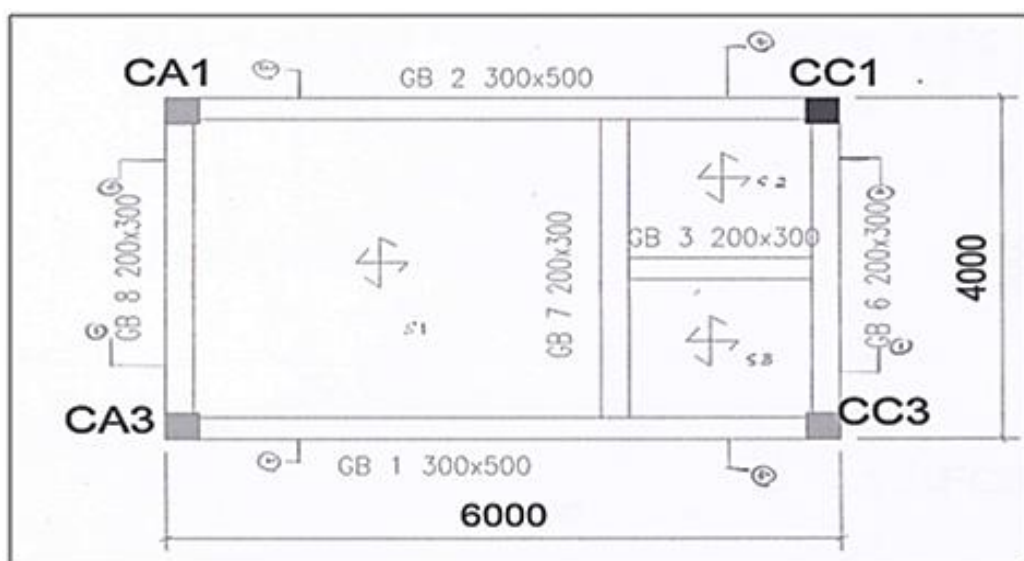


Figure 7.12: Structural key plan designed by student #19

From Table 7.5, it can be seen that the aggregated scores produced by the FIS-based CRA models are in accordance with the expert's (appraiser's) assessment, as compared with the aggregated scores produced by the conventional CRA model. As an example, student #6 was awarded 2, 20, and 6.5 for *skp*, *ase*, and *ds*, respectively. The aggregated scores were 71.2500%, 50.4176%, and 49.2427%, with the conventional CRA model, FIS-based CRA models without and with MFRR, respectively. Using a simple addition method, the conventional CRA model produced an aggregated score of 71.2500%, even though student #6 performed poorly with a score of 2 for *skp* (because the key plan designed did not follow the logic of load transfer). However, the *ase* score was 20 as the analysis and design were complete for the structural elements, but for an inappropriate structure. As the sketches were drawn approximately to scale, a score of 6.5 was awarded for *ds*. While the conventional CRA model did not manage to tackle the issue associated with a low *skp* score, FIS-based CRA could handle it properly. More reasonable aggregated scores, i.e., 50.4176% and 49.2427%, were yielded by the FIS-based CRA models without and with fuzzy rule relabeling, respectively. This outcome is in accordance with the fuzzy rule solicited from the expert (appraiser), i.e. students who achieved excellent and good scores for analysis of structural elements and detailed sketching, respectively, but with an unsatisfactory score for structural key plans should be given an aggregated score of "Very Weak" (see Figure 7.9).

The importance of MFRR can be observed by comparing the aggregated scores of students #6 and #7. Both students were awarded the same scores for *skp* and *ds*, i.e. 2 and 6.5, respectively. Student #6 scored better than Student #7 for *ase*. Thus, student #6 should be awarded a higher aggregated score than student #7. However, owing to the non-monotonically-ordered fuzzy rules from the expert (appraiser), the aggregated score for student #6 (50.4176%) was lower than that of student #7 (56.0532%) with the FIS-based

CRA model without fuzzy rule relabeling. This problem could be solved with the proposed MFRR technique. By using the FIS-based CRA model with fuzzy rule relabeling, the aggregated score for student #6 (49.2427%) was higher than that of student #7 (49.1672%). The same scenario can be observed from the aggregated scores of students #12 and #13.

7.6 Summary

In this chapter, the FIS-based CRA model with a monotone fuzzy rule relabeling technique has been proposed. It is important to ensure the monotonicity property of the FIS-based CRA model, in order to produce meaningful assessment outcomes. A proposed MFRR technique provides a practical solution to solve problems associated with the non-monotonic rule base that has been constructed based on the appraiser's evaluation. A case study related to civil engineering education has been used to evaluate the proposed approach. Promising results have been demonstrated, as discussed and analyzed in the previous section. In the next chapter, concluding remarks will be presented.

CHAPTER 8

CONCLUSION AND SUGGESTIONS FOR FUTURE WORKS

8.1 Conclusion and Contributions

The importance of the monotonicity property in FIS-based models has been highlighted in several recent literatures, whereby a monotonically-ordered fuzzy rule base is an essential building block for constructing a monotone FIS model. However, little attention has been paid to the case that a fuzzy rule base that contains noise, resulting non-monotone fuzzy rules. Drawing conclusion from these non-monotone fuzzy rules might lead to invalid/contradict inference outcomes. To overcome the shortcoming, in this thesis, MFRR has been proposed as a solution to optimally relabel non-monotone fuzzy rules into a set of monotone fuzzy rules. Four MFRR methods have been devised, i.e., MFRR-BF, MFRR-BFBT, MFRR-NF and MFRR-H. The relabeled monotone fuzzy rules have shown to be useful for zero-order TSK FIS modelling. Besides that, a graphical representation of fuzzy rules is proposed. A set of fuzzy rules is represented in a graph view, in which non-monotone relationship between two fuzzy rules is denoted by an edge in the graph. Such representation provides better visualization and understanding on the monotonicity relationship of a fuzzy rule base. A general MFRR framework for practical implementation is then devised, in which a three-stage framework is formulated. Furthermore, the thesis also demonstrated that the developed MFRR framework has effectively improve the performance when applied to the real-world problems.

In conclusion, the major contributions of this research are as follows:

- i. A new method to systematically analyze non-monotone fuzzy rules in a fuzzy rule base is developed. This achieved Objective 1 of this research.
- ii. New methods to systematically relabel non-monotone fuzzy rules (i.e., MFRR) in an optimal way are devised. This achieved Objective 2 of this research.
- iii. A practical MFRR framework is formulated. This achieved Objective 3 of this research.
- iv. The applicability of the MFRR framework to FIS-based RPN model in FMEA and FIS-based education assessment model with benchmark and real-world information. This achieved Objective 4 of this research.

8.2 Suggestions for Future Work

While promising and positive results have been obtained, much works can be done to further improve the work presented in this thesis. Some suggestions for future works are as follows.

- i. For further research, the application of MFRR to a fuzzy rule base with both fuzzy rules gathered from human experts and induced from data sets, will be considered.
- ii. In this research work, the use of MFRR is limited to the zero-order TSK FIS model. It is interesting to investigate on the use of MFRR to other fuzzy models such as the Mamdani models, potential with different relabeling criteria.
- iii. The use of monotone test for fuzzy rules to facilitate MFRR will be investigated. The monotone test could serve as a metric to indicate the degree of a fuzzy rule base fulfilling monotonicity property. Such metric is useful to provide an indication whether a fuzzy rule base should (or should not) be relabeled in practice.

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APPENDICES

List of Publications

Journal Articles

Wui Lee Chang, Lie Meng Pang and Kai Meng Tay. (2017). Application of self-organizing map to failure modes and effects analysis methodology. *Neurocomputing*, 249, 314-320. (Impact factor:3.317)

Lie Meng Pang, Kai Meng Tay and Chee Peng Lim. (2016). Monotone fuzzy rule relabeling for the zero-order TSK fuzzy inference system. *IEEE Transactions on Fuzzy Systems*, 24(6), 1455-1463. (Impact factor: 7.671)

Conference Papers

Yi Wen Kerk, Lie Meng Pang, Kai Meng Tay and Chee Peng Lim, “Multi-expert decision-making with incomplete and noisy fuzzy rules and the monotone test”, IEEE International Conference on Fuzzy Systems (FUZZ), Vancouver, Canada, 2016.

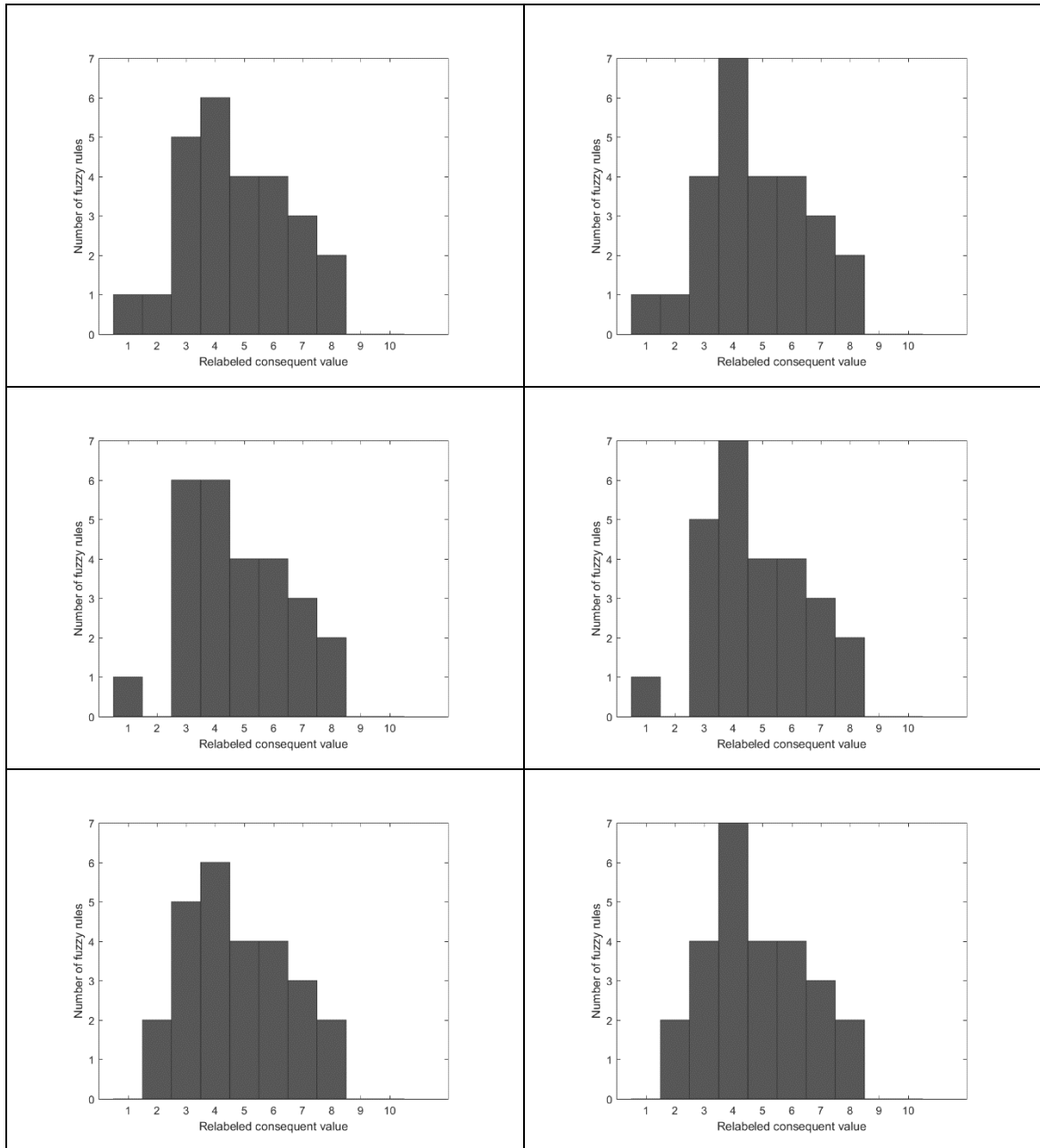
Lie Meng Pang, Kai Meng Tay and Chee Peng Lim, “A new monotonicity index for fuzzy rule-based systems”, IEEE International Conference on Fuzzy Systems (FUZZ), Beijing, China, 2014.

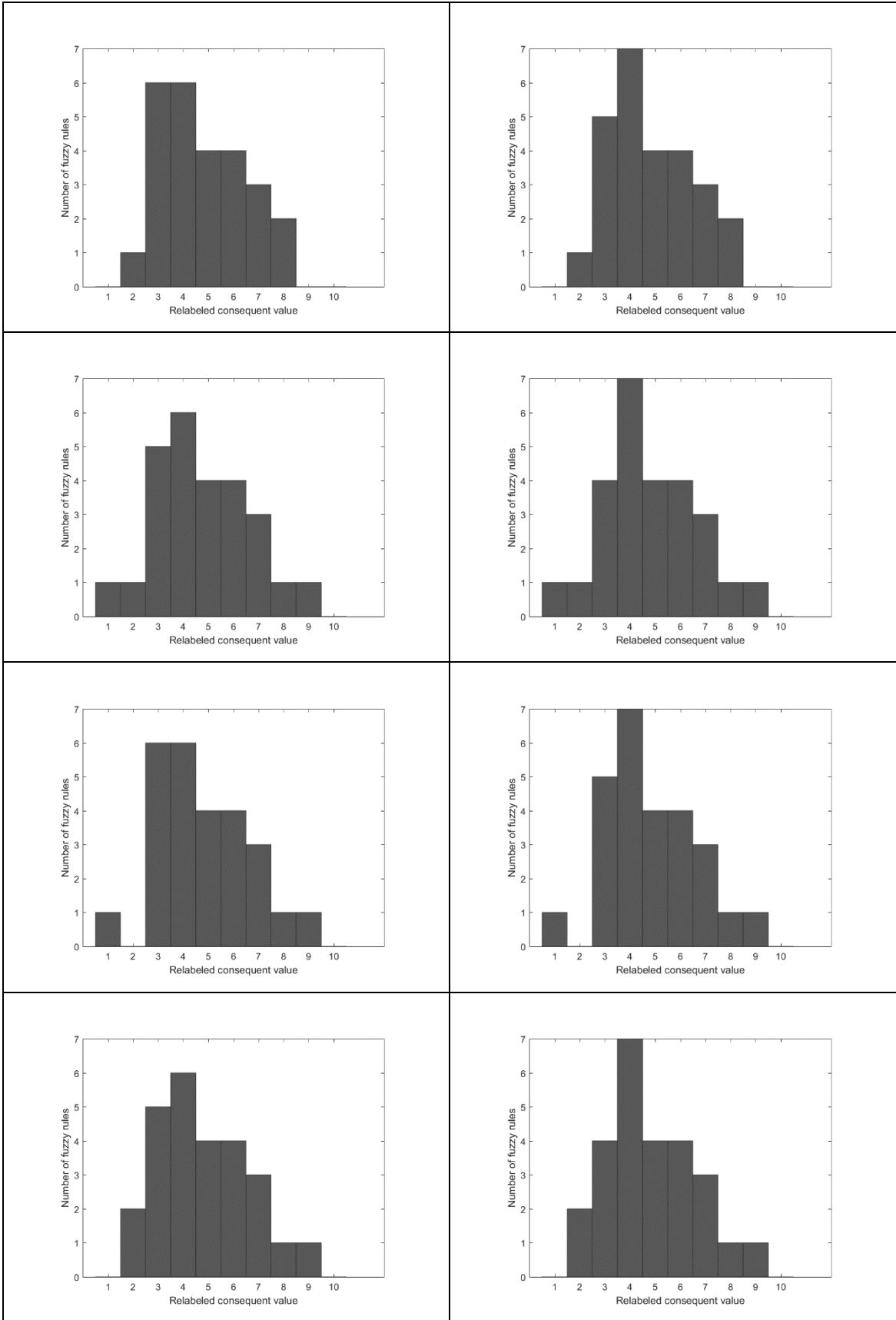
Kai Meng Tay, Lie Meng Pang, Tze Ling Jee and Chee Peng Lim, “A new framework with Similarity Reasoning and monotone fuzzy rule relabeling for Fuzzy Inference Systems,” IEEE International Conference on Fuzzy Systems (FUZZ), Hyderabad, India, 2013.

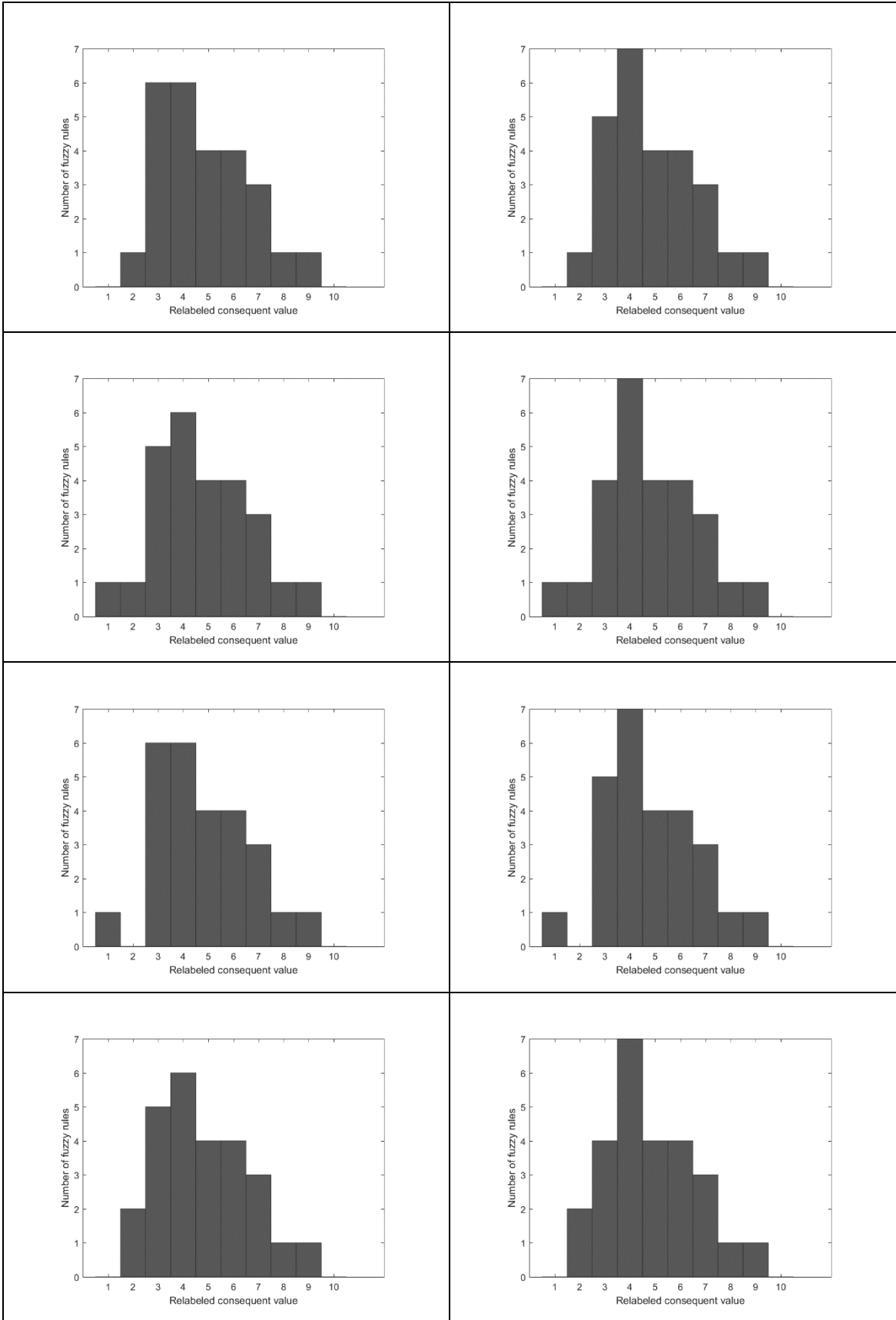
Kai Meng Tay, Tze Ling Jee, Lie Meng Pang, and Chee Peng Lim, “A new online updating framework for constructing monotonicity-preserving Fuzzy Inference Systems,” IEEE International Conference on Fuzzy Systems (FUZZ), Hyderabad, India, 2013.

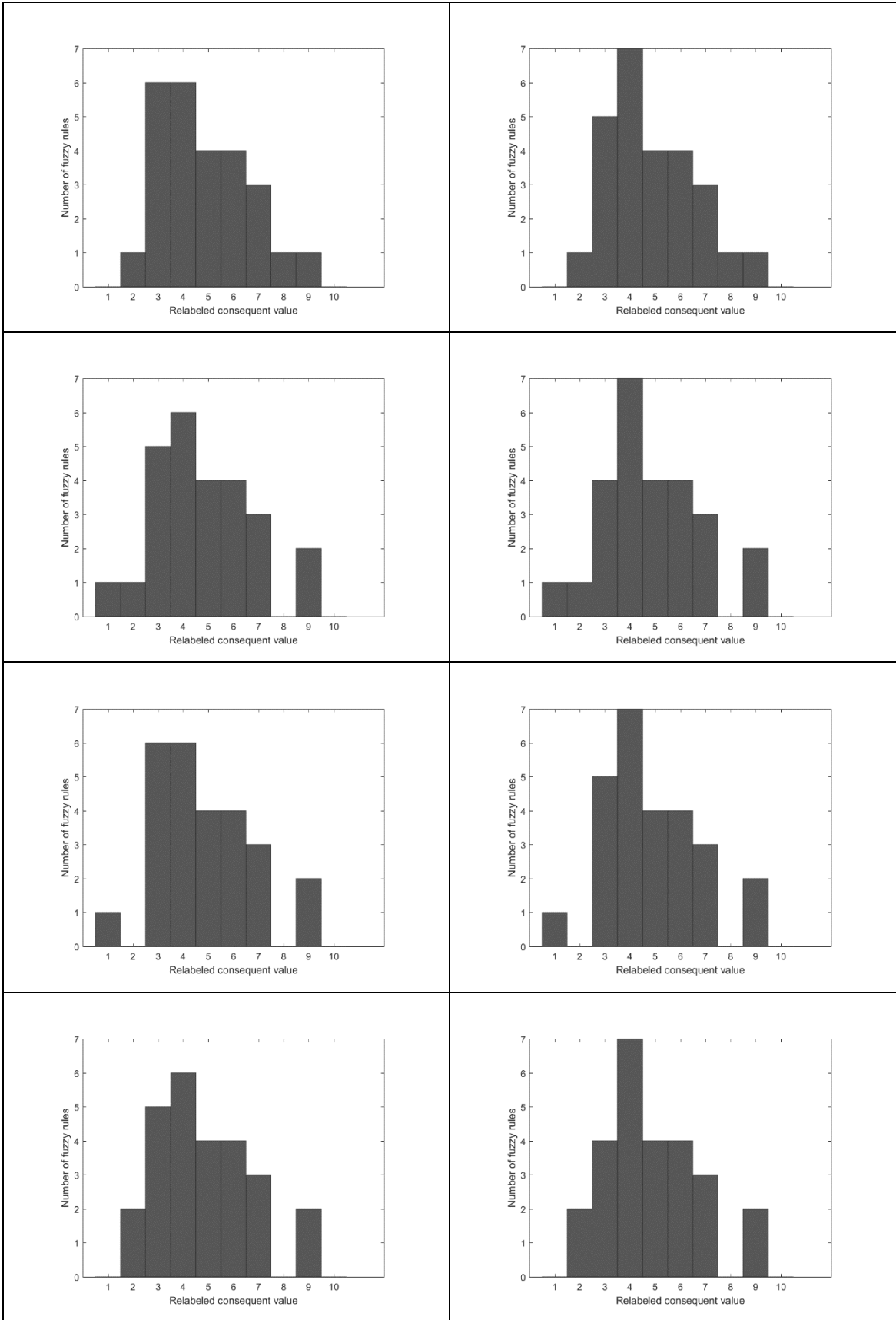
Appendix A

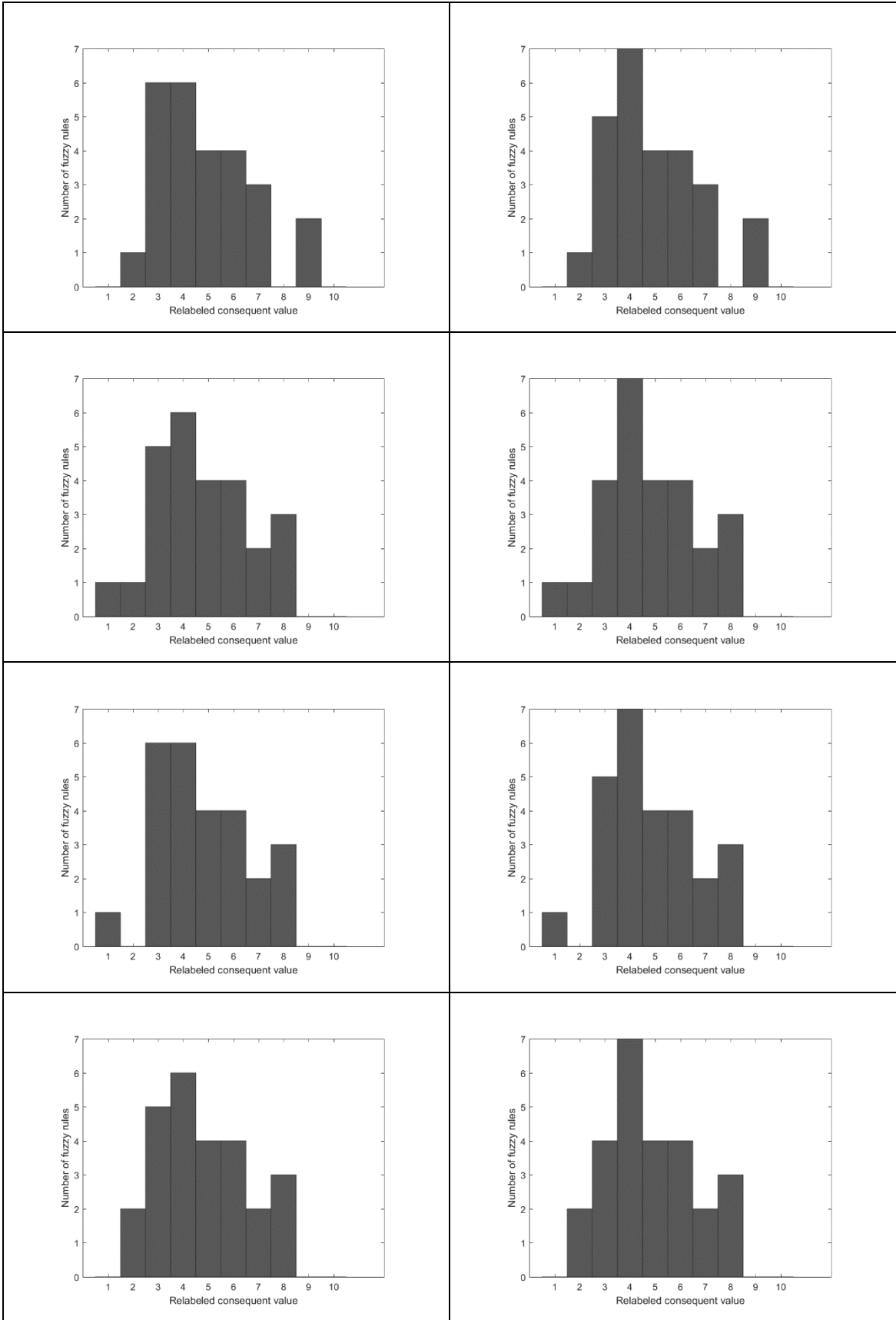
Histogram distribution for the 128 optimal solutions (Chapter 6, Section 6.3.2)

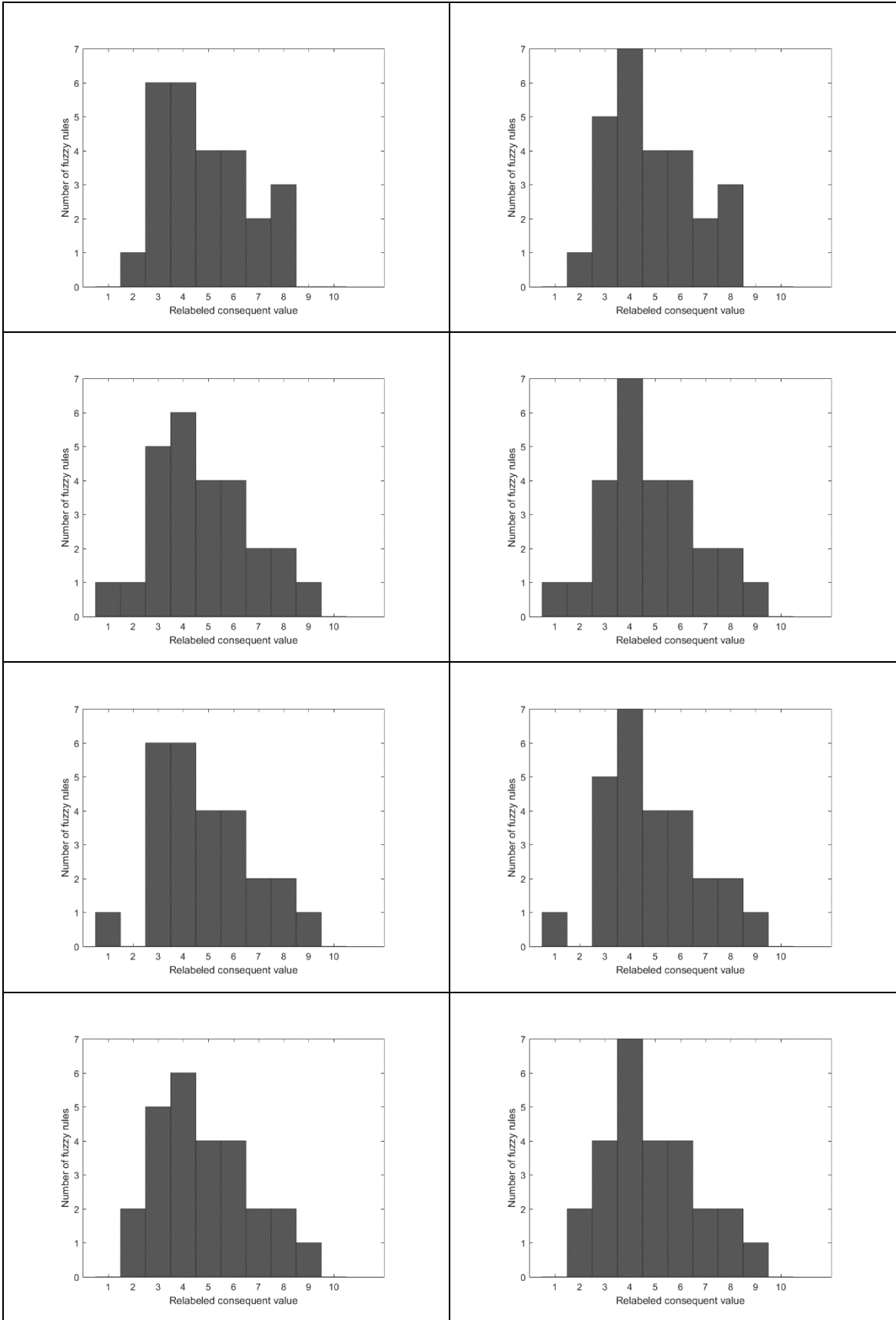


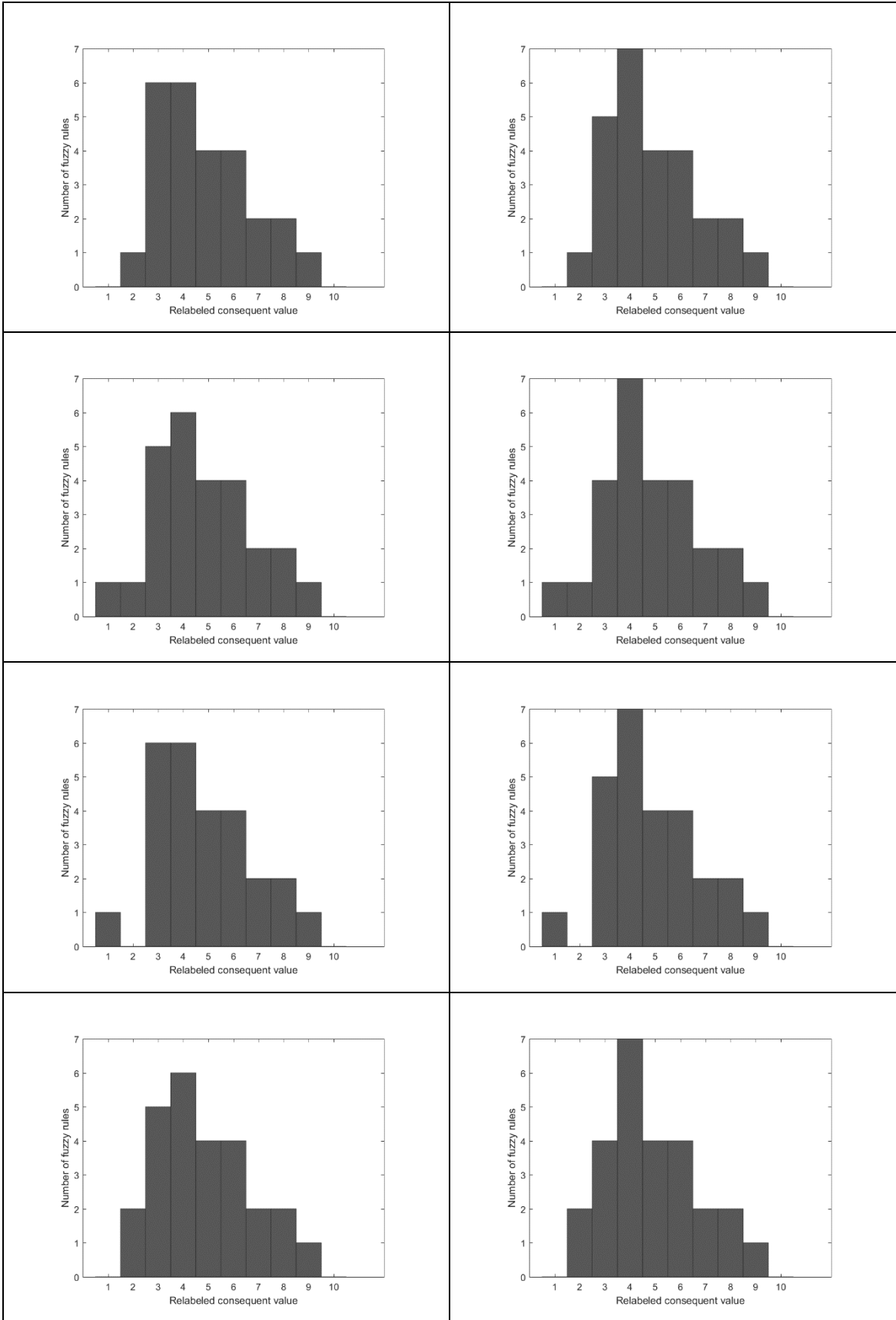


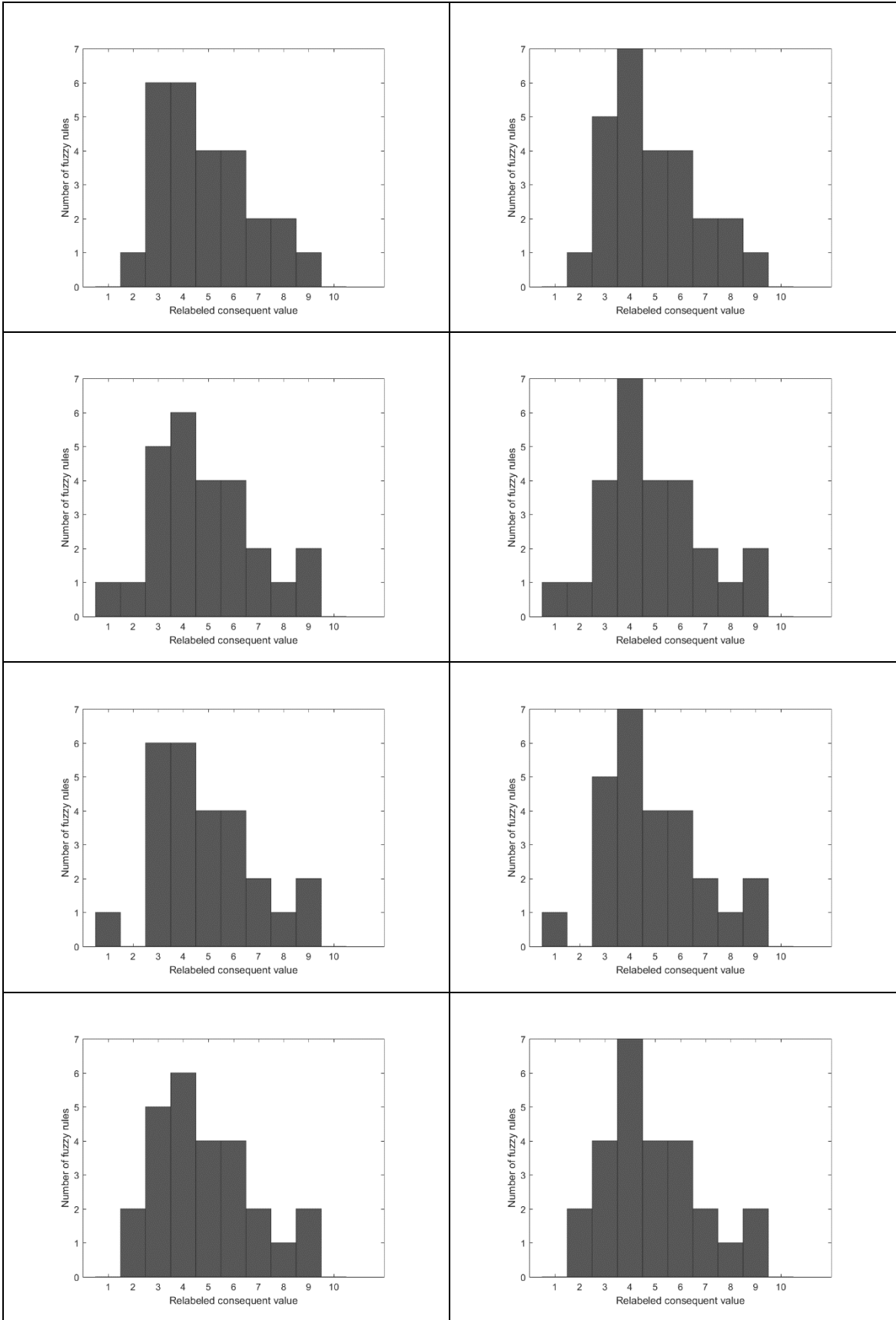


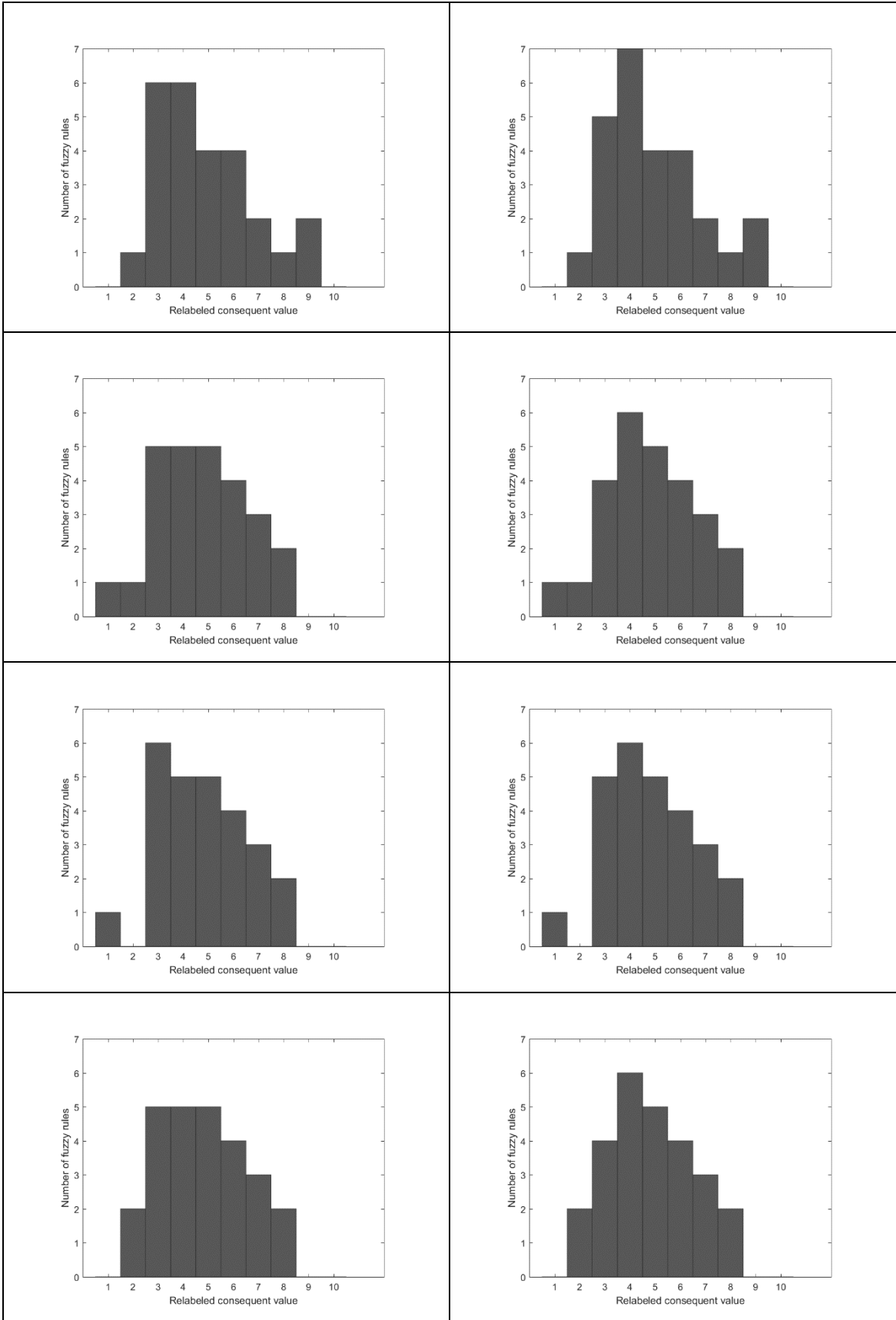


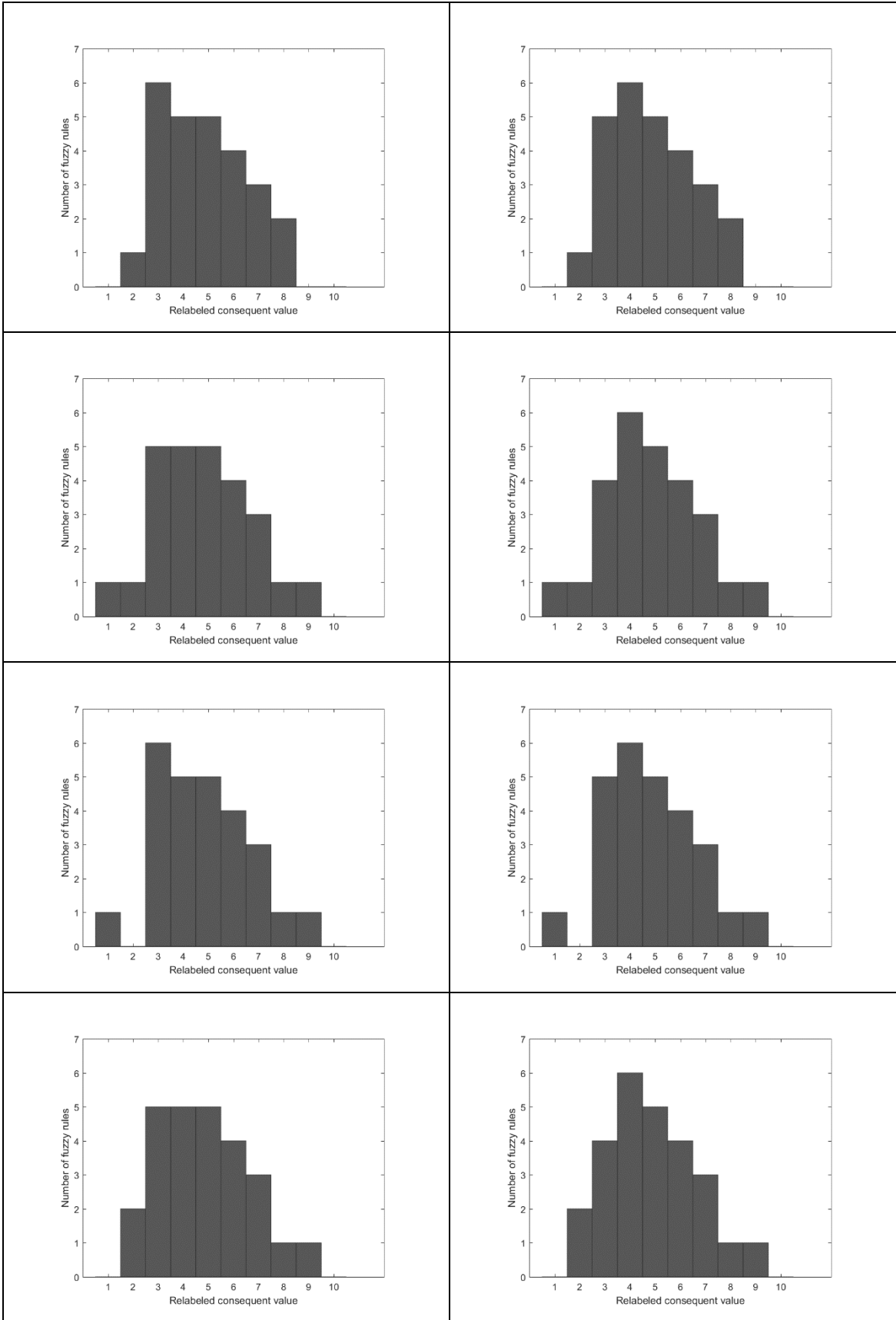


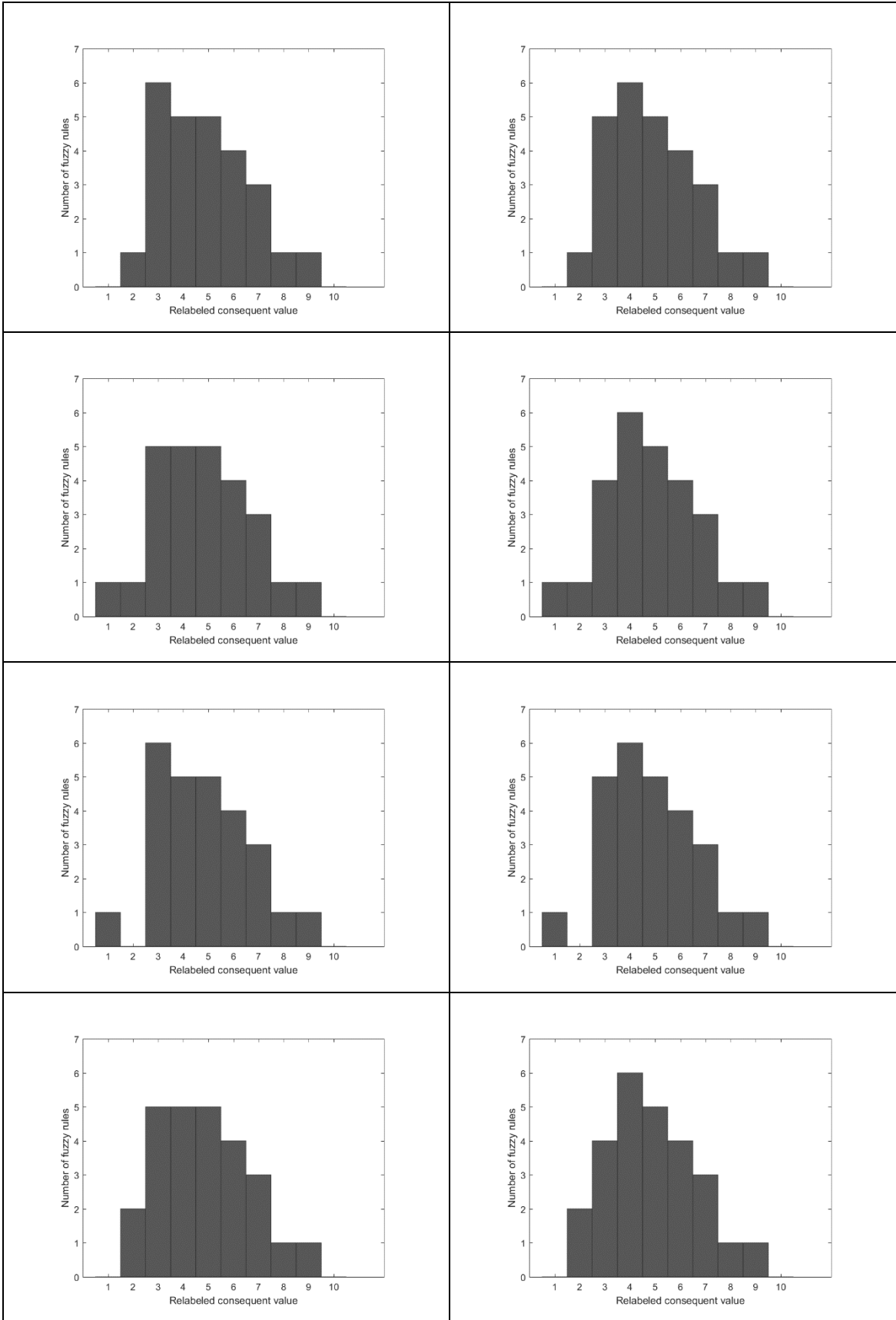


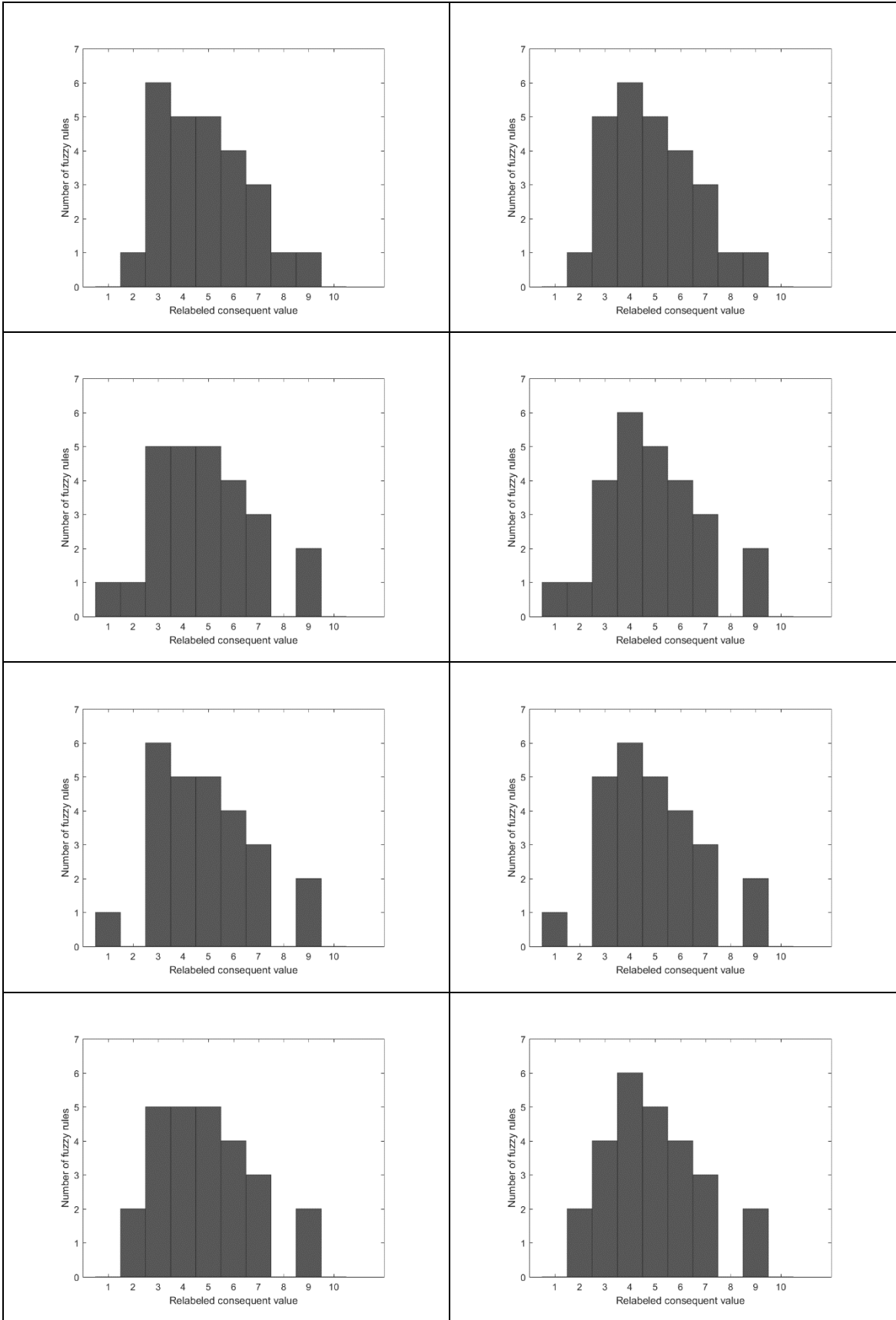


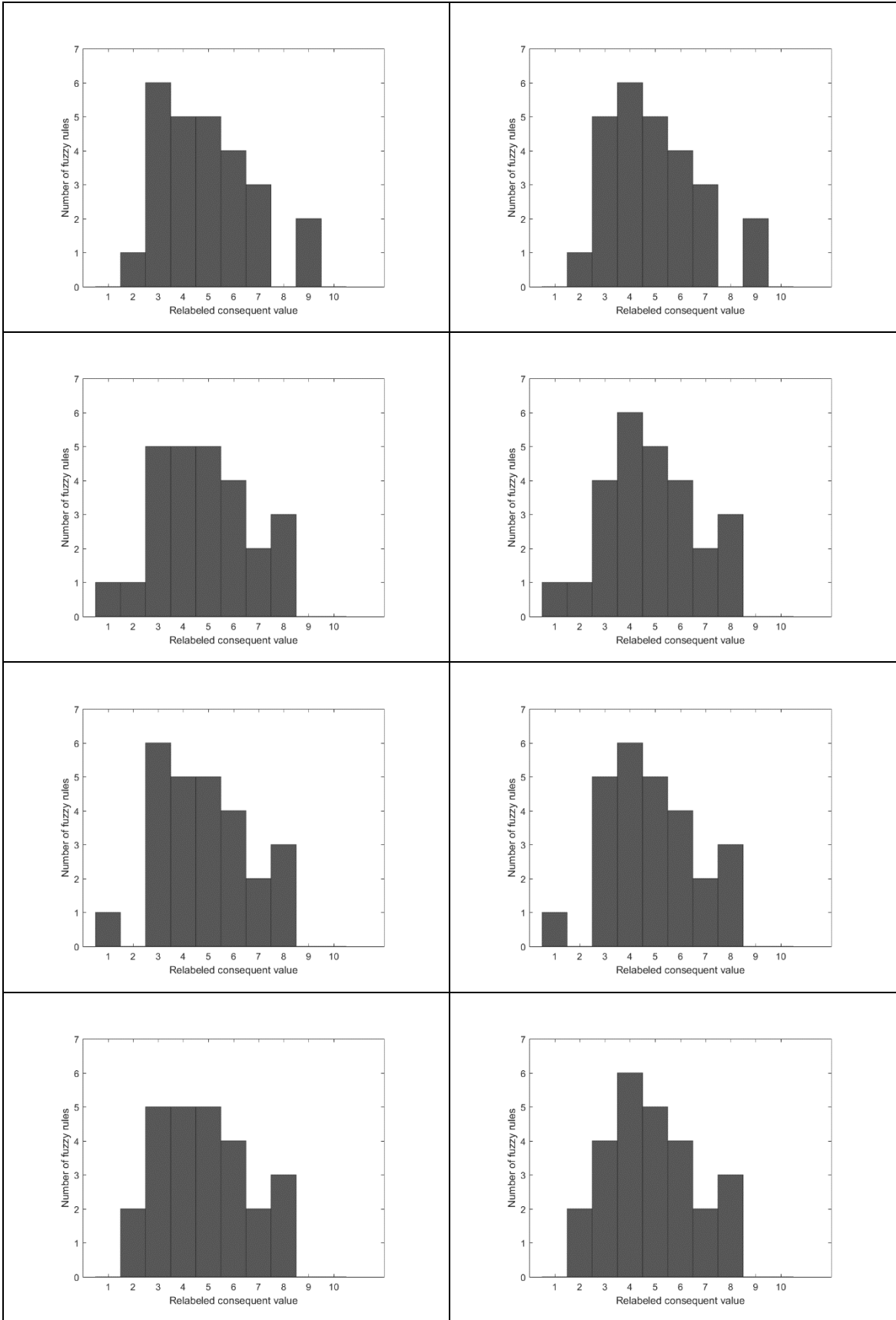


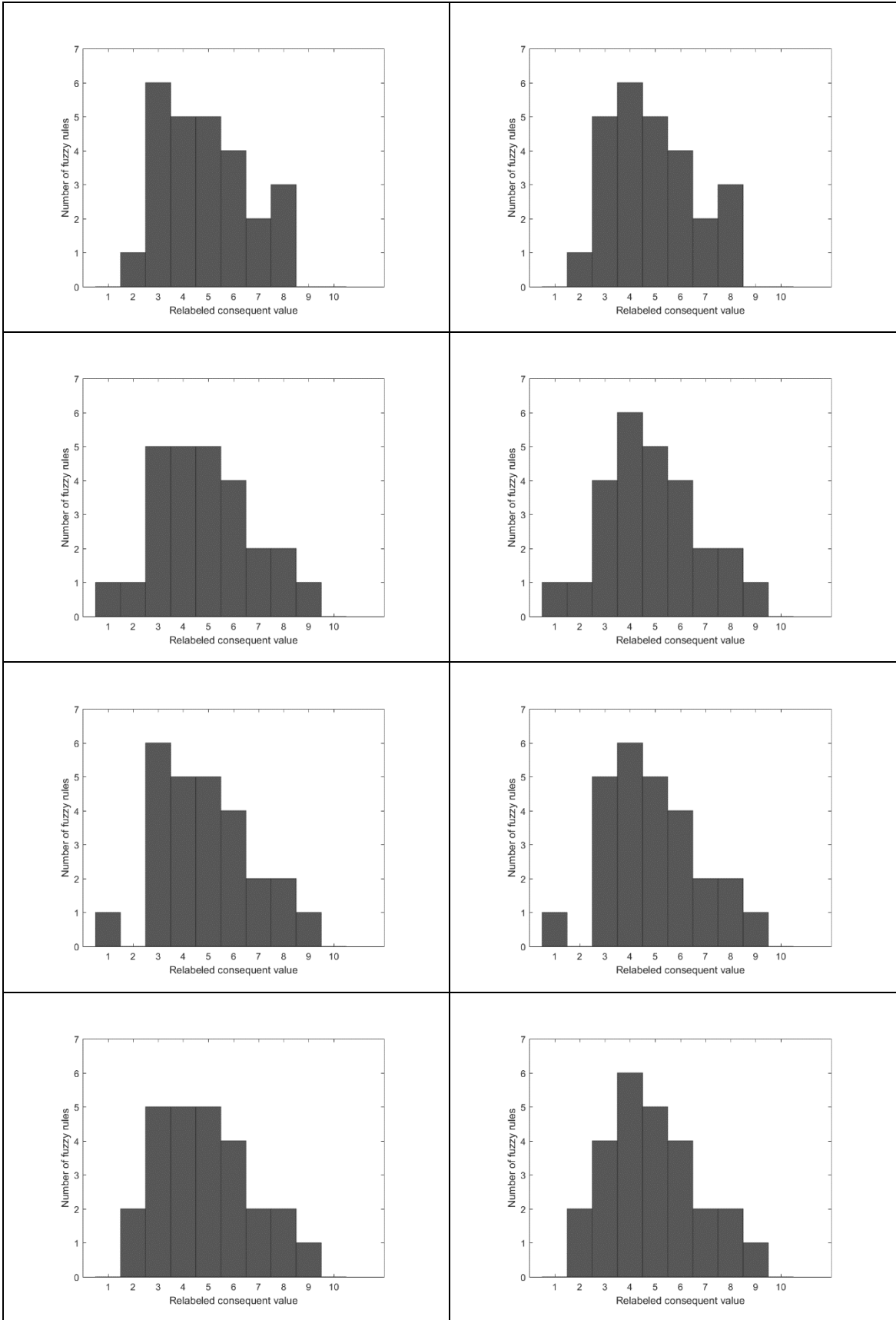


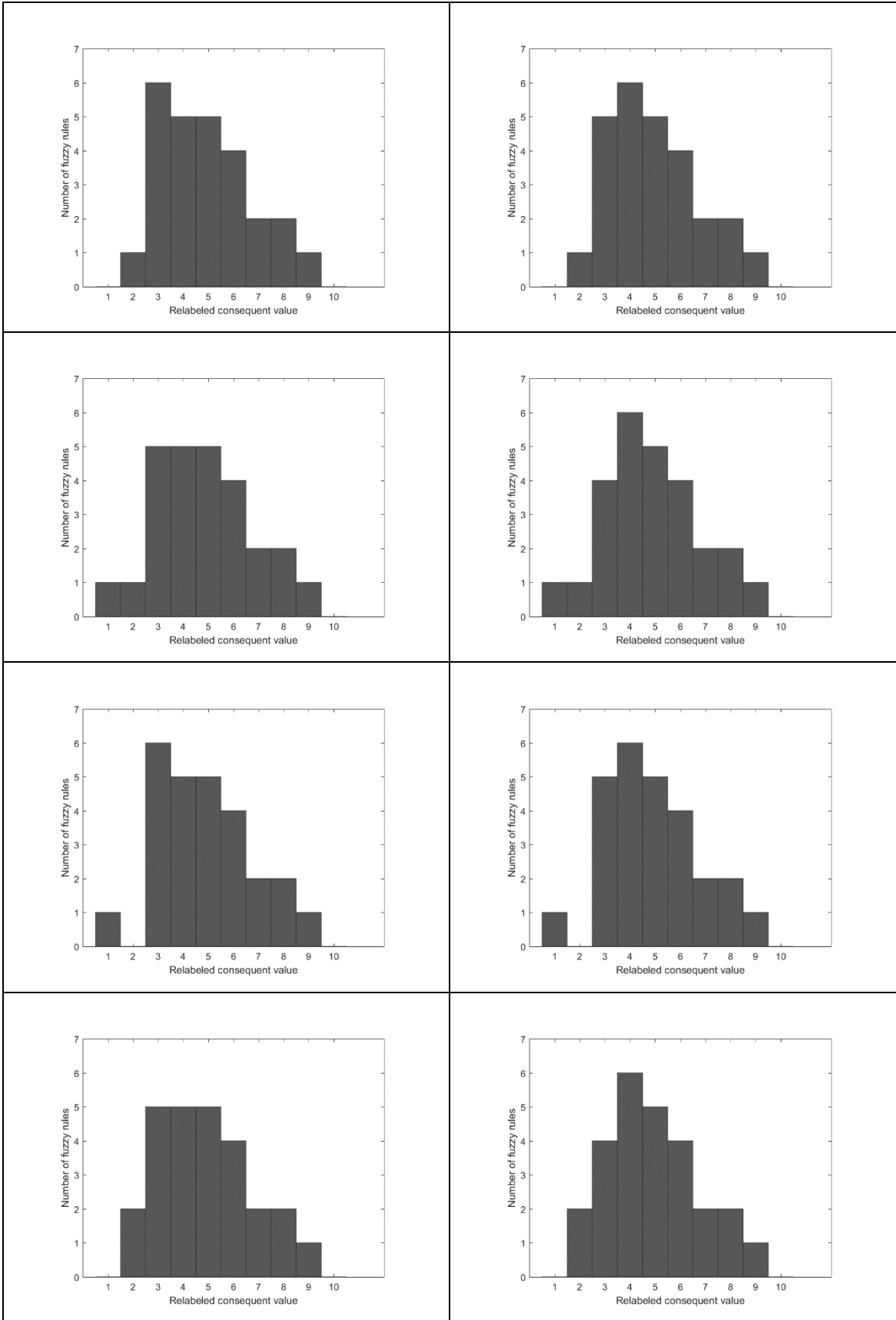


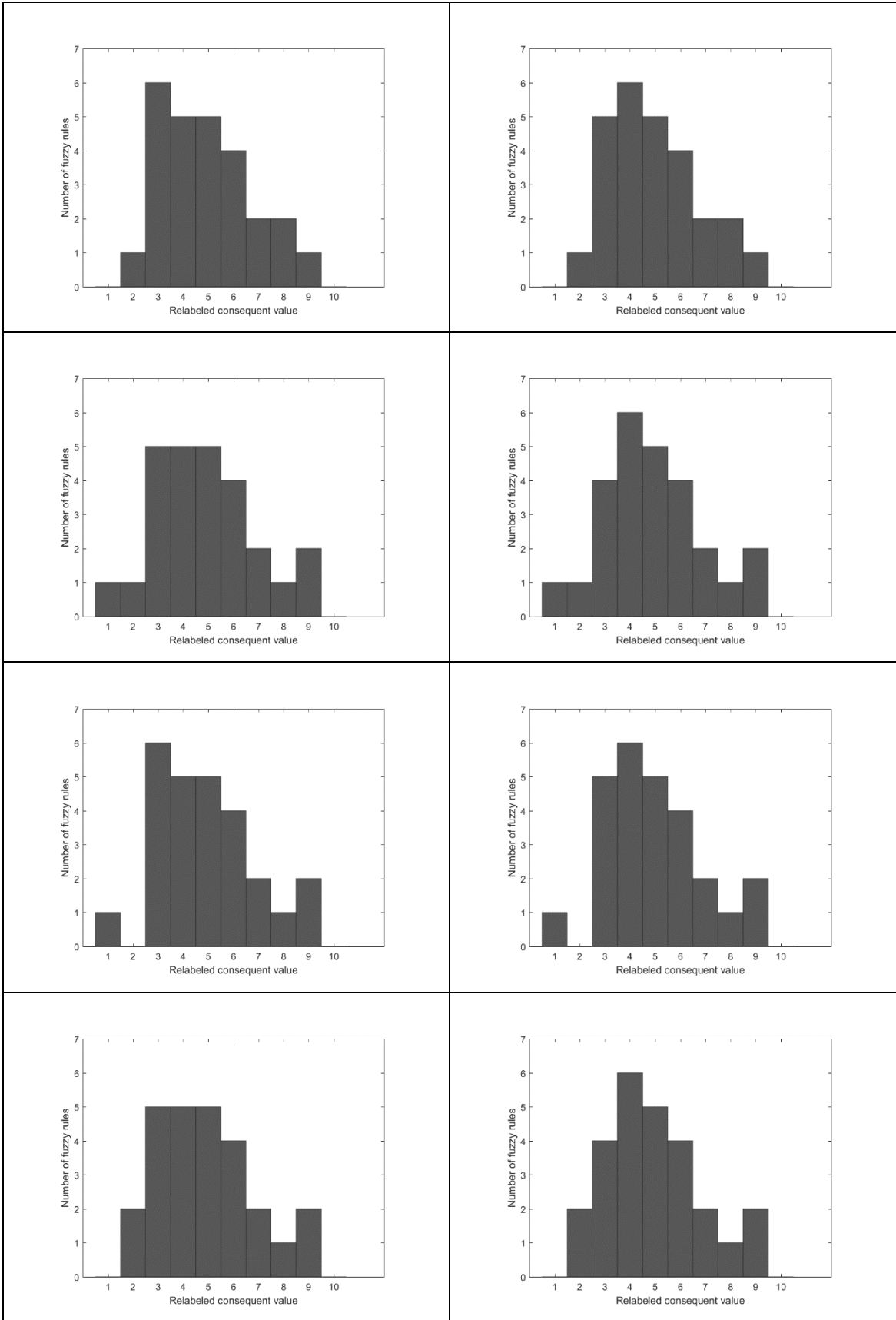


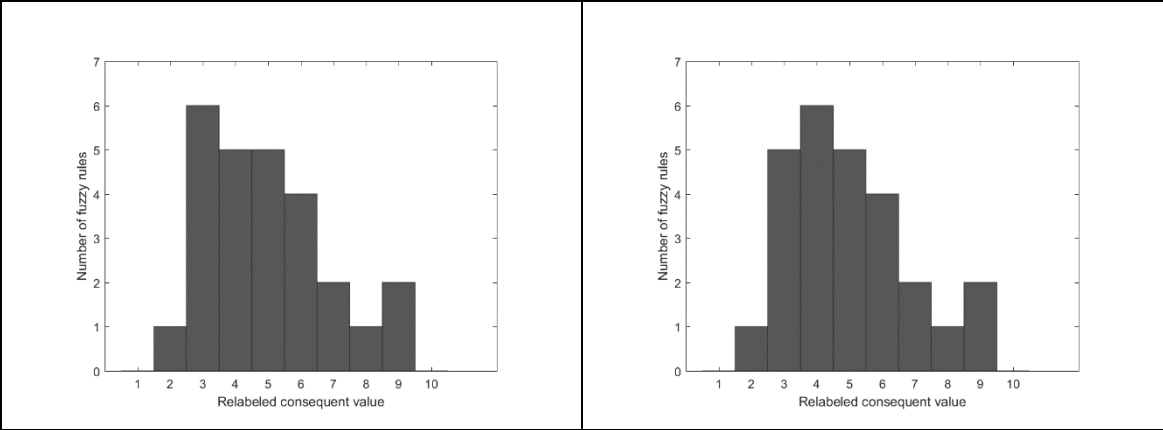












Appendix B

Fuzzy IF-THEN rules for FIS-based CRA model (Chapter 7, Section 7.4.6)

$R_{1,1,1}$: IF *Structural key plans* is **Unsatisfactory** and *Analysis of structural elements* is **Unsatisfactory** and *Detailed sketching* is **Unsatisfactory** THEN aggregated score is **Unsatisfactory**.

$R_{1,2,1}$: IF *Structural key plans* is **Unsatisfactory** and *Analysis of structural elements* is **Satisfactory** and *Detailed sketching* is **Unsatisfactory** THEN aggregated score is **Unsatisfactory**.

$R_{1,3,1}$: IF *Structural key plans* is **Unsatisfactory** and *Analysis of structural elements* is **Good** and *Detailed sketching* is **Unsatisfactory** THEN aggregated score is **Very Weak**.

$R_{1,4,1}$: IF *Structural key plans* is **Unsatisfactory** and *Analysis of structural elements* is **Very Good** and *Detailed sketching* is **Unsatisfactory** THEN aggregated score is **Very Weak**.

$R_{1,5,1}$: IF *Structural key plans* is **Unsatisfactory** and *Analysis of structural elements* is **Excellent** and *Detailed sketching* is **Unsatisfactory** THEN aggregated score is **Very Weak**.

$R_{1,1,2}$: IF *Structural key plans* is **Unsatisfactory** and *Analysis of structural elements* is **Unsatisfactory** and *Detailed sketching* is **Satisfactory** THEN aggregated score is **Unsatisfactory**.

$R_{1,2,2}$: IF *Structural key plans* is **Unsatisfactory** and *Analysis of structural elements* is **Satisfactory** and *Detailed sketching* is **Satisfactory** THEN aggregated score is **Very Weak**.

$R_{1,3,2}$: IF *Structural key plans* is **Unsatisfactory** and *Analysis of structural elements* is **Good** and *Detailed sketching* is **Satisfactory** THEN aggregated score is **Very Weak**.

$R_{1,4,2}$: IF *Structural key plans* is **Unsatisfactory** and *Analysis of structural elements* is **Very Good** and *Detailed sketching* is **Satisfactory** THEN aggregated score is **Very Weak**.

$R_{1,5,2}$: IF *Structural key plans* is **Unsatisfactory** and *Analysis of structural elements* is **Excellent** and *Detailed sketching* is **Satisfactory** THEN aggregated score is **Very Weak**.

$R_{1,1,3}$: IF *Structural key plans* is **Unsatisfactory** and *Analysis of structural elements* is **Unsatisfactory** and *Detailed sketching* is **Good** THEN aggregated score is **Unsatisfactory**.

$R_{1,2,3}$: IF *Structural key plans* is **Unsatisfactory** and *Analysis of structural elements* is **Satisfactory** and *Detailed sketching* is **Good** THEN aggregated score is **Very Weak**.

$R_{1,3,3}$: IF *Structural key plans* is **Unsatisfactory** and *Analysis of structural elements* is **Good** and *Detailed sketching* is **Good** THEN aggregated score is **Very Weak**.

$R_{1,4,3}$: IF *Structural key plans* is **Unsatisfactory** and *Analysis of structural elements* is **Very Good** and *Detailed sketching* is **Good** THEN aggregated score is **Unsatisfactory**.

$R_{1,5,3}$: IF *Structural key plans* is **Unsatisfactory** and *Analysis of structural elements* is **Excellent** and *Detailed sketching* is **Good** THEN aggregated score is **Very Weak**.

$R_{1,1,4}$: IF *Structural key plans* is **Unsatisfactory** and *Analysis of structural elements* is **Unsatisfactory** and *Detailed sketching* is **Very Good** THEN aggregated score is **Unsatisfactory**.

$R_{1,2,4}$: IF *Structural key plans* is **Unsatisfactory** and *Analysis of structural elements* is **Satisfactory** and *Detailed sketching* is **Very Good** THEN aggregated score is **Very Weak**.

$R_{1,3,4}$: IF *Structural key plans* is **Unsatisfactory** and *Analysis of structural elements* is **Good** and *Detailed sketching* is **Very Good** THEN aggregated score is **Very Weak**.

$R_{1,4,4}$: IF *Structural key plans* is **Unsatisfactory** and *Analysis of structural elements* is **Very Good** and *Detailed sketching* is **Very Good** THEN aggregated score is **Weak**.

$R_{1,5,4}$: IF *Structural key plans* is **Unsatisfactory** and *Analysis of structural elements* is **Excellent** and *Detailed sketching* is **Very Good** THEN aggregated score is **Weak**.

$R_{1,1,5}$: IF *Structural key plans* is **Unsatisfactory** and *Analysis of structural elements* is **Unsatisfactory** and *Detailed sketching* is **Excellent** THEN aggregated score is **Very Weak**.

$R_{1,2,5}$: IF *Structural key plans* is **Unsatisfactory** and *Analysis of structural elements* is **Satisfactory** and *Detailed sketching* is **Excellent** THEN aggregated score is **Very Weak**.

$R_{1,3,5}$: IF *Structural key plans* is **Unsatisfactory** and *Analysis of structural elements* is **Good** and *Detailed sketching* is **Excellent** THEN aggregated score is **Weak**.

$R_{1,4,5}$: IF *Structural key plans* is **Unsatisfactory** and *Analysis of structural elements* is **Very Good** and *Detailed sketching* is **Excellent** THEN aggregated score is **Weak**.

$R_{1,5,5}$: IF *Structural key plans* is **Unsatisfactory** and *Analysis of structural elements* is **Excellent** and *Detailed sketching* is **Excellent** THEN aggregated score is **Weak**.

$R_{2,1,1}$: IF *Structural key plans* is **Satisfactory** and *Analysis of structural elements* is **Unsatisfactory** and *Detailed sketching* is **Unsatisfactory** THEN aggregated score is **Very Weak**.

$R_{2,2,1}$: IF *Structural key plans* is **Satisfactory** and *Analysis of structural elements* is **Satisfactory** and *Detailed sketching* is **Unsatisfactory** THEN aggregated score is **Weak**.

$R_{2,3,1}$: IF *Structural key plans* is **Satisfactory** and *Analysis of structural elements* is **Good** and *Detailed sketching* is **Unsatisfactory** THEN aggregated score is **Weak**.

$R_{2,4,1}$: IF *Structural key plans* is **Satisfactory** and *Analysis of structural elements* is **Very Good** and *Detailed sketching* is **Unsatisfactory** THEN aggregated score is **Fair**.

$R_{2,5,1}$: IF *Structural key plans* is **Satisfactory** and *Analysis of structural elements* is **Excellent** and *Detailed sketching* is **Unsatisfactory** THEN aggregated score is **Fair**.

$R_{2,1,2}$: IF *Structural key plans* is **Satisfactory** and *Analysis of structural elements* is **Unsatisfactory** and *Detailed sketching* is **Satisfactory** THEN aggregated score is **Very Weak**.

$R_{2,2,2}$: IF *Structural key plans* is **Satisfactory** and *Analysis of structural elements* is **Satisfactory** and *Detailed sketching* is **Satisfactory** THEN aggregated score is **Weak**.

$R_{2,3,2}$: IF *Structural key plans* is **Satisfactory** and *Analysis of structural elements* is **Good** and *Detailed sketching* is **Satisfactory** THEN aggregated score is **Weak**.

$R_{2,4,2}$: IF *Structural key plans* is **Satisfactory** and *Analysis of structural elements* is **Very Good** and *Detailed sketching* is **Satisfactory** THEN aggregated score is **Fair**.

$R_{2,5,2}$: IF *Structural key plans* is **Satisfactory** and *Analysis of structural elements* is **Excellent** and *Detailed sketching* is **Satisfactory** THEN aggregated score is **Fair**.

R_{2,1,3}: IF *Structural key plans* is **Satisfactory** and *Analysis of structural elements* is **Unsatisfactory** and *Detailed sketching* is **Good** THEN aggregated score is **Very Weak**.

R_{2,2,3}: IF *Structural key plans* is **Satisfactory** and *Analysis of structural elements* is **Satisfactory** and *Detailed sketching* is **Good** THEN aggregated score is **Weak**.

R_{2,3,3}: IF *Structural key plans* is **Satisfactory** and *Analysis of structural elements* is **Good** and *Detailed sketching* is **Good** THEN aggregated score is **Fair**.

R_{2,4,3}: IF *Structural key plans* is **Satisfactory** and *Analysis of structural elements* is **Very Good** and *Detailed sketching* is **Good** THEN aggregated score is **Good**.

R_{2,5,3}: IF *Structural key plans* is **Satisfactory** and *Analysis of structural elements* is **Excellent** and *Detailed sketching* is **Good** THEN aggregated score is **Good**.

R_{2,1,4}: IF *Structural key plans* is **Satisfactory** and *Analysis of structural elements* is **Unsatisfactory** and *Detailed sketching* is **Very Good** THEN aggregated score is **Very Weak**.

R_{2,2,4}: IF *Structural key plans* is **Satisfactory** and *Analysis of structural elements* is **Satisfactory** and *Detailed sketching* is **Very Good** THEN aggregated score is **Fair**.

R_{2,3,4}: IF *Structural key plans* is **Satisfactory** and *Analysis of structural elements* is **Good** and *Detailed sketching* is **Very Good** THEN aggregated score is **Fair**.

R_{2,4,4}: IF *Structural key plans* is **Satisfactory** and *Analysis of structural elements* is **Very Good** and *Detailed sketching* is **Very Good** THEN aggregated score is **Good**.

R_{2,5,4}: IF *Structural key plans* is **Satisfactory** and *Analysis of structural elements* is **Excellent** and *Detailed sketching* is **Very Good** THEN aggregated score is **Good**.

R_{2,1,5}: IF *Structural key plans* is **Satisfactory** and *Analysis of structural elements* is **Unsatisfactory** and *Detailed sketching* is **Excellent** THEN aggregated score is **Weak**.

R_{2,2,5}: IF *Structural key plans* is **Satisfactory** and *Analysis of structural elements* is **Satisfactory** and *Detailed sketching* is **Excellent** THEN aggregated score is **Fair**.

R_{2,3,5}: IF *Structural key plans* is **Satisfactory** and *Analysis of structural elements* is **Good** and *Detailed sketching* is **Excellent** THEN aggregated score is **Fair**.

R_{2,4,5}: IF *Structural key plans* is **Satisfactory** and *Analysis of structural elements* is **Very Good** and *Detailed sketching* is **Excellent** THEN aggregated score is **Fair**.

R_{2,5,5}: IF *Structural key plans* is **Satisfactory** and *Analysis of structural elements* is **Excellent** and *Detailed sketching* is **Excellent** THEN aggregated score is **Good**.

R_{3,1,1}: IF *Structural key plans* is **Good** and *Analysis of structural elements* is **Unsatisfactory** and *Detailed sketching* is **Unsatisfactory** THEN aggregated score is **Very Weak**.

R_{3,2,1}: IF *Structural key plans* is **Good** and *Analysis of structural elements* is **Satisfactory** and *Detailed sketching* is **Unsatisfactory** THEN aggregated score is **Weak**.

R_{3,3,1}: IF *Structural key plans* is **Good** and *Analysis of structural elements* is **Good** and *Detailed sketching* is **Unsatisfactory** THEN aggregated score is **Fair**.

*R_{3,4,1}: IF Structural key plans is **Good** and Analysis of structural elements is **Very Good** and Detailed sketching is **Unsatisfactory** THEN aggregated score is **Fair**.*

*R_{3,5,1}: IF Structural key plans is **Good** and Analysis of structural elements is **Excellent** and Detailed sketching is **Unsatisfactory** THEN aggregated score is **Fair**.*

*R_{3,1,2}: IF Structural key plans is **Good** and Analysis of structural elements is **Unsatisfactory** and Detailed sketching is **Satisfactory** THEN aggregated score is **Weak**.*

*R_{3,2,2}: IF Structural key plans is **Good** and Analysis of structural elements is **Satisfactory** and Detailed sketching is **Satisfactory** THEN aggregated score is **Weak**.*

*R_{3,3,2}: IF Structural key plans is **Good** and Analysis of structural elements is **Good** and Detailed sketching is **Satisfactory** THEN aggregated score is **Fair**.*

*R_{3,4,2}: IF Structural key plans is **Good** and Analysis of structural elements is **Very Good** and Detailed sketching is **Satisfactory** THEN aggregated score is **Fair**.*

*R_{3,5,2}: IF Structural key plans is **Good** and Analysis of structural elements is **Excellent** and Detailed sketching is **Satisfactory** THEN aggregated score is **Fair**.*

*R_{3,1,3}: IF Structural key plans is **Good** and Analysis of structural elements is **Unsatisfactory** and Detailed sketching is **Good** THEN aggregated score is **Weak**.*

*R_{3,2,3}: IF Structural key plans is **Good** and Analysis of structural elements is **Satisfactory** and Detailed sketching is **Good** THEN aggregated score is **Fair**.*

*R_{3,3,3}: IF Structural key plans is **Good** and Analysis of structural elements is **Good** and Detailed sketching is **Good** THEN aggregated score is **Fair**.*

*R_{3,4,3}: IF Structural key plans is **Good** and Analysis of structural elements is **Very Good** and Detailed sketching is **Good** THEN aggregated score is **Good**.*

*R_{3,5,3}: IF Structural key plans is **Good** and Analysis of structural elements is **Excellent** and Detailed sketching is **Good** THEN aggregated score is **Good**.*

*R_{3,1,4}: IF Structural key plans is **Good** and Analysis of structural elements is **Unsatisfactory** and Detailed sketching is **Very Good** THEN aggregated score is **Weak**.*

*R_{3,2,4}: IF Structural key plans is **Good** and Analysis of structural elements is **Satisfactory** and Detailed sketching is **Very Good** THEN aggregated score is **Fair**.*

*R_{3,3,4}: IF Structural key plans is **Good** and Analysis of structural elements is **Good** and Detailed sketching is **Very Good** THEN aggregated score is **Fair**.*

*R_{3,4,4}: IF Structural key plans is **Good** and Analysis of structural elements is **Very Good** and Detailed sketching is **Very Good** THEN aggregated score is **Good**.*

*R_{3,5,4}: IF Structural key plans is **Good** and Analysis of structural elements is **Excellent** and Detailed sketching is **Very Good** THEN aggregated score is **Good**.*

*R_{3,1,5}: IF Structural key plans is **Good** and Analysis of structural elements is **Unsatisfactory** and Detailed sketching is **Excellent** THEN aggregated score is **Weak**.*

*R_{3,2,5}: IF Structural key plans is **Good** and Analysis of structural elements is **Satisfactory** and Detailed sketching is **Excellent** THEN aggregated score is **Fair**.*

*R_{3,3,5}: IF Structural key plans is **Good** and Analysis of structural elements is **Good** and Detailed sketching is **Excellent** THEN aggregated score is **Good**.*

*R_{3,4,5}: IF Structural key plans is **Good** and Analysis of structural elements is **Very Good** and Detailed sketching is **Excellent** THEN aggregated score is **Good**.*

*R_{3,5,5}: IF Structural key plans is **Good** and Analysis of structural elements is **Excellent** and Detailed sketching is **Excellent** THEN aggregated score is **Very Good**.*

*R_{4,1,1}: IF Structural key plans is **Very Good** and Analysis of structural elements is **Unsatisfactory** and Detailed sketching is **Unsatisfactory** THEN aggregated score is **Weak**.*

*R_{4,2,1}: IF Structural key plans is **Very Good** and Analysis of structural elements is **Satisfactory** and Detailed sketching is **Unsatisfactory** THEN aggregated score is **Fair**.*

*R_{4,3,1}: IF Structural key plans is **Very Good** and Analysis of structural elements is **Good** and Detailed sketching is **Unsatisfactory** THEN aggregated score is **Fair**.*

*R_{4,4,1}: IF Structural key plans is **Very Good** and Analysis of structural elements is **Very Good** and Detailed sketching is **Unsatisfactory** THEN aggregated score is **Fair**.*

*R_{4,5,1}: IF Structural key plans is **Very Good** and Analysis of structural elements is **Excellent** and Detailed sketching is **Unsatisfactory** THEN aggregated score is **Good**.*

*R_{4,1,2}: IF Structural key plans is **Very Good** and Analysis of structural elements is **Unsatisfactory** and Detailed sketching is **Satisfactory** THEN aggregated score is **Weak**.*

*R_{4,2,2}: IF Structural key plans is **Very Good** and Analysis of structural elements is **Satisfactory** and Detailed sketching is **Satisfactory** THEN aggregated score is **Fair**.*

*R_{4,3,2}: IF Structural key plans is **Very Good** and Analysis of structural elements is **Good** and Detailed sketching is **Satisfactory** THEN aggregated score is **Fair**.*

*R_{4,4,2}: IF Structural key plans is **Very Good** and Analysis of structural elements is **Very Good** and Detailed sketching is **Satisfactory** THEN aggregated score is **Good**.*

*R_{4,5,2}: IF Structural key plans is **Very Good** and Analysis of structural elements is **Excellent** and Detailed sketching is **Satisfactory** THEN aggregated score is **Good**.*

*R_{4,1,3}: IF Structural key plans is **Very Good** and Analysis of structural elements is **Unsatisfactory** and Detailed sketching is **Good** THEN aggregated score is **Weak**.*

*R_{4,2,3}: IF Structural key plans is **Very Good** and Analysis of structural elements is **Satisfactory** and Detailed sketching is **Good** THEN aggregated score is **Fair**.*

*R_{4,3,3}: IF Structural key plans is **Very Good** and Analysis of structural elements is **Good** and Detailed sketching is **Good** THEN aggregated score is **Good**.*

*R_{4,4,3}: IF Structural key plans is **Very Good** and Analysis of structural elements is **Very Good** and Detailed sketching is **Good** THEN aggregated score is **Very Good**.*

*R_{4,5,3}: IF Structural key plans is **Very Good** and Analysis of structural elements is **Excellent** and Detailed sketching is **Good** THEN aggregated score is **Very Good**.*

*R_{4,1,4}: IF Structural key plans is **Very Good** and Analysis of structural elements is **Unsatisfactory** and Detailed sketching is **Very Good** THEN aggregated score is **Weak**.*

*R_{4,2,4}: IF Structural key plans is **Very Good** and Analysis of structural elements is **Satisfactory** and Detailed sketching is **Very Good** THEN aggregated score is **Fair**.*

*R_{4,3,4}: IF Structural key plans is **Very Good** and Analysis of structural elements is **Good** and Detailed sketching is **Very Good** THEN aggregated score is **Good**.*

*R_{4,4,4}: IF Structural key plans is **Very Good** and Analysis of structural elements is **Very Good** and Detailed sketching is **Very Good** THEN aggregated score is **Very Good**.*

*R_{4,5,4}: IF Structural key plans is **Very Good** and Analysis of structural elements is **Excellent** and Detailed sketching is **Very Good** THEN aggregated score is **Very Good**.*

*R_{4,1,5}: IF Structural key plans is **Very Good** and Analysis of structural elements is **Unsatisfactory** and Detailed sketching is **Excellent** THEN aggregated score is **Weak**.*

*R_{4,2,5}: IF Structural key plans is **Very Good** and Analysis of structural elements is **Satisfactory** and Detailed sketching is **Excellent** THEN aggregated score is **Fair**.*

*R_{4,3,5}: IF Structural key plans is **Very Good** and Analysis of structural elements is **Good** and Detailed sketching is **Excellent** THEN aggregated score is **Good**.*

*R_{4,4,5}: IF Structural key plans is **Very Good** and Analysis of structural elements is **Very Good** and Detailed sketching is **Excellent** THEN aggregated score is **Very Good**.*

*R_{4,5,5}: IF Structural key plans is **Very Good** and Analysis of structural elements is **Excellent** and Detailed sketching is **Excellent** THEN aggregated score is **Excellent**.*

*R_{5,1,1}: IF Structural key plans is **Excellent** and Analysis of structural elements is **Unsatisfactory** and Detailed sketching is **Unsatisfactory** THEN aggregated score is **Weak**.*

*R_{5,2,1}: IF Structural key plans is **Excellent** and Analysis of structural elements is **Satisfactory** and Detailed sketching is **Unsatisfactory** THEN aggregated score is **Fair**.*

*R_{5,3,1}: IF Structural key plans is **Excellent** and Analysis of structural elements is **Good** and Detailed sketching is **Unsatisfactory** THEN aggregated score is **Fair**.*

*R_{5,4,1}: IF Structural key plans is **Excellent** and Analysis of structural elements is **Very Good** and Detailed sketching is **Unsatisfactory** THEN aggregated score is **Good**.*

*R_{5,5,1}: IF Structural key plans is **Excellent** and Analysis of structural elements is **Excellent** and Detailed sketching is **Unsatisfactory** THEN aggregated score is **Good**.*

*R_{5,1,2}: IF Structural key plans is **Excellent** and Analysis of structural elements is **Unsatisfactory** and Detailed sketching is **Satisfactory** THEN aggregated score is **Weak**.*

*R_{5,2,2}: IF Structural key plans is **Excellent** and Analysis of structural elements is **Satisfactory** and Detailed sketching is **Satisfactory** THEN aggregated score is **Fair**.*

*R_{5,3,2}: IF Structural key plans is **Excellent** and Analysis of structural elements is **Good** and Detailed sketching is **Satisfactory** THEN aggregated score is **Fair**.*

*R_{5,4,2}: IF Structural key plans is **Excellent** and Analysis of structural elements is **Very Good** and Detailed sketching is **Satisfactory** THEN aggregated score is **Good**.*

*R_{5,5,2}: IF Structural key plans is **Excellent** and Analysis of structural elements is **Excellent** and Detailed sketching is **Satisfactory** THEN aggregated score is **Good**.*

*R_{5,1,3}: IF Structural key plans is **Excellent** and Analysis of structural elements is **Unsatisfactory** and Detailed sketching is **Good** THEN aggregated score is **Weak**.*

*R_{5,2,3}: IF Structural key plans is **Excellent** and Analysis of structural elements is **Satisfactory** and Detailed sketching is **Good** THEN aggregated score is **Fair**.*

*R_{5,3,3}: IF Structural key plans is **Excellent** and Analysis of structural elements is **Good** and Detailed sketching is **Good** THEN aggregated score is **Good**.*

*R_{5,4,3}: IF Structural key plans is **Excellent** and Analysis of structural elements is **Very Good** and Detailed sketching is **Good** THEN aggregated score is **Very Good**.*

*R_{5,5,3}: IF Structural key plans is **Excellent** and Analysis of structural elements is **Excellent** and Detailed sketching is **Good** THEN aggregated score is **Very Good**.*

*R_{5,1,4}: IF Structural key plans is **Excellent** and Analysis of structural elements is **Unsatisfactory** and Detailed sketching is **Very Good** THEN aggregated score is **Weak**.*

*R_{5,2,4}: IF Structural key plans is **Excellent** and Analysis of structural elements is **Satisfactory** and Detailed sketching is **Very Good** THEN aggregated score is **Fair**.*

*R_{5,3,4}: IF Structural key plans is **Excellent** and Analysis of structural elements is **Good** and Detailed sketching is **Very Good** THEN aggregated score is **Good**.*

*R_{5,4,4}: IF Structural key plans is **Excellent** and Analysis of structural elements is **Very Good** and Detailed sketching is **Very Good** THEN aggregated score is **Very Good**.*

*R_{5,5,4}: IF Structural key plans is **Excellent** and Analysis of structural elements is **Excellent** and Detailed sketching is **Very Good** THEN aggregated score is **Excellent**.*

*R_{5,1,5}: IF Structural key plans is **Excellent** and Analysis of structural elements is **Unsatisfactory** and Detailed sketching is **Excellent** THEN aggregated score is **Weak**.*

*R_{5,2,5}: IF Structural key plans is **Excellent** and Analysis of structural elements is **Satisfactory** and Detailed sketching is **Excellent** THEN aggregated score is **Fair**.*

*R_{5,3,5}: IF Structural key plans is **Excellent** and Analysis of structural elements is **Good** and Detailed sketching is **Excellent** THEN aggregated score is **Very Good**.*

*R_{5,4,5}: IF Structural key plans is **Excellent** and Analysis of structural elements is **Very Good** and Detailed sketching is **Excellent** THEN aggregated score is **Excellent**.*

*R_{5,5,5}: IF Structural key plans is **Excellent** and Analysis of structural elements is **Excellent** and Detailed sketching is **Excellent** THEN aggregated score is **Excellent**.*

